PREFACE

An artificial neural network emulates the operations of biological neural networks, with neurons acting as the basic units for processing information. These networks find extensive use in information processing, healthcare, and weather prediction, playing pivotal roles in diverse applications. Before utilizing any neural network model, evaluating the system's stability and synchronization is essential. The stability and synchronization of neural network models entail multiple dimensions that require thorough examination. This thesis delves into numerous facets of these considerations for integer and fractional order systems, offering valuable perspectives on the stability and synchronization of neural network models.

The thesis has seven chapters those explore stability and synchronization issues in neural networks, covering integer and fractional order systems. Chapter 1 serves as an introduction, providing an overview of artificial neural networks and tracing their historical development. It delves into the various types of neural networks and introduces the concept of delay differential equations. Additionally, this chapter establishes key definitions and methodologies, which have been employed throughout the thesis, laying a solid foundation for the subsequent discussions.

Chapter 2 explores quasi-projective synchronization of non-identical complex-valued neural networks, focusing on the Cohen-Grossberg neural network, a broader class that includes the Hopfield neural network. This chapter employs a direct method instead of the real separation method, offering efficiency and simplicity. A significant contribution lies in estimating the bound of the synchronization error. This bound is influenced by various parameters, including the system parameters and the projective coefficient. During practical implementation, the synchronization error does not approach zero but fluctuates within a small range.

Chapter 3 discusses the quasi-projective synchronization within quaternion-valued neural networks, where the master and response systems exhibit non-identical characteristics. The model accounts for time-varying delays and interaction terms, enhancing its complexity. Despite this complexity, it potentially demands fewer parameters than alternative approaches like the real separation method, the plural separation method, reducing computational burden and memory usage. The direct method establishes various criteria for achieving quasi-projective synchronization, while the Lyapunov stability theory is utilized to estimate error bounds.

Chapter 4 delves into the fixed-time synchronization of the hypercomplex neural network featuring a mixed-time-varying delay. This work specifically addresses higher-dimensional neural networks with $n+1$ dimensions. The core motivation behind this exploration is to provide insight into fixed-time synchronization, where the system achieves synchronization within a known finite time frame, with the upper bound of the settling time remaining unaffected by initial conditions. To facilitate this, a Lyapunov functional is developed alongside a novel controller to obtain the desired synchronization. The chapter also includes a comparative analysis of settling times employing various lemmas, with empirical evidence demonstrating that the settling time obtained from Lemma 1 is more accurate than that from Lemma 2.

Chapter 5 explores the Lagrange stability criteria for hypercomplex neural networks with time-varying delays. To address the challenges posed by the non-commutativity and non-associativity of hypercomplex neural networks, those are decomposed into $n + 1$ equivalent real-valued neural networks. By applying Lyapunov's theory, Lagrange stability is achieved, which ensures that the system's solutions remain bounded over time. Additionally, a globally attractive exponential set is obtained, indicating the regions of convergence within the system.

Chapter 6 delves into the concept of function-projective Mittag-Leffler synchronization among non-identical fractional-order neural networks. The stability analysis utilizes a pre-existing lemma about Lyapunov functions within fractional-order neural network systems. To achieve function-projective Mittag-Leffler synchronization, a non-linear controller is formulated. This chapter provides a detailed examination of how the Mittag-Leffler function, which generalizes the exponential function and captures memory effects in fractional-order calculus, plays a crucial role. By defining the Caputo derivative and applying Lyapunov stability theory, the chapter elaborates on the stability outcomes concerning the function projective Mittag-Leffler synchronization scheme. It shows how adjusting the scaling factor enables the master and response systems to synchronize towards a predefined scaling function for fractional-order neural networks.

In Chapter 7, the focus is on the Lagrange α-exponential synchronization of non-identical fractional-order complex-valued neural networks. The study explores various conditions conducive to achieving Lagrange α-exponential synchronization and α-exponential convergence of these networks, leveraging additional inequalities and the Lyapunov method. A significant aspect of the chapter is its detailed analysis of the structure of the α-exponential convergence ball, demonstrating how the convergence rate correlates with the system's characteristics such as the network's connectivity, weight distributions and the fractional order of the differential equations.