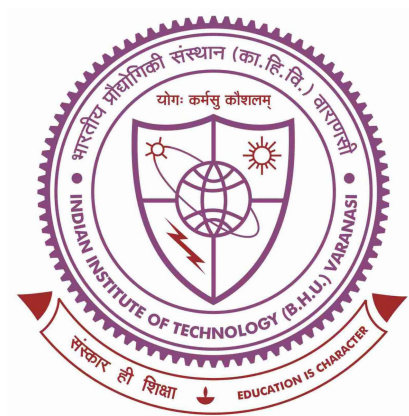


Unveiling Phases and Phase Transitions in Inhomogeneous Active Systems



Thesis submitted in partial fulfillment for the Award of

Doctor of Philosophy

in

Physics

by

Pawan Kumar Mishra

DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY
(BANARAS HINDU UNIVERSITY)
VARANASI - 221 005

ROLL No. 19171019

YEAR OF SUBMISSION:2024

Table of contents

List of figures	xix
-----------------	-----

List of tables	xxv
----------------	-----

1	Introduction	1
1.1	Historical foundations of modern science	1
1.2	The Emergence of Statistical Mechanics	2
1.2.1	Challenges in Defining Hamiltonian for Nonequilibrium Systems	3
1.2.2	Energy Dissipation	4
1.2.3	Detailed Balance Condition	4
1.2.4	Time-Reversal Symmetry	5
1.2.5	Entropy Production	7
1.2.6	Non-Equilibrium Steady States (NESS)	7
1.2.7	Fluctuation Theorems	8
1.2.8	Examples of Nonequilibrium Systems	9
1.3	Active Matter: A Subclass of Nonequilibrium Systems	9
1.4	Types of Active particles	10
1.5	Models to Study Active Systems	12
1.5.1	Microscopic Models	12
1.5.2	Coarse-Grained Models	14
1.6	Inhomogeneity in Active systems	15
1.6.1	Extrinsic Inhomogeneity	16
1.6.2	Intrinsic Inhomogeneity	17
1.7	Observables to characterize the system:	18
1.7.1	Ordering in the System	18
1.7.2	Ordering in Different Dimensions	19
1.7.3	Phase Transition	19
1.8	Organisation of the thesis	21

1.9	Technical details	21
2	Active polar flock with birth and death	23
2.1	Introduction	23
2.2	Model and Numerical Methodology	24
2.3	Results	27
2.4	Hydrodynamic equations of motion for slow variables	31
2.4.1	Perturbative calculation for effective free energy for local polarisation	35
2.5	Conclusion	39
3	Directional cues affect the collective behaviour of Self propelled particles in one dimension	41
3.1	Introduction	41
3.2	Model and Numerical Methodology	43
3.3	Results	46
3.4	Conclusion	53
4	Active Brownian particles can mimic the pattern of the substrate	55
4.1	Introduction	55
4.2	Model and Numerical Methodology	57
4.2.1	Moment Equations	57
4.3	Results	60
4.3.1	Coarse-grained model (CG)	60
4.3.2	Microscopic model (MIC)	65
4.4	Miscellaneous shapes	67
4.5	Conclusion	69
5	Polar particles induce strings in liquid crystal	71
5.1	Introduction	71
5.2	Model and Numerical Methodology	72
5.3	Numerical Details	75
5.4	Results	75
5.4.1	Artificial Defects in Coupled System	80
5.4.2	Confined System	81
5.5	Conclusion	84

Table of contents	xvii
6 Summary and Future Prospects	87
6.1 Summary	87
6.2 Future scope	89
Appendix	91
References	97
List of Publications	i

Abstract

The active matter systems can be recognized as a collection of particles in which the individual components possess nonzero motility by converting the energy from their surroundings and also from the medium. The active particles spontaneously self-organize when present in large numbers reveal fascinating properties like persistent motion, phase separation, clustering, self organisation and so on. Understanding these unusual properties exhibited by active systems on different length scales, has been a focus of research in recent decades. This thesis presents the projects related to various phenomena in active matter systems e.g. birth and death, disorder in one dimensional active systems, inhomogeneous substrates and mixture of polar and nematic systems. All the projects are presented in following chapters:

Chapter 1 is the historical context and description of active systems. The definition and fundamental characteristics of active systems are covered in this chapter. We also go through various methods for researching active systems, such as agent-based modelling. The remaining chapters are structured as follows.

In Chapter 2, we study a collection of self-propelled polar particles on a two-dimensional substrate with birth and death. We introduce a minimal lattice model for the system using active Ising spins, where each particle can have two possible orientations. The activity is modeled as a biased movement of the particle along its direction of orientation. The particles also align with their nearest neighbors using Metropolis Monte Carlo algorithm. The system shows a disorder-to-order transition by tuning the temperature of the system.

The chapter 3, explores the effect of quenched disorder on the characteristic of self-propelled particles in one-dimension. Here, particles interact with disorder which serve as directional cues. The study investigates how the density of the disorder influence the emergence of ordering and clustering in the collection of the self propelled particles. We introduce the microscopic model as well as corresponding coarse-grained equations of motion for the local density and the orientation of particle. Disorder affects the macroscopic ordering in the system, the size of the ordered clusters decays algebraically with disorder.

In chapter 4, we discuss how the active Brownian particles (ABPs) can be identified as a successful way of modeling the moving microorganism on the substrate. We studied the dynamics and the steady state of ABP moving on a two-dimensional substrate with space-dependent activity. On the substrate, some regions are marked as high in activity, and other regions are such that particles behave as passive Brownian particles. The final steady state of particle density profile, polarization and flux very much follows the structure of

the inhomogeneous activity, and the density in high activity region is lower, maximum at the interface and nearly constant with mean density in the passive region. The steady state density profile for various shapes and designs on two-dimensional substrates is studied. Hence, the collection of ABPs on an inhomogeneous substrate can mimic the inhomogeneity of the substrate.

Further in 5 we couple the liquid crystal with external species present in it and these external species have polar symmetry unlike the liquid crystal which are apolar in nature. The strings are visible in the order parameter field of both polar and apolar species. For polar species, the magnitude of the order parameter is close to zero along the string which is acting like domain wall separating two oppositely oriented domains of polar species. We observe the defect pairs connected through strings are not necessarily opposite in nature, it can be same $(+1/2, +1/2)$, $(-1/2, -1/2)$, as well as opposite $(+1/2, -1/2)$.

The thesis is finally concluded in chapter 6 with a synopsis of all the chapters mentioned above with future prospects.

Acknowledgements

I would like to express my special gratitude to all those people for their constant support, love, and affection throughout this journey. Here, I have made a mere attempt to name a few.

First of all, I would like to express my sincere thanks to my respected supervisor, **Dr. Shradha Mishra, Department of Physics, IIT(BHU), Varanasi**, for her excellent guidance, continuous encouragement, patience, advice, and moral support throughout my Ph.D. journey. I feel very fortunate to have a supervisor who cared deeply about my work and overall personal growth, shaped my understanding of the subject, and instilled in me the confidence to work independently. I have learned from her far more than one could ordinarily expect from a supervisor.

I express my heartfelt gratitude to **Prof. D. Giri**, a member of my RPEC, for his constant support and invaluable advice throughout my Ph.D. journey. I am equally grateful to **Prof. Sandip Chatterjee**, Head of the Department of Physics, IIT(BHU), for his encouragement and leadership, which greatly contributed to my academic environment. I extend my sincere thanks to all the faculty members of the Department of Physics, IIT(BHU), for their guidance, moral support, and valuable insights that have enriched my learning experience. I am also deeply appreciative of the office staff and non-teaching staff of the Department of Physics for their administrative assistance and logistical support, which ensured a smooth and conducive environment for my research.

My sincere thanks also go to **Dr. Manoranjan Kumar**, S.N. Bose National Centre for Basic Science, Kolkata, and **Dr. Sanjay Singh**, a member of my RPEC from the School of Materials Science and Technology, IIT(BHU), for their continued support and guidance throughout my research work.

I wish to thank my fellow lab mates **Pratikshya, Partha, Divya, Anish, Abhra, Arun, Mohit, and Riya** of the Soft-matter group in the Department for their help and support. Further, I would like to extend my thanks to my seniors **Dr. Rakesh Das, Dr. Sudipta Pattanayak, Dr. Sameer Kumar, Dr. Jay Prakash Singh, Dr. Vivek Semwal, and Dr. Shambhavi Dikshit** for their tremendous help. I am also grateful to my Institute, IIT(BHU), for providing the necessary resources, institute computing facility, and supercomputing facility **PARAM SHIVAY** throughout my research. I express my thanks to all teaching and non-teaching staff members of the department for their support.

I gratefully acknowledge the UGC for providing the fellowship in the form of Junior Research Fellowship and Senior Research Fellowship.

The person with the greatest indirect contribution to this work is my grandmother, **Smt. Amarawati Mishra** and grandfather, **Late. Shri Shobh Nath Mishra**, for their deepest love, endless patience, and continued support throughout my research work. I express my sincere and cordial gratitude to my father **Shri. Anjani Kumar Mishra**, my mother **Smt. Keshri Mishra**, my younger sister **Poonam Mishra**, my younger brother **Mr. Pratik Mishra**, my uncle **Late. Shri Krishna Chandra Mishra**, my aunt **Smt. Asha Mishra** (chachi) and **Smt. Vijay Kumari Shukla** (bua), and cousins **Kunta Didi, Vijay bhaiya, Vinay, Prachi, Sakshi, Priyam, Pritam, Preeti, Priya, Chahat, Payal, and Ansh** who always stood by my decisions and provided all kinds of support, love, care, and patience.

I thank my special friend **Miss. Samiksha Srivastava** for her togetherness, love, and support. I thank all my friends, including **Ashish, Sachin, Anuvrat, Kaustubh, Harsh, Monia, Kartika, Shubham, Santosh, Shweta, Akanksha, Neha, Shrish, Gagan, Shivanshu, Praveen, Shailesh, Sachin Sharma, and Brijesh**, for their unwavering love, care, and support—be it emotional, moral, or financial—that encouraged me to move forward. The list of friends who stood by me during this journey is long, and I sincerely apologize for not being able to mention all their names here. Their contribution is equally cherished and remembered.

I extend my heartfelt gratitude to all my **teachers and mentors, and to the schools and institutions** I have been privileged to be a part of throughout my academic journey. Their dedication, wisdom, and encouragement have been instrumental in nurturing my curiosity, instilling values, and guiding me towards this achievement. I would like to acknowledge one of my early educators, **Shri Satya Prakash Shrivastava**, whose guidance played a pivotal role in shaping my foundation.

I would like to sincerely acknowledge **ChatGPT, Google, online databases, forums, and preprint archives** for its invaluable support in writing, organizing, and preparing my thesis and providing access to a wealth of knowledge and tools critical for conducting research in this digital era.

This acknowledgment would be incomplete without expressing my deepest regards to the great visionary, **Pt. Madan Mohan Malaviya**, whose foresight and dedication created this divine center of knowledge. My heartfelt gratitude and reverence go to **Ma Saraswati**, the goddess of wisdom and learning, **Baba Vishwanath**, the eternal protector and guide, and **Ma Ganga**, the nurturer and purifier. Their divine blessings have been my unwavering source of strength, inspiration, and guidance throughout this journey, making everything possible.

Certificate

It is certified that the work contained in the thesis titled **Unveiling Phases and Phase Transitions in Inhomogeneous Active Systems** by **Mr. Pawan Kumar Mishra**, Roll Number **19171019**, has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of **Ph. D. Degree** in **Physics**.


Dr. Shradha Mishra

Assistant Professor
(Supervisor)
Department of Physics
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005

Copyright Transfer Certificate

Title of the Thesis : Unveiling Phases and Phase Transitions in Inhomogeneous Active Systems.

Name of the Student : Pawan Kumar Mishra

Copyright Transfer

The undersigned hereby assigns to the Institute of Technology (Banaras Hindu University) Varanasi all rights under copyright that may exist in and for the above thesis submitted for the award of the **Doctor of Philosophy (Ph.D.) in Physics**.

Date 14th August 2024

Place Varanasi



Pawan Kumar Mishra

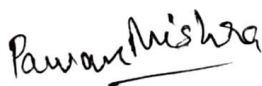
Note: However, the author may reproduce or authorize others to reproduce material extracted verbatim from the thesis or derivative of the thesis for author's personal use provided that the source and the Institute's copyright notice are indicated.

Declaration

I, **Pawan Kumar Mishra**, certify that the work embodied in this thesis is my own bona-fide work and carried out by me under the supervision of **Dr. Shradha Mishra** from July 2019 to August 2024 at the **Department of Physics**, Indian Institute of Technology (BHU), Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers whenever and wherever their works have been cited in my work in this thesis. I further declare that I have not wilfully copied any others' work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, theses, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

Date 14th August 2024

Place Varanasi


Pawan Kumar Mishra

Certificate by the Supervisor

It is certified that the above statement made by the student is correct to the best of my knowledge.



Dr. Shradha Mishra
Assistant Professor
(Supervisor)
Department of Physics
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005



Head of the Department

HEAD (Physics)
भौतिकी विभाग / Deptt. of Physics
भा.प्रौ.सं. / (का.वि.वि.) / IIT (BHU)
वाराणसी / Varanasi-221005

Dedicated
to
My grandfather

Late. Shri Shobh Nath Mishra

List of figures

- 1.1 Visualization of polar and apolar particles along with their ordering. **Apo-
lar particle:** Apolar particle is shown with green double-headed arrows
and green ellipses. The double-headed arrows indicate bidirectional align-
ment of the apolar particles, while the ellipses highlight their positions
without directional preference. **Polar particle:** Polar particle is depicted
with blue arrow and ellipse representing their head and tail respectively.
The ellipses are oriented along the direction of the arrows, showing the
alignment of the polar particles. **Nematic ordering:** showcasing the spa-
tial distribution and alignment of polar particles in the system. **Polar
ordering** revealing the arrangement of polar particles within the system.
Nematic ordering of polar rods: Apolar ordering for polar particles. . . . 11
- 1.2 A cartoon of a particular type of Extrinsic and Intrinsic inhomogeneity
in panel (a) and (b) respectively. The red arrows (in (a)) are the random
quenched directions and blue arrows (in (a)) are orientation of particles.
The colors of the circle in (b) represents their size and arrows on the circle
are the orientation of particles. 17
- 2.1 (a) Sketch of a part of system carrying spins $S = +1$ (red (triangle right))
and $S = -1$ (dark green (triangle left)) along with vacant sites $S=0$ (black
circle) on a two-dimensional lattice, (b) represents the flipping rate at fixed
temperature, (c) represents the probability of movement of the spins to the
neighboring sites and in (d) the birth and death parameter γ is added in the
model in which the particles disappearing from random sites is shown by
magenta square and appearing at the sites which were vacant represented
by orange triangle right for $S = +1$ and green triangle left for $S = -1$ 25

- 2.2 A loop of four configurations that breaks Kolmogorov's criterion [Kolmogorov (1936)] of detailed balance for dIm , flipping and movement is shown for the spin represented by partially filled (red (triangle right)) and (dark green (triangle left)). The clockwise loop ($C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4 \rightarrow C_1$) gives the total probability $D^2 e^{-4\beta}$ while the anticlockwise loop ($C_1 \rightarrow C_4 \rightarrow C_3 \rightarrow C_2 \rightarrow C_1$) gives the total probability $D^2 e^{-2\beta}$, thus showing that the system does not satisfy detailed balance. The numbers associated to the arrows are the transition rates and other symbols have the same meaning as in Fig.[2.1]. 26
- 2.3 The plot of order parameter M vs. temperature T (a) for the system size 800×50 and various values of γ i.e. The black (circle), red (square), green (diamond), blue (triangle right) and maroon (triangle left) for $\gamma = 0$, $\gamma = 0.01$, $\gamma = 0.1$, $\gamma = 0.5$ and $\gamma = 1$ respectively. The panels (b), (c) and (d) represent M vs. T for fixed $\gamma = 0$, $\gamma = 0.1$ and $\gamma = 1$ respectively. The different curves in each panel (b)-(d) represent different system sizes i.e. the indigo (triangle down), magenta (triangle right) and green4 (triangle right) for 400×50 , 600×50 and 800×50 . The lines are guide to the eyes. 28
- 2.4 Time snapshots of local density ρ_{loc} (a-c) and local magnetisation m (d-f). From left to right (a-c) or (d-f) is for $\gamma = 0.0$, 0.01 and 0.1 and (t_1 - t_3) is for 1.4×10^5 , 1.5×10^5 , 1.6×10^5 respectively. Color bars represent the value of local density ρ_{loc} and magnetisation m . System size is 400×50 and only a part of the system is shown for better resolution. 29
- 2.5 Plot of PDF $P(\rho_x)$ for different values of γ i.e. The black (circle), green (square), maroon (diamond), indigo (triangle right), cyan (triangle left) and orange (triangle down) symbols represent $\gamma = 0$, $\gamma = 0.02$, $\gamma = 0.04$, $\gamma = 0.06$, $\gamma = 0.08$ and $\gamma = 0.1$ respectively. The lines are guide to eyes. Inset: plot of $\Delta\rho$ vs. γ where $\Delta\rho$ represents the difference between two peaks of the distribution of local density. The system size is same as in Fig.[2.4] 31
- 2.6 Plot of PDF of order parameter $P(M)$ for $\gamma = 0$ and $\gamma = 0.1$ in (a) and (b) respectively, where the black (circle), red (square), green (diamond), blue (triangle left) and maroon (triangle up) symbols represents the temperatures (a) $T = 2.15$, $T = 2.20$, $T = 2.25$, $T = 2.30$ and $T = 2.35$ respectively and (b) $T = 1.65$, $T = 1.70$, $T = 1.75$, $T = 1.80$ and $T = 1.85$ respectively near the disorder-to-order transition. Lines are guide to eyes. The system size is same as in Fig.[2.4] 32

- 3.1 In this illustrative diagram, all the possible scenarios of the flipping of SPPs are showcased in different panels. (I-III) represent when there is a disorder in the path of SPPs, (IV) when there is no disorder is present. SPPs and loactions of disorders are depicted as a circular entity connected with an arrow, right-moving SPPs are marked in blue, while left-moving ones are portrayed in brown. Similarly, quenched disorders within the system are illustrated with an arrowhead; left and right-oriented disorders are marked in green and red, respectively. The SPPs and disorders in the top row are shown at a particular time t before the interaction with SPPs/disorders and in the bottom row at time $t + 1$ after the interaction with SPPs/disorders. 43
- 3.2 In this series of spatio temporal snapshots (a)-(d) for $\rho_d = 0, 0.002, 0.008$ and 0.02 respectively, the one-dimensional system unfolds over time, where the x-axis represents the spatial coordinates of SPPs and disorders. The y-axis signifies the progression of time. SPPs with leftward orientation are depicted in red, while those with rightward orientation are shown in blue. Left-oriented obstacles are represented in green, and right-oriented obstacles in brown. 46
- 3.3 The spatio-temporal snapshots (a-d) illustrates the effect of increasing the disorder strength $\Delta_d = (0, 0.001, 0.01, 0.04)$ respectively in the coarse-grained model. The x -axis denotes the product of local magnetization and density $g(x, t)$ and the y -axis represents the increasing time direction. The color bar represents the value of $g(x, t)$ 46
- 3.4 (a) Two-point Spatial correlation function $C_m(r)$ vs. r for different disorder densities ($\rho_d = 0.010, 0.025, 0.050, 0.075$ and 0.100) for microscopic model in the steady state at time $t = 12000$. The symbols are data from numerical simulation and lines are fit to the proposed form of the $C_m(r)$ as given in Eq.(5.1). *Inset*: The plot of scaled $C_m(r$ vs. scaled distance $r/L(\rho_d)$ for different disorder densities. Plot (b) shows the power exponent $\alpha(\rho_d)$ vs. ρ_d on linear scale. The two symbols show the power exponent for different packing fraction of the SPPs/disorders along the fixed length, violet square is for density 1.0 while brown triangle for particle density 2.0. (c) Demonstrates the cluster size $\zeta(\rho_d)$ vs. ρ_d on log – log scale. Symbols have usual meaning as in (b). 48

- 3.5 (a)-(e) show the distribution neighbours for $\rho_d = 0.010, 0.025, 0.050$ and 0.100 respectively. (a)-(e) insets show the fitting of corresponding distribution of neighbours by power law (red) and exponential (blue). (f) Plot of number of cells with non-zero densities N_{nz} with respect to disorder density ρ_d 49
- 3.6 Two point correlation function $C_m(r)$ vs. r at different times $t = 500, 1000, 1500$ shown by black (solid line) green (dashed line) and mazenta (dotted line) respectively and disorder densities $\rho_d = 0.010, 0.025, 0.050$ shown in different panels from $a - d$ respectively. *Insets*: The scaled $C_m(r)$ vs. scaled distance $r/L(\rho_d)$ for all the four cases. 51
- 3.7 (a) Shows the plot of two-point correlation function $C_{cg}(r)$ for various disorder strengths, $\Delta_d = 0.001$ (black, circles), 0.002 (red, squares), 0.004 (green, diamonds), 0.007 (pink, triangles). *Inset* : The scaled $C_{cg}(r)$ vs. scaled distance $r/L(t)$. (b) shows the correlation function at different times for a particular disorder strength $\Delta_d = 0.001$. *Inset*: The scaled $C_{cg}(r)$ vs. scaled distance $r/L(t)$ 52
- 4.1 The figure shows the activity profiles in the first row for different shapes of distributions of activity of Active Brownian Particles (ABPs). Each subplot in the columns represents a different shape of distribution: Step function shape, Sigmoid function shape, Gaussian function shape, and Cone shape from left to right respectively. The activity profiles are represented by color intensity, with warmer colors indicating higher activity levels and cooler colors indicating lower activity levels. In the second, third, and fourth rows, the density $\rho(\mathbf{r}, t)$, polarization $\mathbf{p}(\mathbf{r}, t)$, and flux/current $\mathbf{J}(\mathbf{r}, t)$ of the ABPs are displayed from top to bottom, respectively. The color intensity in the second row represents the density levels, with red indicating higher densities and blue indicating lower densities. In the third and fourth rows, the color intensity represents the magnitude of polarization and flux, with warmer colors indicating higher magnitudes and cooler colors indicating lower magnitudes. The stream of arrows in the third and fourth columns illustrate the direction of polarization and flux, respectively. The rest of the system parameters are given in the main text. 62

- 4.2 This figure shows the three dimensional representation of step, sigmoid, Gaussian and cone shapes of activity profile in the first row. Activity profile is represented by speed of the particles on the z -axis on two dimensional xy plane. The density corresponding to different shapes of activity profiles is shown on the z -axis in second row. These 3d snapshots are shown for system size, $L=128$ at time, $t=10000$. Note: that the density profile in the steady state exactly opposite to that of activity profile. 63
- 4.3 Time evolution snapshots of the model with Gaussian shape distribution of speed at time steps $t_1=1$, $t_2=1000$, $t_3=40000$ and $t_4=200000$ is shown from left to right. Density, polarisation and flux is shown in first, second and third row of the figure, respectively at the given times. The color bars represent the value of the density, polarisation and flux of the particles from low at bottom to high at the top. The stream of arrows represent the direction of polarisation and flux in the model. 64
- 4.4 The figure illustrates the time-evolution of particle densities on substrates for the microscopic model (*MIC*) for two different types of activity profiles i.e. step and Gaussian shape, represented in the top and bottom rows, respectively. Each column corresponds to a specific time point ($t_1 = 15000$, $t_2 = 30000$, $t_3 = 40000$, and $t_4 = 50000$). The particle density is visualized using color maps, with colors ranging from red to blue, indicating low to high density, respectively. 66
- 4.5 Radial density as a function of distance from the center for the activity profile of step function, illustrating the behaviour of density profile in CG model (main plot) and the MIC model (inset plot). 66
- 4.6 This figure shows the various distributions we have considered. a) E shaped potential b) VA (Hindi) shaped potential c) LA (Telugu) shaped potential d) Flower shaped potential. 67
- 4.7 Steady state snapshot of the particles density distribution for different potentials. a) English Alphabet 'E' b) English Alphabet 'T' c) English Alphabet 'U' d) English Alphabet 'J' e) Telugu Alphabet [Rao & Ajitha (1995)] 'la' f) random shape 'flower' g) Hindi Alphabet 'pa' h) Hindi Alphabet 'na' i) Hindi Alphabet 'va'. The red region represents low density and blue region represents high density of particles. All the snapshots are taken at an activity of 2.0 and at the time $t=50000$. The box size is 128×128 68

5.1	Visualization of the Nematic order parameter as γ varies from bottom to top and time progresses from left to right. Each row represents a different value of γ : (a-e) $\gamma = 0$, (f-j) $\gamma = 0.40$, (k-o) $\gamma = 0.80$, and (p-t) $\gamma = 1.00$. The progression from left to right in each row illustrates the temporal evolution (time steps: 1, 8000, 16000, 24000 and 30000) of the Nematic order parameter within each γ condition. The same description is valid for the Polar order parameter	76
5.2	Snapshots of the polar order parameter at a fixed time step i.e. 10000 showing variations in the average density of polar particles from left to right ($\rho_{p0} = 0.05, 0.10, 0.15$ and 0.20) and the coupling strength from bottom to top ($\gamma = 0.50, 0.55, 0.60$ and 0.65).	77
5.3	Phase diagram illustrating the width of strings formed between pairs of defects as a function of density and coupling parameter. The color map represents the width of the strings. Snapshots of the polar order parameter field illustrate variations in the width of the strings with respect to density and coupling parameters. Each panel corresponds to a different combination of density and coupling values: (a) Low density and weak coupling; (b)-(e) low density and strong coupling; (f) High density and strong coupling.	79
5.4	Probability of strings connecting defects of the same nature. Different curves represent different values of density.	80
5.5	Snapshots for random and polar order initial conditions of polar particles for $\gamma = 0$ and $\gamma = 1.0$	82
5.6	Mean square displacement for random and polar order initial conditions of polar particles from $\gamma = 0$ to $\gamma = 1.0$	83
5.7	Comparison of nematic order parameters and polarization fields across different γ values. Each row of subplots presents data from three different folders, corresponding to $\gamma = 0.00, 0.60$, and 1.00 . The left column shows the nematic order parameter (Q) with Roman numerals Q I, Q II, and Q III indicating different states of the nematic order. The right column illustrates the polarization fields (P) for the same γ values, with Roman numerals P I, P II, and P III representing different polarization states. Each plot uses the jet colormap to visualize the variation in order parameter and polarization fields. Color bars to the right of each subplot indicate the range of values.	83
5.8	The figure illustrates the plot of width $\frac{W}{D}$ vs coupling strength γ for three values of average density of polar particles $\rho_p = (0.05, 0.10, 0.15)$	84

List of tables

1.1	The table presented provides a detailed comparison of ordering phenomena in equilibrium systems $T \neq 0$, clean active systems, and active systems with disorder across different spatial dimensions. The ordering behavior is categorized into Long-Range Order (LRO), Quasi Long-Range Order (QLRO), and Short-Range Order (SRO).	19
-----	--	----