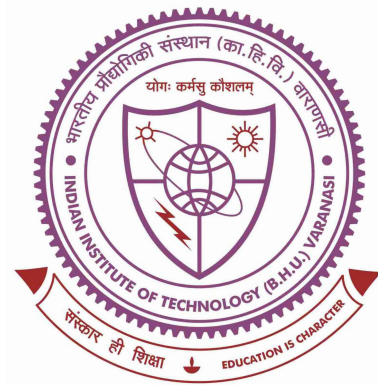


Ordering kinetics, phase separation and role of noise in active Brownian particles



Thesis submitted in partial fulfillment for the Award of

Doctor of Philosophy

in

Physics

by

Shambhavi Dikshit

DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY
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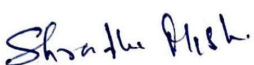
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Department of Physics, IIT (BHU)

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