CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

1.1 Thermoelasticity: Definition and Applications

Thermoelasticity is an appealing scientific discipline that explores the mutually reinforcing influences of thermal and mechanical fields within an elastic body, with a focus on accurate estimation of the thermomechanical responses of the material. This is an extension of the theory of elasticity as well as the theory of heat conducton. The phenomenon in which the mechanical properties of a material undergo changes in responses to variations in temperature is commonly known as thermal deformation. Therefore, thermoelasticity theory predicts the thermomechanical interactions in the elastic body. The theory of thermoelasticity differs from the classical theory of elasticity by considering the influence of internal forces on the temperature field as well as the impact of temperature variations on deformation. Although the theory of thermoelasticity has a long history, its conceptual framework have been established in the near middle of the 19^{th} century by Duhamel (1837) and Neumann (1841), widespread interest in this area of science did not gain significant attention until the period following the Second World War conflict. There exist compelling justifications for the abrupt and relentless revival of this field. First, in aeronautical engineering, it has been observed that the higher velocities of contemporary aircraft lead to the occurrence of aerodynamic heating. Consequently, this phenomenon generates large thermal stresses and decreases the endurance of the aircraft structure by lowering its elastic limit.

After the devastating use of atom bomb in World War II, the global community encouraged the development of nuclear energy for peaceful civilian purposes and use it for commercial operation. In the nuclear power plant, the biggest challenges were maintaining strict safety standards and structural design to deal with exceptionally high temperature and temperature gradients originating from nuclear reactors. Similar to this, undesirable thermal stresses are caused by combustion processes at high temperatures in modern propulsion systems like jet and rocket engines, spacecraft and missiles, in the functioning of big steam turbines, and furthermore in the construction of ships, where, unexpectedly, ship cracks are usually caused by thermal stresses of modest intensity. The exploration of these and analogous issues has resulted in an unexpected variety of theoretical and experimental research publications describing many different aspects of thermoelasticity theory in engineering structures. Thermoelasticity covers a broad spectrum of events. It represents a generalized theory of heat conduction, generalized theory of thermal stresses, an analysis of the temperature distribution resulting from deformation, and an investigation of thermoelastic dissipation, which contributes to internal damping in elastic bodies. The theory, therefore, holds significant cognitive importance. Regardless of its mathematical complication, the field of thermoelasticity offers a profound understanding of the mechanisms underlying deformation processes associated with thermal phenomena in elastic materials.

1.2 Classical Coupled Thermoelasticity Theory (CTE)

The influence of distortion on temperature distribution and the impact of temperature on strain and stress distribution are both included in the conventional thermoelasticity theory. The classical heat conduction theory serves as the foundation to this theory, therefore, it is also known as classical coupled thermoelasticity theory. The uncoupled theory, in contrast, is built on the simple premise that the effect of strain on the temperature field may be avoided. In cases when the major cause of temperature variations on a body is due to heat input, it is generally assumed that the mechanical coupling factor in the energy balance equation may be ignored. However, the impact of temperature fluctuations becomes significant when they primarily arise from the structural deformation of the object. In the realm of scientific inquiry, coupled theory involves the simultaneous evaluation of both stress and temperature distribution. This approach allows for a comprehensive understanding of the interplay between these two crucial factors. On the other hand, uncoupled theory takes a sequential approach, where physical fields are evaluated one after the other. While both approaches have their merits, coupled theory offers a more holistic perspective on the intricate relationship between stress and temperature distribution. The coupled thermoelasticity theory, in contrast to the classical "uncoupled theory of thermoelasticity," addresses the limitation wherein changes in elasticity do not impact temperature, and vice versa.

Going into the historical development of the concept of thermoelasticity, one can find in literature that on February 23, 1835, Duhamel presented the first investigation on thermoelasticity to the French Academy of Sciences in Paris; the article was later published (Duhamel (1837)). For the first time, this paper proposed the concept of thermal and mechanical fields being coupled and deduced the equations for the strain in an elastic body with temperature gradients. Additionally it contains the formulation of boundary value problems. In subsequent studies, Neumann (1841) was able to carry out and confirm the aforementioned findings. The theory focused on the distinct thermal and mechanical effects, treating them as independent factors. The overall strain was calculated by combining the elastic strain with the thermal expansion resulting from the temperature distribution exclusively. Consequently, the existing theory failed to incorporate the connection between strain and temperature distributions in a precise and well defined way. In a significant development, Thomson (1857) astutely incorporated thermodynamic principles into his analysis. Notably, he became the pioneer in utilizing the laws of thermodynamics to calculate the distribution of stresses and strains within an elastic material when subjected to fluctuating temperatures. In subsequent studies, Voigt (1928) and Jefferys (1930) carried out the task of thermodynamically documenting the equations proposed by Duhamel (1837). Subsequently, the field of coupled thermoelasticity experienced significant advancements due to the remarkable contributions made by Biot (1956) in his work. In this study, Biot effectively utilized the fundamental principles and laws of thermomechanics through a rigorous and justified approach, and thereby derived the essential governing equations and constitutive relations for the field of coupled thermoelasticity. Biot's theory, commonly referred to as the classical coupled thermoelasticity theory, has been recognized for its significant contributions in the field. In his field of conventional thermoelasticity theory, Biot (1956) made a significant contribution by introducing the variational principle. This principle serves as a valuable tool for deriving Lagrangian equations, thereby enhancing our understanding of the subject. Moreover, the author has eloquently expounded upon the methodologies employed to derive comprehensive solutions for the thermoelasticity equations within an isotropic homogeneous medium.

The following are the basic fundamental relations for the linear classical coupled thermoelasticity theory for a general anisotropic medium due to Biot (1956):

The equation of balance of momenta:

$$\rho \ddot{u}_i = \sigma_{ij,j} + \rho F_i. \tag{1.2.1}$$

Law of conservation of energy :

$$\rho T_0 \dot{S} - \rho Q = -q_{i,i}. \tag{1.2.2}$$

The constitutive relations

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - \beta_{ij}\theta, \tag{1.2.3}$$

$$\rho S = \frac{\rho c_E}{T_0}\theta + \beta_{ij}\varepsilon_{ij} \tag{1.2.4}$$

Strain-displacement relation

$$\varepsilon_{ij} = \frac{u_{ij} + u_{ji}}{2} \tag{1.2.5}$$

The Classical Fourier's Law

$$q_i = -K_{ij}\theta_{,j} \tag{1.2.6}$$

Using above relation, the following equations can be obtained:

$$K_{ij}\theta_{,ij} + \rho R = \rho c_E\dot{\theta} + T_0\beta_{ij}\dot{u}_{i,j}, \tag{1.2.7}$$

$$C_{ijkl}u_{k,lj} - \beta_{ij}\theta_{,j} + \rho H_i = \rho\ddot{u}_i. \tag{1.2.8}$$

Here, Eq. (1.2.7) and Eq. (1.2.8) represent the heat transport equation and equation of motion, respectively. The above-mentioned equations, which are interconnected, provide a comprehensive set of field equations within the framework of linear conventional thermoelasticity theory applied to homogeneous and anisotropic solids.

1.3 Drawback of Fourier's law

The theory of Biot has been substantiated as an advanced structure that enabled remarkable investigation of the interconnected influences of elastic and thermal fields. Noteworthy contributions to this area of study include the works of Chadwick (1960), Boley and Wiener (1960), Nowacki (1975), Parkus (2012), Nowinski (1978), and Dhaliwal and Singh (1980), which provide extensive explanations and applications of the theory. Nevertheless, it is important to note that this theory, while possessing noteworthy features, is limited by its dependence on the Classical Fourier's law (Eq. (1.2.6)) and hence, it consists of a parabolic type partial differential equation for thermal distribution and a hyperbolic partial differential equation for mechanical distribution (Eq. (1.2.8)). This hyperbolic-parabolic system of partial differential equations results in infinite speed of thermal disturbance. It is seen that among the four field equations that set up by Eq. (1.2.7) and Eq. (1.2.8), three of them, denoted as scalar equations of motion in Eq. (1.2.8), exhibit hyperbolic characteristics. On the other hand, the fourth equation, known as the heat transport Eq. (1.2.7), has parabolic behavior. In accordance with the theory, it is predicted that elastic disturbances possess a limited speed, whereas thermal disturbances exhibit an infinite speed. These two types of disturbances are coupled. The observed conclusion indicate that when the material undergoes a thermomechanical disturbance, the resulting effect will be immediately experienced at infinitely far distance from the source of disturbance. This unnatural outcome of the classical theory therefore drew huge attention of researchers in subsequent years. However, by surveying into the literature (see Chandrasekharaiah (1986; 1998)), one can note that prior to this, attempts had been made to modify the classical theory of heat conduction to eradicate the paradox of infinite heat propagation speed. In fact, in mid 19th century, Maxwell (1867) hypothesized the appearance of wave-type heat flow for the first time and indicated the need of a modification to Fourier's law (Chandrasekharaiah (1986)). The phenomenon of wave like thermal signals is now called as the "second sound" effect. We further recall the work reported by several pioneering researchers including Landau (1941), Tisza (1947), Ward and Wilks (1951) who remarked about the possibility of wave type heat flow in materials. Experimentally, Peshkov (1944) discovered the second sound for the first time in liquid helium and determined that its speed was 19 m/s at 1.4 K. Furthermore, the experimental verification on the prediction of wave type heat flow made by Landau (1941) and Tisza (1947) was conducted by Maurer and Herlin (1949), Pellam and Scott (1949), and Atkins and Osborne (1950). The occurrence of a second sound in fluid helium had further been established at temperatures below 2.2 K, as reported by Lifshitz (1958). The effect was subsequently noted in some other crystals (see McNelly et al. (1970), Jackson et al. (1970), Jackson and Walker (1971), Rogers (1971)). Based on the available evidences, it was therefore apparent that the obtained result with respect to classical thermoelastic models lacks physical validation. Moreover, numerous studies conducted within the context of classical thermoelasticity theory have analyzed certain limitations that yield unsatisfactory outcomes in the context of short laser pulses and low temperatures (see Lord and Shulman (1967), Green and Lindsay (1972), Francis (1972), Chandrasekharaiah (1986), Ignaczak and Ostoja-Starzewski (2010), and the relevant citations therein). Furthermore, the progress in micro-scale technology enables the transmission of thermal field motion in the form of a wave, therefore countering the infinite propagation of heat by diffusion. Over the years, experts have devoted significant efforts in changing the notion of this theory, as seen by the many studies conducted on the subject. Several valuable improvements/modifications have been suggested in accordance with limitations of classical thermoelasticity theory. The altered thermoelasticity theories are often denoted as generalized thermoelasticity theories.

1.4 Generalized Thermoelasticity Theory

Generalized thermoelasticity theories encompass modified versions of the conventional thermoelasticity theory, primarily developed to address discrepancies that arise in classical theory and to account for the thermal wave behavior which was ignored in the Fourier's law. The theories presented can be broadly classified into two distinct categories. The initial approach was centered around a modified version of the heat conduction law, specifically replacing Fourier's law utilized in classical thermoelasticity with a suitable adjustment in the constitutive relationship between heat flux and temperature gradient. The modified constitutive relations primarily consist of novel constitutive variables or phase-lag parameters that pertain to either time, space, or both. The second category pertains to thermoelasticity theories, wherein the conventional theory is enhanced through an alternative approach to derive reconditioned constitutive equations. This is achieved by employing thermodynamic principles as a basis for the improvements made. Nevertheless, it is worth noting that numerous theories falling under the second category maintain the fundamental principles of Fourier's law without any alterations. Below, we present a concise overview of several widely recognized and extensively researched generalized theories, as well as some recently proposed ones.

1.4.1 Overview of generalized thermoelasticity theories based on non-Fourier models

This section covers a historical developement of various generalized thermoelasticity theories that involve modified versions of Fourier's law as given below.

1.4.1.1 Thermoelasticity with one relaxation time (LS or ETE)

The first theory of generalized thermoelasticity suggested by Lord and Shulman (1967), has emerged as one of the most explored modified theories in the field of thermoelasticity. The issue of infinite propagation speed resulting from diffusion equation was first overcome by Cattaneo (1948). This concern was also examined separately by Morse and Feshbach (1953) as well as Vernotte (1958). Cattaneo (1958) and Vernotte (1958; 1961) each proposed an altered version of the unsteady heat conduction equation. This modified equation serves as a linear extension of the conventional Fourier equation. Notably, an additional parameter, denoted as τ_q, is introduced into Eq. (1.2.6) to incorporate the behavior of temperature waves that are not captured by Fourier's law.

The modified Fourier's law of heat conduction presented by Cattaneo and Vernotte for the case of anisotropic and homogeneous material can be given as follows:

$$q_i + \tau_q \frac{\partial q_i}{\partial t} = -K_{ij}\theta_{,j}, \tag{1.4.1}$$

Here, the symbol τ_q represents the time delay required for the establishment of a steady-state condition in heat conduction, after the rapid imposition of a temperature gradient. It is sometimes referred to as a thermal relaxation time.
Combining Eq. (1.4.1) with the energy equation

$$\rho c_E \dot{\theta} = -q_{i,i} + \rho Q \tag{1.4.2}$$

yields the corresponding heat conduction equation as

$$K_{ij}\theta_{,ij} = \left(1 + \tau_q \frac{\partial}{\partial t}\right)\left(\rho c_E \dot{\theta} + T_0 \beta_{ij} \dot{u}_{i,j} - \rho Q\right). \tag{1.4.3}$$

Eq. (1.4.3) is the governing partial differential equation of the heat conduction theory based on the law given by Eq. (1.4.1). This equation is of hyperbolic-type and characterizes the combined diffusion and wave-like behavior of heat transport. It predicts the finite speed of heat propagation as

$$V_T = \sqrt{\left(\frac{K_{ij}}{\rho c_E \tau_q}\right)} \tag{1.4.4}$$

The objections raised by Cattaneo and Vernotte regarding the concept of diffusion have not encountered significant opposition. It is widely accepted that the transportation of heat pulses should be facilitated through the propagation of waves. This didn't mean that there are big movements afoot to discard Fourier's law. The relaxation time in Eq. (1.4.1) is thought to be very small in nearly all practical and even exotic applications, so several authors have argued that the last component in Eq. (1.4.1) might be avoided. In fact, in our view, an understanding of time scale is the central object of scientific investigations of heat waves, and is only imperfectly understood at that time. Chester (1963) provided a comprehensive explanation of the heat conduction equation, which encompasses the physical interpretation of the variable τ_q (as denoted by Eq. (1.4.5)). He has estimated the value of τ_q and has put forth the subsequent expression:

$$\tau_q = \frac{3K}{\rho c v_s^2}, \tag{1.4.5}$$

The variable v_s represents the speed of regular sound. The investigation conducted by several researchers has postulated the potential values of τ_q, denoting the relaxation time, for both metals and gases, ranging from 10^{-14} seconds to 10^{-10} seconds. These findings can be found in the works of Nettleton (1960), Chester (1963), Maurer (1969), Mengi and Turhan (1978) as well as other relevant sources cited within these articles. In view of the above expression, the value of τ_q being very small has created an urge among the researchers to neglect the second term on the left side in Eq. (1.4.1). Although, the more rigorous studies by several researchers including Baumeister and Hamill (1969), Chan et al. (1971), Maurer and Thompson (1973), Sadd and Cha (1982) have proved the relevance of the Eq. (1.4.1) in case of very high heat-flux and very short time intervals. It has been reported that the hyperbolic type heat conduction equation (Eq. (1.4.3)) corresponding to modified Fourier's law (Eq. (1.4.1)) presents more physically relevant results in these cases as compared to the parabolic type diffusion equation corresponding to Fourier's law of heat conduction.

Lord and Shulman (1967) have worked upon developing an extension of Classical thermoelasticity theory around the modified Fourier's law given by Eq. (1.4.1) and have arrived at a generalized thermoelasticity theory. This theory supports the finite speed of heat propagation. This theory is also termed as extended thermoelasticity theory (ETE) or thermoelasticity with thermal relaxation. The heat conduction equation of this theory in the context of anisotropic and homogeneous material can be presented as follows:

$$K_{ij}\theta_{,ij} = \left(1 + \tau_q \frac{\partial}{\partial t}\right)\left(\rho c_E \dot{\theta} + T_0 \beta_{ij} \dot{u}_{i,j} - \rho Q\right). \tag{1.4.6}$$

When the relaxation parameter (τ_q) is set to zero, the above theory precisely reduces to Classical theory (Biot's theory) i.e hyperbolic type heat transport equation reduces to Classical parabolic type heat transport equation which corresponds to physically absurd case of $V_T = \infty$ (Eq. (1.4.4)).

1.4.1.2 Dual-phase-lag thermoelasticity theory (DPL)

As pointed out in previous section, the CV law given by Eq. (1.4.1) can be regarded as an approximation of the relation (1.4.7). Tzou (1992) has proposed the following natural generalization of that law termed as single phase-lag model:

$$q_i(\boldsymbol{x}, t + \tau_q) = -K_{ij}\theta_{,j}(\boldsymbol{x}, t). \tag{1.4.7}$$

According to above equation, τ_q may be interpreted as the time-lag between the temperature gradient and the resulting heat flux vector in a transient process, it being presumed that heat flux is the result of a temperature gradient. If $\tau_q = 0$, the Eq. (1.4.7) reduces to Fourier's law. Furthermore, if we expand the left hand side of the Eq. (1.4.7) by Taylor's series about t and retain only the first order term in τ_q, we recover the CV model Eq. (1.4.1) according to which the sum $q_i + \tau_q \frac{\partial q_i}{\partial t}$ is the instantaneous result of a temperature gradient. Thus, if the relation (1.4.7) is taken as a heat conduction law, then this law is consistent with and includes as particular cases the Fourier's law (1.2.6) and the CV Eq. (1.4.1). Subsequently, Tzou (1995a; 1995b) has further generalized the concept of time-lag and put forward a unique heat conduction relationship, which may be expressed in the following manner:

$$q_i(\boldsymbol{x}, t + \tau_q) = -K_{ij}\theta_{,j}(\boldsymbol{x}, t + \tau_\theta). \tag{1.4.8}$$

According to Eq. (1.4.8), the delay time denoted as τ_θ is primarily due to the microstructural interactions. These interactions encompass the small-scale heat transport mechanisms that take place within the microscale, as well as the small-scale effects of heat transport in space. Examples of such interactions include phonon-electron interaction and phonon scattering. Consequently, this delay time is commonly referred to as the phase lag of the temperature gradient. The second delay time, denoted as τ_q can be recognized as the relaxation time resulting from the rapid transient influences of thermal inertia. In the presence of a temperature gradient at a specific point in the medium at time $t + \tau_\theta$, the resulting heat flux will be experienced at time $t + \tau_q$ at that same point, provided that if $\tau_q > \tau_\theta$. However, the outcomes exhibit an inverse relationship when $\tau_\theta > \tau_q$. Furthermore, one may express that this theory describes the microscopic effects in both space and time, while the classical Fourier's law adopts a macroscopic perspective in terms of space and time. Tzou (2014; 1993) had proved that both of these models Single phase lag and Dual phase lag are deemed acceptable within the theoretical framework of the second law of extended irreversible thermodynamics.

Through the utilization of Taylor series expansion, Tzou (1995a; 1995b) has introduced two distinct relations for heat-flux and temperature gradient in the aforementioned Eq. (1.4.8):

1. Tzou has derived the following constitutive relation of heat conduction known as Dual-phase-lag heat conduction model-I by expanding both sides of Eq. (1.4.8) by Taylor's series expansion and keeping just the first-order terms in τ_q and τ_θ (Tzou (1995b)):

$$\left(1 + \tau_q \frac{\partial}{\partial t}\right) q_i = -K_{ij} \left(1 + \tau_\theta \frac{\partial}{\partial t}\right) \theta_{,j}. \tag{1.4.9}$$

   The above equation pertaining to heat flow is also referred to as the Jeffreys-type equation, which was introduced by Joseph and Preziosi (1989a).

2. The Dual-phase-lag heat conduction model-II is obtained by retaining the second-order terms in Taylor's series expansion around time t on the left-hand side (expressed in terms of τ_q) and the first-order term on the right-hand side (expressed

in terms of τ_θ) of Eq. (1.4.8) (Tzou 1995a) as follows:

$$K_{ij}\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\theta_{,j}=-\left(1+\tau_q\frac{\partial}{\partial t}+\tau_q^2\frac{\partial^2}{\partial t^2}\right)q_i. \tag{1.4.10}$$

The Dual-phase-lag heat conduction model is consequently extended by Chandrasekharaiah (1998), who introduced a new generalized thermoelasticity theory, namely Dual-phase-lag thermoelasticity theory based on the above two modified Fourier's laws given by Tzou (1995a; 1995b). Chandrasekharaiah (1998) has conducted the analysis of Eq. (1.4.10) with respect to the thermoelasticity theory. The author combined Eq. (1.4.10) with energy law (given by Eq. (1.2.2)) and Eq. (1.2.4) to derive the following governing relation of heat transportation for DPL thermoelasticity theory in the context of anisotropic medium:

$$\left(1+\tau_\theta\frac{\partial}{\partial t}\right)K_{ij}\theta_{,ij}=\left(1+\tau_q\frac{\partial}{\partial t}+\tau_q^2\frac{\partial^2}{\partial t^2}\right)\left(\rho c_E\dot{\theta}+T_0\beta_{ij}\dot{u}_{i,j}-\rho Q\right). \tag{1.4.11}$$

The above Eq. (1.4.11) and the equation of motion (1.2.8) together constitute a coupled, complete and fully hyperbolic system of field equations for an anisotropic thermoelasticity theory that includes the dual-phase-lag effects. Thus, the thermoelastic theory based on the heat conduction law (Eq. (1.4.10)) is a hyperbolic thermoelastic model. If we set $(\tau_q)^2=\tau_\theta=0$, this theory reduces to the LS theory. In addition to the work reported by Tzou (2014), one can also consult the work on the topic of phase-lagging heat transfer (see Tien et al. (1998); Sharma (2005); Zhang et al. (2007)).

1.4.1.3 Green-Naghdi thermoelasticity theory (GN I, II, and III)

Green and Naghdi (1991b) proposed an alternate approach that differs considerably from the traditional approach. This methodology involves the formulation of constitutive equations by using a modified equation of energy balance which is combined effect of both the traditional energy balance equation and the entropy balance equation. A notable characteristic of this procedure is that the derivation of the fundamental equations didn't utilize an entropy production inequality. Instead, the inequality is employed solely to implement additional restrictions on the constitutive variables only after the establishment of fundamental equations. Green and Naghdi (1992; 1993) presented three theories that were referred to as GN-I, GN-II, and GN-III theories. In Green and Naghdi (1993) the authors have introduced a new constitutive variable ν, where, $\frac{\partial \nu}{\partial t} = \theta$ where, ν is termed as thermal displacement. The linear version of type-I model becomes identical with classical coupled theory of thermoelasticity, possessing the paradox of infinite speed of heat propagation. The type-II model shows that there is no dissipation of thermal energy in the body because the internal rate of production of entropy is considered to be identically zero here. This model allows undamped thermoelastic waves in the thermoelastic body. Therefore, this model is known as the theory of thermoelasticity without energy dissipation. The type-III model includes the previous two models as special cases. From Eq. 1.4.12, it is apparent that if Fourier conductivity is dominant $(K_{ij} \gg K^*_{ij})$, the equation reduces to the GN-I model. Conversely, when the Fourier conductivity is considerably lower $(K_{ij} \ll K^*_{ij})$, it reduces to GN-II model. This model shows dissipation of energy in general, and involves damped thermoelastic waves.

The laws of heat conduction employed in these thermoelasticity theories for the anisotropic medium can be stated as follows:

$$q_i = -K_{ij}\theta_{,j} - K^*_{ij}\nu_{,j}. \tag{1.4.12}$$

The anisotropic version of displacement equation of motion given by Eq. (1.2.8) along with heat conduction equation obtained on combining energy equation with Eq. (1.4.12) gives the set of governing equations for the thermoelasticity theory of type GN III. In the context of isotropic case, the linearized forms of the equations under GN II and GN III have been presented in Green and Naghdi (1992; 1993), respectively. Subsequently, qualitative analysis on the solution of equations for the case of anisotropic medium in the contexts of both the GN II and GN III thermoelasticity theories have been reported by Qunitanilla (1999; 2001; 2002a)

1.4.1.4 Three-phase-lag (TPL) thermoelasticity theory

The idea of phase-lag has been further generalized by Roychoudhuri (2007a) based on Green-Naghdi thermoelasticity theory. This extension involves the inclusion of three distinct phase-lag factors in the constitutive relation for heat conduction, as proposed by Green and Naghdi in their GN-III model. In this case, an additional phase-lag parameter is included to account for the gradient of thermal displacement. Furthermore, phase-lag parameters are also considered for the heat flux and temperature gradient terms. The heat conduction law, as adjusted in accordance with the TPL theory for a homogeneous and anisotropic medium is mathematically represented as follows:

$$q_i(\boldsymbol{x}, t+\tau_q) = -K_{ij}\theta_{,i}(\boldsymbol{x}, t+\tau_\theta) - K^*_{ij}\nu_{,i}(\boldsymbol{x}, t+\tau_\nu). \tag{1.4.13}$$

where τ_v represents the phase lag for the temperature gradient vector. The TPL model has significance in addressing issues pertaining to nuclear boiling, exothermic catalytic processes, phonon scattering, and phonon-electron interactions, among others.

1.4.1.5 Quintanilla-Moore-Gibson-Thompson (QMGT) thermoelasticity theory

In a recent publication, Quintanilla (2019) introduced a novel thermoelasticity theory known as the Moore-Gibson-Thompson (MGT) thermoelasticity theory. The newly suggested thermoelastic model may be seen as an extension of the Green-Naghdi (GN-III) model and the Lord-Shulman (LS) model. The present theory incorporates the new heat conduction model in the following manner:

$$q_i(\boldsymbol{x},t)+\tau_q\dot{q}_i(\boldsymbol{x},t)=-\left[K_{ij}\theta_{,j}(\boldsymbol{x},t)+K^*_{ij}\nu_{,j}(\boldsymbol{x},t)\right]. \tag{1.4.14}$$

The present theory has been enhanced by the inclusion of a relaxation time inside the heat equation of Green-Naghdi's III type theory. The relaxation parameter denotes the duration of slackening caused by the rapid transient impacts of thermal inertia. In their study, Quintanilla (2019) successfully established the well posedness and demonstrated the stability of the solutions in the context of three-dimensional cases. This thesis provides evidence for this theory which supporting the finite speed of heat propagation as shown in chapter 6.

1.4.2 Overview of generalized thermoelasticity theories based on classical Fourier's Law

Here we describe some generalized thermoelasticity theories which are developed by deriving new sets of constitutive relations where the Fourier's law of heat conduction is in general kept unchanged.

1.4.2.1 Green-Lindsay thermoelasticity theory (GL)

In the preceding sections, we examined that generally, a new thermoelasticity theory include thermal relaxation terms is derived based on different modifications of the Fourier's equation. However, Green and Lindsay (1972) proposed a unconventional theory of thermoelasticity including the concept of second sound, which is not based on any predetermined form of the heat conduction law. This theory is characterized by its simplicity and explicitness, and closely related to Muller's theory (1971). In this theoretical formulation, the preservation of the classical form of the entropy source and flux is uphold. Additionally, analogous to Muller's theory, the temperature rate is incorporated within the constitutive variables. It's formulation based on the concept of entropy production inequality, as suggested by Green and Laws (1972). This theory maintains the traditional form of the entropy flow and source, and, similar to Muller's theory that incorporates the temperature-rate as one of the constitutive variables. One notable characteristic of this theory is its adherence to the classical Fourier's law, provided that the material exhibits a center of symmetry at every location. Furthermore, it should be noted that even in the context of general anisotropy, the heat conduction equation within this theory does not include the term representing the rate of heat flow. Suhubi (1975) has independently devised a hypothesis that refers to as "***temperature-rate dependent thermoelasticity***". The derivation of this theory results in modified constitutive relations involving temperature-rate terms and two relaxation times. The constitutive relations for the anisotropic homogeneous medium for GL theory (Green and Lindsay (1972)) are given as follows:

$$T_0 \rho S = \rho c_E(\theta + \tau_0 \dot{\theta}) + \beta_{ij} T_0 e_{ij}, \tag{1.4.15}$$

$$\sigma_{ij} = C_{ijkl} e_{kl} - \beta_{ij}(\theta + \tau_1 \dot{\theta}), \tag{1.4.16}$$

$$q_i = -\left(c_i \dot{\theta} + K_{ij} \theta_{,j}\right). \tag{1.4.17}$$

Here, τ_0 and τ_1 are the two thermal relaxation time parameters included in the constitutive relations to incorporate the temperature rate terms. Using the aforementioned Eqs. (1.2.1), (1.2.2) and (1.2.6), often referred to as the governing equations for GL theory.

It is noteworthy that in this context, in contrast to LS model, the magnitudes of stresses, entropy, and heat flux are influenced not only by the change in temperature but also by the temperature rate term. In addition, the process of heat conduction in this framework does not include the concept of flux rate term. The two theories, namely LS and GL, exhibit distinct structural characteristics, and it is not possible to derive one theory as an individual case of the other. The incorporation of the second sound phenomena into LS model is primarily achieved via the flux-rate term. On the other hand, in the GL model, it is the temperature rate-term that assumes a crucial role. It has been observed that by neglecting the flux-rate term from the constitutive Eq. (1.4.1), the CTE model is recovered by LS model. By neglecting the term $\dot{\theta}$ from the constitutive equations Eqs. (1.4.15-1.4.17), GL model also reduces to CTE model.

1.4.2.2 Modified Green-Lindsay thermoelasticity theory (MGL)

In the development of thermoelasticity theories, such as classical thermoelastic model, the Green-Lindsay model, and several other models, it has often been considered by researchers that the strain rate component are relatively of negligible magnitude. Consequently, this term has been neglected from the constitutive equations. Nevertheless, in some scenarios, such as explosive welding, projectile penetration, rock blasting, and high-speed machining, there is a significant occurrence of high strain rate, often on the scale of $10^4 s^{-1}$. Consequently, it is imperative to consider the strain rate term and not disregard its influence. In a recent study, Yu et al. (2018) proposed a modification to the GL model by including a second-order tensor in the formulation. This tensor is dependent on both strain and its rate component. The constitutive equations are dependent on both temperature rate and strain rate factors. The present model addresses the issue of discontinuity in the displacement field. Several investigations have found that the displacement field for transient motion in GL theory exhibits discontinuity (see Chandrasekharaiah and Srikantiah (1986), Dhaliwal and Rokne (1989), Chatterjee and Roychoudhuri (1990), Ignaczak and Mr' owka-Matejewska (1990)). The presence of discontinuity in the displacement field indicates that there is a penetration of one component of the matter into another, therefore violating the continuum hypothesis as described by Chandrasekharaiah (1998). The theory developed by Yu et al. (2018), the resulting modifications in constitutive relations are described below:

$$T_0 \rho S = \rho c_E(\theta + \tau_0 \dot{\theta}) + \beta_{ij} T_0 (e_{ij} + \tau_0 \dot{e}_{ij}), \tag{1.4.18}$$

$$\sigma_{ij} = C_{ijkl}(e_{kl} + \tau_1 \dot{e}_{kl}) - \beta_{ij}(\theta + \tau_1 \dot{\theta}). \tag{1.4.19}$$

Eq. (1.4.18) and Eq. (1.4.19) denote the incorporation of the strain-rate term and temperature-rate term into the stress-strain-temperature and entropy constitutive relations, correspondingly. Moreover, in their article, through the utilization of a one-dimensional problem, the authors have effectively revealed the substantial implications of this novel theory. This theory possesses the potential to rectify the inherent limitation associated with the occurrence of a discontinuity in displacement field observed in LS and GL models. In that article, a comprehensive analysis has been conducted to compare the outcomes obtained within the framework of MGL thermoelasticity theory with those derived from GL theory and GN (II, III) thermoelasticity theories.

1.5 Bio-thermoelasticity Theory

In the context of biological tissue, even minor fluctuations in stress and temperature have the potential to induce deviations in the production rate of hormones, render proteins inactive, and even influence the immune system (Lau et al. 1995). In the field of modern medicine and everyday activities, biological tissues encounter numerous thermal shock loads, such as laser exposure, ultrasound treatment, and burn injury. These thermal shock loads induce distinct thermal and mechanical reactions in biological tissues, consistent with predictions by generalized thermoelastic models. These models account for a correlation between temperature and volumetric strain rate. Without any uncertainty, a precise understanding of the mechanism behind thermoelastic responses in biological tissue is crucial in order to enhance the treatment effectiveness of thermal therapies and prevent excessive heat injuries. The process of heat transfer in biological tissue, such as the skin, is a complex phenomenon due to the involvement of various mechanisms. These mechanisms include heat conduction within solid tissues, convection between the blood and solid tissues, metabolic heat generation, and blood perfusion. The comprehensive investigation into the intricate mechanisms of bio-heat transfer within the skin tissue holds immense importance and offers valuable insights for the application of heat therapy in the realm of medical treatment. The initial formulation of bio-heat transfer in biological tissue was introduced by Pennes, who formulated a model based on the principles of Fourier's law. The widespread application of Pennes model in the field of bio-heat transfer can be attributed to its inherent simplicity in analysis. Nevertheless, the bio-heat conduction model possesses certain inherent disadvantages, primarily stemming from its assumption of an infinite thermal propagation speed. In the context of living biological tissues, it has been consistently observed that heat exhibits a finite speed of propagation. This phenomenon is particularly noteworthy due to the presence of highly non-homogeneous inner structures within these tissues. According to Pennes model (1948), the energy conservation equation of classical bio-heat transfer could be written as

$$\rho_t c_t \frac{\partial T}{\partial t} = -q_{i,i} + \rho_b w_b c_b \left(T_b - T\right) + Q_{met} + Q_{ext}.$$

The symbols ρ_t and c_t denote density and specific heat, respectively and T is the temperature of tissue. w_b is the perfusion rate of blood, which serves as a measure of the volume of blood that enters a unit volume of skin tissue within a given unit of time. c_t and ρ_b represent specific heat and density of blood, respectively. T_b is the reference temperature of arterial blood entering skin tissue is maintained at a constant value equivalent to the inner body temperature. The heat exchange between arterial blood and the local tissue is represented by the second term on the right-hand side of the equation, $\rho_b w_b c_b \left(T_b - T\right)$. Q_{met} and Q_{ext} represent the quantities of metabolic heat generation in the skin tissue and the heat source caused by external heating, respectively.

1.6 Literature Review

The field of thermoelasticity theory has made remarkable progress, leading to extensive research conducted in both experimental and theoretical contexts. In order to address the limitations of conventional thermoelasticity theory and get accurate outcomes due to thermomechanical loading, researchers have made progressive and continuous improvements in the development of several generalized thermoelasticity theories. Numerous academic review articles and books have documented these improvements to certain extents. Here are a few of them as being worth mentioning: Nowacki (1969; 1975), Chandrasekharaiah (1986; 1998), Joseph and Prezios (1989b), Straughan (2011), Parkus (2012), Hetnarski and Ignaczak (1999), and Ignaczak and Ostoja-Starzewski (2010). Additionally, Picard has established the corresponding solution theory for these problems. To access further research and analysis pertaining to a variety of issues within different thermoelasticity theories, it is recommended to consult the doctoral thesis of Roushan Kumar (2010), Rajesh Prasad (2012), Shweta Kothari (2013), Rakhi Tiwari (2017), Bharti Kumari (2017), Shashi Kant (2018), Anil Kumar (2018), Manushi Gupta (2021), Om Namah Shivay (2021), Harendra Kumar (2022), Komal Jangid (2023). However, in the following, we attempt to provide a literature review conducted to ascertain the current level of knowledge in the field of generalized thermoelasticity theories, relevant to the present study.

The framework of generalized thermoelasticity theory proposed by Lord and Shulman (1967), is regarded as one of the most widely accepted suitable improvements to classical theory. Subsequently, Fox (1969) and Lord and Lopez (1970) have examined the phenomena of thermoelastic disturbances and wave propagation in materials subjected to very low temperatures. The study by Puri (1973), an analysis had been conducted on the phase velocity, specific loss, and amplitude ratio of plane waves propagating through an unbounded elastic medium. The author has derived the approximations for these quantities for both extremely low and high-frequency values and recognized that these fundamental quantities shows the behaviour of wave-like characteristics of elastic and thermal disturbances. The analysis of thermoelastic reactions inside a cylindrical media has been conducted by Wadhawan (1973) who has considered a two-dimensional infinite circular cylinder of isotropic medium and solved the harmonic problem of thermoelasticity under small vibration. Furthermore, the issue of thermal shock in the context of an infinite plate and a long bar is examined by Kolyano and Semerak (1973), Shashkov and Yu (1977) and Szekeres (1980). Dhaliwal and Sherief (1980) has provided comprehensive evidence to support the LS model's applicability to anisotropic media. The uniqueness of the problem of stress-heat-flux initial boundary conditions is shown by Ignaczak (1979). Chandrasekharaiah (1986) provided a detailed description for the domain of influence results, uniqueness of the solution, variational principle, and reciprocity theorem within the framework of LS model. The study additionally dealt with the thermal shock issue within the LS model. Sherief (1987) demonstrated the uniqueness of the LS thermoelasticity theory over a broad range of anisotropic media and conducted an investigation into the stability of the proposed model. Chandrasekharaiah (1988) incorporated the thermo-piezoelectricity effect within the LS theory, highlighting the uniqueness of the solution within the same framework. Anwar and Sherief (1988b) addressed a one-dimensional problem within the framework of the LS model. They used the state space technique to analyze the problem and provided numerical findings for two distinct media: a layered domain and a half-space domain. In contrast, Furukawa et al. (1990) derived the transient solution for a one-dimensional problem occurring in an unbounded media with a circular hollow cylinder. Their study highlighted the influence of relaxation time on the physical fields involved. Sharma and Singh (1989) performed a detailed investigation on plane harmonic thermoelastic waves in an anisotropic homogeneous material. Their research aimed to demonstrate the presence of four distinct waves: a quasi-longitudinal wave, two quasi-transverse waves, and a thermal wave. The research conducted by Chand et al. (1990) included an integrated investigation of the thermal, mechanical, and magnetic field aspects inside a uniformly rotating elastic half-space. Furthermore, Mukhopadhyay et al. (1991) conducted a study in an infinite solid containing a spherical cavity, when the internal boundary of the cavity undergoes a sudden increase in temperature and dynamic pressure. The investigation of half-space issues has been conducted by various researchers, such as Chattopadhyay et al. (1982) and Ramamurthy and Sharma (1991),

within the framework of the same model subjected to different boundary conditions. The fundamental solution for this case in which an impulsive body force and heat source are applied at a single location in an infinite domain is obatined by Wang and Dhaliwal (1993). Subsequently, several other problems on this model of thermoelasticity have been discussed by Sharma and Chand (1991; 1996), Wadhawan (1972; 1973), Roychoudhuri and Bhatta (1983), and Sharma (1987). Mukhopadhyay et al. (1991), Misra et al. (1987), Banerjee and Roychoudhuri (1995), and Mukhopadhyay and Bera (1989) have tackled the challenges associated with viscoelastic mediums using the LS theory are also worth to be mentioned here. Picard (2009) has proposed a structural formulation for linear material laws inside the framework of classical mathematical physics. This formulation incorporated a variety of evolutionary problems, including several initial boundary value problems in classical mathematical physics. Later on Mukhopadhyay and Kumar (2016) continued to expand this work.

Green and Lindsay (1972) suggested a new approach to extend Biot's theory of thermoelasticity. This technique is centered around the modified Clausius inequality (Müller (1967)) and firmly rooted in the principles of irreversible thermodynamics. Since the beginning, this hypothesis has garnered significant academic attention. In their study, Green and Lindsay (1972) have also examined the propagation of acceleration waves within the framework of linear thermoelasticity of their model. Moreover, Boschi and Ieşan (1973) employed this model to a homogeneous micropolar continuum. This extension was achieved by the use of derivations that relied on invariance criteria under superposed rigid body movements. In the research conducted by Agarwal (1978), a homogeneous and isotropic thermoelastic half space was examined in order to investigate the behavior of surface waves within the framework of LS and GL theories. Agarwal (1979) conducted further research on the propagation and stability of time-dependent planar harmonic waves within the context of these models. The domain of influence theorem for linear in the context Green-Lindsay thermoelasticity theory was established by Ignaczak (1978). The variational principle, uniqueness theorem, and Betti-Rayleigh-type reciprocity theorem for anisotropic media were established by Chandrasekharaiah and Srikantaiah (1983). In their investigation, Tao and Prevost (1984) used the perturbation approach to examine wave propagation within (GL) theory. They further investigated the impact of the relaxation time parameters on the observed phenomena. It is also worth mentioning the work conducted by Ignaczak (1985) in the presence of a heat source in an unbounded media. The investigation revealed the presence of a jump discontinuity in the displacement function. Additionally, Chandrasekharaiah and Srikantiah (1984) conducted an investigation on the decay coefficient, energy loss, and phase velocity pertaining to an isotropic homogeneous rotating solid. Erbay and Suhubi (1986) investigated the behavior of longitudinal waves in the context of an infinite cylinder with a stress-free lateral surface and maintained at a constant ambient temperature. Chen and Wang (1988) used a hybrid approach using finite element analysis and Laplace transform to investigate a dynamic problem within the frameworks of LS and GL thermoelasticity theories. Ignaczak (1991) conducted a comprehensive review of domain influence theorems within the framework of LS and GL thermoelasticity theories. In contrast, Hetnarski and Ignaczak (1993) studied the one-dimensional Green's function for a planar heat source in both infinite and semi-infinite spaces. Sherief (1992) examined the fundamental solution in the context of a spherically symmetric point heat source within the framework of the GL theory. Additionally, he conducted a detailed comparison between the results under classical and LS thermoelastic models by using the data of copper material. Chandrasekharaiah and Murthy (1994) proposed a formulation for a mixed initial and boundary value problem via the unified governing equations of the Lord-Shulman and Green-Lindsay models. Sinha and Elsibai (1996) conducted a study to investigate the impact of two relaxation times on the reflection of two distinct kinds of incident waves. Through numerical analysis, they examined the changes in the reflection coefficient and energy partition in relation to the angle of incidence. Misra et al. (1996) conducted a study on the thermoelastic interactions within the framework of the LS theory. They used a state-space technique to investigate the effects of ramp-type heating. Sharma (1997) conducted a study to investigate the effects of two boundary conditions, namely thermal shock and normal load, on the LS and GL models. The state-space technique was used to analyze these impacts. Ezzat and El-Karamany (2002) employed the existing framework of generalized thermoelasticity proposed by Lord-Shulman and Green-Lindsay to present a comprehensive theory of thermoviscoelasticity. The authors examined the theory pertaining to anisotropic media in order to establish the uniqueness of solutions and the reciprocity theorem. Othman (2003) used two different approaches to analyze the GL thermoelasticity model. In the first study, the author demonstrated the impact of temperature-dependent elastic modulus by using the state-space technique. Subsequently, the author gave the precise formulation of physical fields for two distinct situations utilizing normal mode analysis within the context of a rotating elastic medium under linearized theory. El-Maghraby (2005) conducted a study on a two-dimensional thick plate that had a traction-free top surface. The plate was exposed to a known temperature distribution, while the bottom surface was thermally insulated.

The analysis was carried out by using the Lord-Shulman theory and the Green-Lindsay theory. In addition, Youssef (2006) conducted a numerical analysis on a two-dimensional half space problem involving an infinite medium with a cylindrical cavity, as here the medium was subjected to ramp-type heating. In their study, Bagri and Eslami (2007a) extended the examination of GL thermoelasticity theory to encompass a functionally graded sphere. This sphere was subjected to thermal shock on its inner surface, which was loaded symmetrically. On the other hand, Abbas and Abo-el-nour (2008) employed the finite element method to explore thermoelastic variations in an infinite orthotropic elastic medium containing a cylindrical cavity. In the study conducted by Lotfy (2012), an investigation was carried out on an issue involving a two-dimensional mode-I fracture in a material that is thermoelastic and reinforced with fibres, using the GL theory. The impact of temperature-rate on the thermoelasticity theory in the presence of the gravitational field was investigated by Othman et al. (2013). Youssef and El-Bary (2014) conducted a study to investigate thermomechanical interactions within the framework of the GL theory. The authors compared the behavior of field variables across four distinct thermoelasticity theories. Filopoulos et al. (2014) have proposed an advanced generalized linear theory for the thermoelastic medium with microstructures. In a study conducted by Zenkour (2015), an analysis was performed to investigate the impact of thermal shock inside a three-dimensional thermoelastic medium. This study assessed the outcomes of this analysis under various thermoelasticity theories. The study conducted by Aouadi and Moulahi (2015) focused on investigating the best decay rate within the context of the one-dimensional thermoelastic issue, specifically within the framework of GL thermoelasticity theory. In a research conducted by Abbas (2015), an analytical solution was obtained to investigate the free vibration issue of a thermoelastic hollow sphere. The study conducted by Ailawalia et al. (2016) investigated the impact of an internal heat source on a microelongated thermoelastic half space using the GL theory. The researchers generated analytical equations for the field variables.

The authors also emphasized the distinctions between the GL theory and several other thermoelasticity theories. The investigation conducted by Wei et al. (2016) focused on the analysis of the reflection and refraction phenomena of P waves occurring at the interface between a thermoelastic media and a porothermoelastic medium. Chyr and Shynkarenko (2017) have presented a rigorous analysis of the well-posedness of the dynamical issue of thermoelasticity within the framework of the GL theory. In recent years, there has been a significant amount of research conducted on various thermo-mechanical issues with the aim of investigating the potential application of GL theory. The studies conducted by Ezzat and El-Bary (2017), Abd-alla et al. (2017), Kumar et al. (2017), Magaña et al. (2018), Aouadi et al. (2019), Guo et al. (2019), Quintanilla et al. (2023), Marin et al. (2020), Sherief et al. (2022), and Sarkar (2021) are relevant to the topic at hand.

The thermoelasticity theories proposed by Green and Naghdi (1991b; 1992; 1993) have garnered significant attention from researchers seeking to comprehend the novel generalized thermoelasticity theories (GN-I, GN-II, and GN-III theories). These frameworks include thermal displacement as an additional constitutive variable. Chandrasekharaiah (1996a) established an alternate formulation of the GN-II thermoelasticity theory, which utilizes entropy heat flux as a means to demonstrate the uniqueness theorem. Chandrasekharaiah (1996c) has proposed an alternative methodology for examining the uniqueness of solution in the GN thermoelasticity theory. Additionally, Chandrasekharaiah (1996b) has conducted a study on a linear thermoelastic problem in one dimension to explore the propagation of plane waves within the framework of thermoelasticity theory without energy dissipation (GN-II theory). In their work, Chandrasekharaiah and Srinath (1997c) analyzed the thermoelastic interactions that arise from a point heat source within the framework of the GN-II theory. The investigation of wave propagation in a rotating thermoelastic body was also conducted by Chandrasekharaiah and Srinath (1997b). Additionally, Chandrasekharaiah and Srinath (1997a) used the GN-II thermoelasticity theory to investigate the thermoelastic properties inside an axisymmetric unbounded medium with a cylindrical cavity. In their study, Dhaliwal et al. (1997) successfully addressed the thermoelastic interactions within the framework of the GN-III theory. However, it was observed that this theory also exhibits the limitation of unlimited heat propagation speed. Ieşn (1998) and Quintanilla (1999; 2002a) established some theoretical findings within the framework of the GN thermoelasticity theory. Ciarletta (1999) proposed a micropolar thermoelasticity theory that is based on GN theory. Additionally, Ciarletta presented a solution using a Galerkin-type approach inside this theoretical framework. Several other studies have been conducted by various researchers by using the theories of Green and Naghdi. We specially mention the work carried out by Svanadze et al. (2006), Chandrasekharaiah and Srinath (2000), Misra et al. (2000), Wang and Slattery (2002), Sharma et al. (2003), Quintanilla and Straughan (2004), Roychoudhuri and Bandyopadhyay (2005), Bagri and Eslami (2007b), Mallik and Kanoria (2008), Abbas and Othman (2009), Chiriţă and Ciarletta (2010), etc. In their study, Mukhopadhyay and Kumar (2010) used a state-space methodology to address a thermoelastic problem within the framework of the GN-III model. The investigation conducted by Tiwari and Mukhopadhyay (2017) focused on the analysis of wave propagation inside an electromagneto-thermoelastic medium using the GN-II theory. In a study conducted by Abbas (2018), the investigation focused on the analysis of free vibration characteristics shown by a nanobeam resonator. The study specifically considered the use of the GN thermoelasticity theory . El-Attar et al. (2019) conducted a study to obtain the impact of phase-lag on the GN model for an electro-thermoelastic material. Abouelregal (2020) has proposed an extension to the existing GN thermoelasticity theory by including additional factors related to higher order temporal differentials and phase-lags. The research conducted by Sarkar et al. (2020b) explored the impact of laser pulses on transient waves within the framework of GN thermoelasticity theory by taking into account the non-local effect.

In recent years, the GN thermoelasticity theory has been explored by Zenkour (2022), Chirilă et al. (2022), and El-Attar (2019) and Hendy et al. (2021) to discuss the results due to various thermomechanical interactions.

Chandrasekharaiah (1998) proposed the DPL thermoelasticity theory, which represents an extension of the heat conduction theory with the incorporation of two phase-lag parameters (Tzou (1995b; 1995a)) in the governing equations. In the review paper, Chandrasekharaiah (1998) provided a comprehensive analysis of the DPL theory and other theories pertaining to hyperbolic thermoelasticity. In their study, Hetnarski and Ignaczak (1993) presented an analytical methodology for the generalized thermoelasticity theories. They proceeded to compare the outcomes obtained from previously established theories with the DPL thermoelasticity theory. Quintanilla (2002b; 2003) conducted qualitative analysis on the DPL theory in order to examine the stability of solution under the theory. The findings of their research demonstrated that the DPL heat conduction model does not possess unconditional stability. Additionally, the stability condition for the one-dimensional problem has been developed by the author in these studies. Furthermore, Horgan and Quintanilla (2005) conducted a detailled study to examine the spatial behavior of solutions within the context of the DPL model. The thermoelastic reactions inside a composite plate under the DPL thermoelasticity theory were analyzed by Al-Huniti and Al-Nimr (2005). In a study conducted by Roychoudhuri (2007b), the thermoelastic wave inside an elastic half space was examined using the DPL theory. The investigation revealed the presence of two distinct waves in the solution. Ghazanfarian and Abbassi (2009) investigated the impact of phonon scattering on the boundary within the framework of the DPL theory, with the aim of simulating heat conduction at the micro and nano scales. Prasad et al. (2010) examined the harmonic plane waves in the context of dual-phase-lag thermoelasticity theory. The authors presented the asymptotic expressions of wave characteristics, and numerically expressed the results for very high and low frequency values. Mukhopadhyay et al. (2011a) established the finite domain of influence and proved the occurence of finite speed of waves under the DPL model. They showed the dependence of the domain of influence on two phase-lag parameters and thermoelastic coupling constant. Abouelregal (2011) explored thermoelastic variations in the isotropic solid sphere with constrained boundary subjected to constant heat-flux. They showed the effects of phase-lags, τ_q and τ_θ, and compared the results predicted by GL and DPL theories through graphical representation. Singh (2013) analyzed thermoelastic plane wave propagation in a transversely isotropic medium under DPL thermoelastic model. Zenkour et al. (2013) considered the solid half-space with temperature-dependent material properties to study the reflection of thermoelastic waves under dual-phase-lag thermoelasticity theory. Further, Kothari and Mukhopadhyay (2013) and Mukhopadhyay et al. (2014) have studied uniqueness, reciprocity theorem, and variational principle for the linear theory of this model for anisotropic medium and showed the behavior of thermoelastic waves when the medium is subjected to thermal shock. On the other hand, El-Karamany and Ezzat (2011) discussed the same theorems with a different approach for inhomogeneous anisotropic solid and also proved a continuous dependence of the solution on initial data and supply terms by using dissipative inequality. Abouelregal and Abo-Dahab (2015) picked up a two-dimensional problem for the rigidly fixed surface when subjected to thermal shock and compared the results with that of LS and classical thermoelasticity theories. In the context of DPL thermoelasticity, Sarkar (2017) used normal mode analysis and an eigenvalue approach to investigate a three-dimensional half space problem with temperature-dependent material properties and a stress-free boundary subjected to a time-dependent heat source.

Modified Green-Lindsay (MGL) theory or strain and temperature-rate dependent theory (2018) is a recently proposed thermoelastic model that attempts to overcome the drawback of discontinuity in the displacement field and involving the effect of stain and temperature rate terms in the governing equations. This theory has gained attention from the researchers who have investigated this theory in the contexts of different thermoelastic problems and reported some interesting observations on this theory. Quintanilla (2018) presented some qualitative results on this theory, including continuous dependence of the solution on initial conditions and Phragman-Lindelof alternative for the spatial behavior of the solution. Gupta and Mukhopadhyay (2019) derived the general solution of the thermoelastic system of governing equations in terms of metamorphic functions with the help of the representation theorem of Galerkin type solution under MGL theory. Sarkar et al. (2020a) investigated the reflection and wave propagation in an isothermal stress free surface of a thermoelastic medium in this context. Sarkar and De (2020) have investigated the reflection and time harmonic wave propagation under different thermoelastic mediums and observed the existence of longitudinal wave and one vertically shear type wave (SV wave). Gupta and Mukhopadhyay (2021) have considered three different theories: LS, GL and MGL to study the harmonic plane wave propagation and highlighted the differences under these thermoelasticity theories. Shakeriaski et al. (2021a) have considered the MGL theory to demonstrate the implementation of a nonlinear numerical method for solving the coupled thermoelastic problems and also validated the numerical results by comparing them with the analytical solution of the corresponding problem. Recently, Shakeriaski et al. (2021b) have reported the advancement of the recent thermoelasticity theories (including MGL theory) and their applications. Mohamed et al. (2021) examined the thermal effect on elastic solid due to absorption of the Laser pulse radiation under MGL theory. Kalkal et al. (2022) investigated the reflection of thermoelastic plane waves in an anisotropic, rotating, fiber-reinforced medium having variable thermal conductivity and compared the result with GL model. Zhang et al. (2023) studied the effects of the relaxation times and the interface effect on the reflection and transmission natures of coupled thermoelastic waves at the imperfect interface between two elastic, isotropic, isothermal solid half-spaces based on the modified Green-Lindsay model. Although the MGL has been studied for various thermoelastic problems. However, it is a topic of active interest of researchers, and several thermoelastic problems are yet to be investigated under this theory.

A recently published advancement in the field of thermoelasticity has been introduced by Quintanilla (2019; 2020). This recent development involves the merger and generalization of the Lord-Shulman and Green-Nagdi thermoelasticity theory of type III. The theoretical framework presented in this study is derived from the Moore-Gibson-Thompson (MGT) heat conduction model, which has been linearized. Consequently, the thermoelastic model proposed herein is referred to as the **Quintanilla-Moore-Gibson-Thompson** (QMGT) model. The MGT model is a widely employed theory for the mathematical representation and analysis of high amplitude sound waves. The comprehensive investigation conducted by Kaltenbacher et al. (2011) focused on the examination of the fully non-linear variant of the (MGT) equation. The researchers successfully established the well-posedness of the equation and determined the exponential decay rates that appear in the context of high-intensity ultrasound. The spectral analysis for this model was also conducted by Marchand et al. (2012). Lasiecka and Wang (2015) conducted a comprehensive investigation into the influence of memory on linear MGT systems. In their study, Kumar and Mukhopadhyay (2020) employed the (QMGT) model to gain insights into the dynamic characteristics exhibited by a microbeam resonator with a simply supported configuration. Singh and Mukhopadhyay (2023) developed the variational formulation of the microstructure-dependent Timoshenko beam model by considering Hamilton's principle and modified couple stress theory (MCST) in the context of QMGT model. Jangid and Mukhopadhyay (2021) established the domain of influence theorem for a natural stress-heat-flux problem in the QMGT thermoelasticity theory. Kumar and Mukhopadhyay (2020) presented an analytical method for analyzing thermoelastic damping and dynamic behavior of microbeam resonators based on the Moore-Gibson-Thompson (MGT) generalized thermoelasticity theory. Furthermore, Kumar and Mukhopadhyay (2023) intended to examine thermoelastic damping in nanobeam resonators using the modified couple stress theory and Eringen's nonlocal elasticity theory within the context of QMGT model.

For a comprehensive understanding of the thermal behavior observed during thermal treatments, the experiment emerges as the fundamental approach (Xu and Lu (2009)). However, performing a comprehensive experiment generally poses challenges due to the various kinds of tissues and the complicated dynamics of the physical and biochemical processes involved. Consequently, in order to study the variation of temperature and the resulting thermal damage in tissues subjected to instantaneous heating, researchers have employed a theoretical approach accompanied by mathematical models (Zhou et al. (2007); Liu (2015); Liu and Chen (2016)). The Pennes model of bio-heat transfer model (Pennes (1948)) has been used extensively in initial research due to its simplicity. This model has its foundation on the classical Fourier's law. Several modified models have been proposed to assess the thermal behavior occurring in living tissues, including the CV model and the dual-phase-lag model (DPL). These models aim to provide a more comprehensive understanding of the complex thermal processes within biological systems. Numerous studies have been carried out to investigate the thermal behavior within skin tissues using other modified models of bioheat transfer (Liu et al. (1995; 1999); Ahmadikia et al. (2012); Fazlali and Ahmadikia (2013); Hobiny and Abbas (2018); Dutta and Kundu (2018); Liu and Chen (2010; 2016); Lin and Li (2016); Askarizadeh and Ahmadikia (2014; 2015); Zhang et al. (2017); Zhang (2009)). When considering the structure of the skin, it can be imagined as a layered material. Xu et al. ((2008b; 2008a; 2008c)) formulated a detailed theoretical framework to better understand the connection between thermal and mechanical phenomena in the skin. Particularly, the authors remarked here that thermal stress could play a potential role in the sense of thermal pain. Further, in a series of studies, Li et al. ((2018a; 2018b; 2019)) investigated more thoroughly the influence of temperature-dependent properties on heat transfer and the resulting mechanical response in skin tissue. To accurately describe the thermoelastic behavior occurring in the tissue, the researchers employ various theoretical models, namely the generalized thermo-elastic theory incorporating the GN model, fractional order model, and DPL model. Each model is employed separately to govern the thermoelastic phenomena observed in skin tissue. Numerous analytical efforts have been undertaken in previous studies (Liu (2015); Liu and Chen (2016); Antaki (1998a; 1998b); Liu et al. (1999); Fazlali and Ahmadikia (2013); Lin and Li (2016); Li et al. (2018a)) to ascertain the efficacy of employing exact solutions for bioheat transfer models and generalized biothermoelastic models in describing thermal and related thermo-mechanical responses. Some studies (Liu et al. (1999); Ahmadikia et al. (2012); Fazlali and Ahmadikia (2013); Askarizadeh and Ahmadikia (2014); Askarizadeh and Ahmadikia (2015)) have also been made to obtain the exact solutions in certain situations under the assumption of constant properties.

1.7 Objective of the Thesis

The major objective of this thesis is to investigate the impact of various heat sources in thermoelastic medium within the framework of some newly established generalized thermoelasticity theories. The thermo-mechanical investigation through different theories is worthwhile to gain a better understanding of the thermoelasticity theories from the perspective of many features that pertain to their applicability to real-world situations. This work can be categorized into two primary portions based on different thermoelastic models. The beginning of this portion discusses the MGL model based on generalized dissipation inequality and its application to bio-thermoelasticity field. In the last portion, the characteristic features of another newly developed non-Fourier heat conduction based thermoelastic model (QMGT) are illustrated by considering the problem of line heat source on a thermoelastic body.

The QMGT and MGL thermoelasticity theories are presented very recently and are in their early developmental phase. From a mathematical perspective, these theories are the generalized versions of GN-III and GL models. The study is focused on identifying the numerous aspects of these theories and aims to explore the advantages and disadvantages in connection with various coupled thermoelasticity theories found in existing literature. The formulation of boundary integral equations for MGL model is derived in physical domain by using Betti's reciprocity relation approach. Such formulation plays important role in the implementation of boundary element method in finding the numerical approximation of the boundary value problems. Furthermore, a comprehensive investigation has been conducted to examine the impact of various types of heat sources and emphasizing comparisons between the predictions made by these current theories and those established earlier in literature. In addition, we have developed the mathematical framework corresponding to MGL model to examine the heat transmission mechanism as well as the heat-induced mechanical reaction on skin tissue when subjected to thermal load.