2 Literature Survey

This chapter provides information about the work that is happened in past in free convection. The literature review examines the salient findings in this area of free convection over a flat plate whose orientation varies from horizontal to vertical has been presented. The gaps in the literature that require additional investigation are mentioned, and the motivation for the current research has been outlined in light of the above.

2.1 Flow over a heated flat plate

Free convection is involved in the various applications mentioned in chapter 1, and developing those systems requires a fundamental understanding of this phenomenon. The phenomenon of free convection above a heated flat plate is recognized as the most fundamental topic; therefore, it is available in most textbooks on convective heat transfer [4,17]. Hence, it is necessary to examine the flow over a heated flat plate in order to comprehend the mechanism of free convection. In this section, we have discussed the mechanics of free convection when fluid flows along a heated flat plate. Based on the orientation of the plate, the present section is divided into three sub-sections which deal with: vertical ($\phi = 90°$), inclined ($30° \leq \phi \leq 90°$), and slightly inclined to horizontal ($0° \leq \phi \leq 10°$).

2.1.1 Flow over a vertical flat plate

This section examines a simple case of boundary layer (BL) formation on a heated vertical flat plate (VFP). As the plate heats up, the fluid near the plate also heats up more than the fluid further away. So, fluid close to the plate is less dense than fluid further away. As a result of the density difference, buoyancy forces induce in the vertical direction, as shown in figure 2.1. The buoyancy forces cause hot fluid to ascend vertically, entraining fluid from the quiescent region. The thickness of thermal boundary layer (TBL) and velocity boundary layer (VBL) increases in the vertical direction. The resulting velocity distribution is different from that of forced convection BLs as presented in figure 2.1. The streamwise u-velocity is zero at y = 0 (due to no slip condition) and at far away from plate i.e. y tending to infinity. The plate is situated in a considerably larger fluid reservoir and vertical velocity of bulk fluid far from the plate is assumed to be zero (i.e. v = 0 at y→ ∞).

In the previous studies, researchers analyzed the BL formed over a VFP using the integral and similarity analysis method. H.B. Squire [18] applied the integral method to derive the approximate solution for laminar free convection BL over a VFP for the constant heat flux condition. To simplify the analysis, they assumed that the thickness of VBL is equal to TBL thickness. Due to this assumption, their formulation does not provide correct flow characteristics behavior. However, Sparrow et al. [19] used the same geometrical and flow conditions as Squire and obtained the exact solution of the BL equations by the similarity method. A well-known mathematical formulation of the BL equations for steady laminar free convection over a heated VFP given by Sparrow et al. [19] is presented below.

The coordinate system used for the formulation of BL equation is also shown in figure 2.1. The streamwise distance along the plate from the leading edge and normal distance from the plate are taken as x and y respectively. Similarly, the normal and parallel components of velocity along the plate are considered as v and u respectively. The thermal and velocity BL thicknesses are represented as δ_t and δ_v. A plate of length L is subjected to a uniform wall heat flux of q''_w has been considered. The following assumptions have been used for mathematical formation of BL equations:

- Steady and laminar flow.
- The flow variation in z-direction is negligible i.e. 2-D flow.
- Heat transfer due to radiation is neglected.
- The fluid obeys Newton's viscosity law.

- The gravity acts vertically in the downward direction.
- The effect of viscous dissipation is very small in free convection; hence it is neglected.
- In the body force term of the Navier-Strokes equation, the density varies according to the Boussinesq approximation, while all other thermo-physical properties remain constant.

The BL equations for flow over a VFP based on above mentioned assumptions can be written as [17,20]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \vartheta\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \tag{2.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{2.3}$$

The boundary conditions are as follows:

At $y = 0$: $u, v = 0$ (no slip and impermeable wall) and q_w'' =constant (2.4)

At $y \to \infty$: $u \to 0$ (stagnant bulk fluid) and $T \to T_\infty$ (isothermal bulk (2.5) fluid)

Equations (2.1) – (2.3) are non-linear, coupled partial differential equations in which temperature and velocity are dependent on more than one variable. These governing equations with the boundary condition (2.4) - (2.5) are converted into non-dimensional form by using following non-dimensional variables:

$$u = \frac{\vartheta}{L}\left(\frac{Gr_L^*}{5}\right)^{2/5}U\ ; \qquad v = \frac{\vartheta}{L}\left(\frac{Gr_L^*}{5}\right)^{1/5}V\ ; \qquad \bar{\theta} = \frac{T - T_\infty}{q_w'' L/k}\left(\frac{Gr_L^*}{5}\right)^{1/5};$$

$$X = \frac{x}{L}; \qquad Y = \frac{y}{L}\left(\frac{Gr_L^*}{5}\right)^{1/5}; \tag{2.6}$$

where, $Gr_L^* = \frac{g\beta q_w'' L^4}{k\vartheta^2}$ is a modified Grashof number based on the heat flux (q_w'') and the characteristic length (L).

Following are the non-dimensional equations obtained by substituting equation 2.6 into equations (2.1) - (2.5):

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad (2.7)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + \bar{\theta} \qquad (2.8)$$

$$U\frac{\partial \bar{\theta}}{\partial X} + V\frac{\partial \bar{\theta}}{\partial Y} = \frac{1}{Pr}\frac{\partial^2 \bar{\theta}}{\partial Y^2} \qquad (2.9)$$

At $Y = 0$: $\quad U, V = 0 \quad$ and $\quad \frac{\partial \bar{\theta}}{\partial Y} = 1 \qquad$ (2.10)

At $Y \to \infty$: $\quad U \to 0 \quad$ and $\quad \bar{\theta} = 0 \qquad$ (2.11)

An exact solution of equations (2.7) - (2.9) with corresponding boundary condition equations (2.10) - (2.11) has been obtained by introducing a similarity variable $\eta = YX^{\frac{-1}{5}}$, stream function $\psi = X^{\frac{4}{5}} f(\eta)$ and non-dimensional temperature $\theta = X^{-\frac{1}{5}}\bar{\theta}$.

(2.12)

Here, $f(\eta)$ is a non-dimensional stream function whose derivative yields the non-dimensional u-velocity.

Now, using equation 2.12, the BL equations (2.7) - (2.11) are transformed to

$$f''' - 3(f')^2 + 4ff'' + \theta = 0 \qquad (2.13)$$

$$\frac{1}{Pr}\theta'' + 4f\theta' - F'\theta = 0 \qquad (2.14)$$

subject to the boundary conditions:

At $\eta = 0$: $\quad f = f' = 0,\ \theta' = -1 \qquad$ (2.15)

At $\eta \to \infty$: $$f' = 0, \ \theta = 0 \tag{2.16}$$

The wall shear stress is calculated as

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\mu\vartheta}{L^2}\left(\frac{Gr_L^*}{5}\right)^{3/5}X^{\frac{2}{5}}f''(0) \tag{2.17}$$

The local skin friction coefficient is defined as

$$c_{fX} = \frac{\tau_w}{\frac{1}{2}\rho(u_r)^2} = 2X^{\frac{2}{5}}\left(\frac{Gr_L^*}{5}\right)^{-1/5}f''(0) \tag{2.18}$$

here, $u_r = \frac{\vartheta}{L}\left(\frac{Gr_L^*}{5}\right)^{2/5}$

The local heat transfer coefficient is given by

$$h_X = \frac{k}{L}\frac{1}{\theta(0)}X^{\frac{-1}{5}}\left(\frac{Gr_L^*}{5}\right)^{1/5} \tag{2.19}$$

The local Nusselt number is defined as

$$Nu_X = \frac{1}{\theta(0)}X^{\frac{4}{5}}\left(\frac{Gr_L^*}{5}\right)^{1/5} \tag{2.20}$$

Equations (2.13) and (2.14) together with the boundary conditions supplied by equations (2.15) and (2.16) formed a boundary value problem (BVP). The numerical solution of this BVP is complex due to its coupled and non-linear form, which requires a simultaneous solution of the equations. Shooting method can be employed to obtained the analytical solution of these types of equations [21–23]. The VBL and TBL obtained from the similarity solution are asymptotic in nature. As the bulk fluid is stagnant in free convection, the edge of the VBL cannot be defined as a fraction of the bulk fluid velocity (like in forced convection). It can be seen in the previous research works that there is a lack of VBL edge definition in free convection using velocity profile [24]. Therefore, it is necessary to identify the edge of the VBL for quantitative investigation. In the present thesis, identification of the edge of the free convection VBL has been made to quantify its thickness.

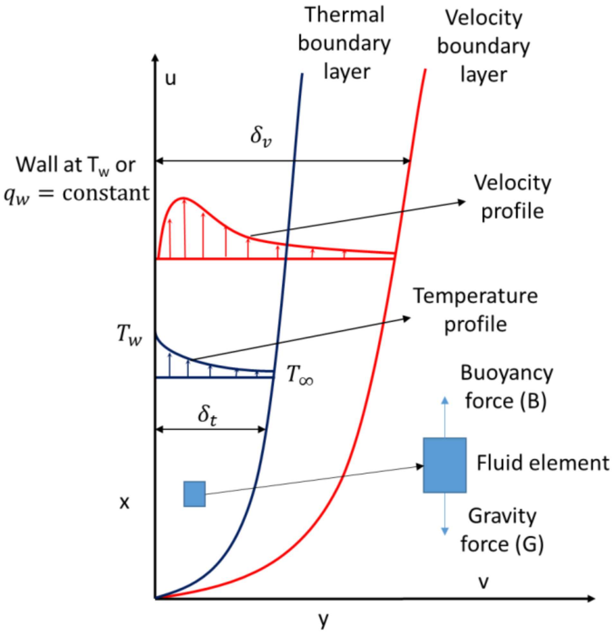

Figure 2.1: Free convection TBL and VBL over a heated vertical flat plate

Previously, there are several experiment on free convection over a VFP have been performed. However, most of these research only focused on only study of thermal characteristics. Rajan et al. [25] studied the free convection from a uniform heated VFP experimentally. They used a very thin metallic strip which is electrically heated. They presented transient and steady state heat transfer co-efficient for various fluids. Goldstein et al. [26] investigated the time evolution of free convection TBL formed on a uniformly heated vertical plate. They used Mach-Zehnder interferometer to obtained temperature field within the TBL for both the unsteady and steady condition. They found that the TBL thickness first increases with time, reaches a maximum value, and then decreases to reach a steady state condition. Vliet et al. [27] experimentally investigated free convection BL on a VFP subjected to uniform heat flux. They estimated local heat transfer based on the measurement of surface temperature for laminar, transitional and turbulent regime. The flow structure was also studied using the hydrogen bubble method. Using this method, they found that in the large eddies exist in both near wall and the outer region of the turbulent

BL. An experimental study on the behavior of free-convective flow near a VFP with a heat-flux jump was carried out by Burak et al. [28]. The temperature field within BL were measured by a Mach-Zehnder interferometer and the velocity field were studied using particle visualization. At the initial stage of formation of free convective flow, a 2-D vortex is observed within a BL for higher heat flux. Tsuji et al. [8] studied the turbulent free convection BL around a VFP in air using hot-wire and cold-wire probes. They found that near the wall, the distributions of streamwise turbulent heat flux and Reynolds stress are different as compared to forced convection. However, normal turbulent heat flux distribution similar to that observed in forced convection. Hattori et al. [29] investigated the free convection BL around a heated VFP at high temperatures. The turbulence characteristics are measured in the BL using a particle image velocimetry and a cold wire technique. They observed that small influence of high heat on the turbulent behavior within the boundary layer. Additionally, even under high wall temperatures, the structure of large-scale fluid motions in the outer layer of free convection BL is maintained. Baudoin et al. [30] studied the free convection flow on a VFP that is caused by a variety of heat sources. Experimental results for velocity measured by particle image velocimetry and temperature measured by micro-thermocouples were presented for various heat source distribution. They found that the vertical spacing of heaters compared to horizontal spacing influences more on the flow and the heat transfer.

There are several numerical studies on free convection over a VFP have also been performed. However, these numerical studies mainly focus on the instability phenomenon. Abedin et al. [31] studied transitional and turbulent free convection BL along a heated VFP using direct numerical simulation (DNS). They were used time-developing DNS approach with Boussinesq approximation in the governing equations to obtained the turbulent characteristics of the free convection BL. They showed that the space-developing approach is less computationally efficient compared to the time-developing approach. Aberra et al. [32] employed DNS to investigate instability of BL in free convection on a uniformly heated VFP. They employed Boussinesq fluid with Prandtl numbers of 0.733 (air) and 6.7 (water). They demonstrated that the Prandtl and Rayleigh numbers and the excitation frequency have more influence on the behavior of the disturbance. Nakao et al. [33] investigated spatially evolving free convection BL along a VFP using Large-eddy simulation (LES) approach. They reported spatial variation of wall shear stress, heat transfer and laminar to turbulence transition. The instability was observed in both inner and outer BL in between the laminar and the transition region while in the turbulent region, a strong vortex motion was observed only in outer layer.

2.1.2 Flow over inclined plates

The previous section discussed the free convection phenomenon when the plate is held vertically. In this section, the plate inclined from the vertical position, as depicted in figure 2.2, has been considered. The plate inclination thickens the BL and gives the wall jet a propensity to separate from the wall [4]. In this case, the buoyancy force (B) has two components: one that acts along the length of the plate (B_x) and another that normally acts on the plate surface (B_y). If ϕ is the angle of inclination of the plate from horizontal, then $B_x = B \sin \phi$ and $B_y = B \cos \phi$. The buoyancy component B_x is responsible for the motion of the fluid along the plate. On the other hand, B_y has a tendency to separate the flow from the wall.

Due to the flow separation laminar to turbulent transition occurs. Hence, the BL in free convection can be categorized into three regimes: laminar, transitional and turbulent [4,20]. A small region adjacent to the surface with little or no randomness is referred to as the laminar regime [6,17]. The transition from a laminar to a turbulent regime is frequently triggered by an instability phenomenon [34]. The instability causes the large diffusion of energy into the bulk fluid from the BL. The large diffusion causes turbulence in fluid motion, indicating that the transition from laminar to turbulence has begun [6]. Since the hydrodynamic and thermal parameters like wall shear stress and heat transfer coefficient vary based on the flow regimes, determining the location of the transition from one regime to another is crucial for obtaining an accurate information about these parameters.

Numerous studies have been conducted on free convection over a vertical flat plate because of the fundamental nature of the problem [21,26,27,35,36]. The thermal characteristics of the flow were studied by most researchers to distinguish the laminar and turbulent regions of fluid flow over a flat plate. R. Cheesewright [35] investigated turbulent free convection using a vertical flat plate and concluded that the start of transition can be indicated by significant temperature fluctuation within the BL. Vliet et al. [27] studied turbulent free convection experimentally on a vertical plate subjected to constant heat flux. The downstream position where the surface temperature begins to decline from a maximum value was defined as the start of transition. All of these research focused solely on the commencement of thermal transition and revealed that it occurs at a specific Grashof number. Gebhart and his colleagues [36–40] studied in detail the free convection transition mechanism in water over a vertical flat plate. They showed that the hydrodynamic or velocity transition occurs before the thermal transition. Therefore, in addition to thermal transition, the onset of hydrodynamic transition needs to be investigated which has been attempted in the present work. They further demonstrated that the transition criteria could not be determined solely by the Grashof number. By analysing the variation in some observable parameters for free convection over a vertical flat plate, a few authors were able to pinpoint the location of the hydrodynamic transition. Jaluria et al. [36] employed a hot wire anemometer to detect velocity at several locations within the velocity boundary layer

(VBL), determining the commencement of velocity transition based on the variance in intermittency factor. Mahajan et al. [40] employed hot wire anemometry and concluded that the onset of the velocity transition occurs at the locations where the highest stream wise velocity begins to decrease. With the use of a hot wire anemometer, Vitharana et al. [41] investigated the velocity transition in free convection along a vertical flat plate for mercury. They assumed that the fluctuation in velocity within the VBL indicated the start of velocity transition.

As opposed to a vertical plate, free convection over an inclined flat plate has received less attention. Vliet et al. [42] investigated free convection over inclined plates with uniform heat flux in water and air. They classified three distinct regimes as laminar, transitional and turbulent based on the surface temperature data. However, their study was limited to only thermal transitions in free convection. Few authors with the help of visual inspection describe the flow patterns over an inclined plate. Fujii et al. [14] examined free convection in water for various plate inclinations. They showed how the inclination angle changed the characteristics of the flow pattern and the heat transfer coefficient. They visualized the flow pattern by illumination of aluminium particles suspended in the flow. Sparrow et al. [43] carried out an experimental analysis of a secondary flow superimposed upon the main flow produced by free convection on an inclined plate. In their research, the flow pattern is seen by local changes in the colour of fluid due to a pH change. These visualization methods give only a qualitative estimation of the flow patterns but for a better understanding of the commencement of transition, a quantitative measurement of the entire flow field is required [9,15,16]. Recently, with the help of PIV, Goodrich et al. [6] investigated free convection VBL transition over vertical cylinders. They illustrated that variations in velocity and VBL thickness measured by PIV can be used to detect transition.

There are few analtyical and numerical studies also conducted for the case of free convection in the inclined plates. Kierkus [44] studied laminar free convection above an inclined plate having uniform temperature plate using perturbation analysis. They estimated velocity and temperature fields using first order perturbation for the Prandtl number of 0.7 and various inclination angles. Guha et al. [45] investigated thermo-fluid-dynamics of free convection above a heated inclined plates using CFD simulation. The effect of various inclination of the plate on the flow structures were analyzed in their work. They found that as the inclination angle from the horizontal increases, a small decrease in average Nusselt number is seen initially. After that it reaches a minimum value and then onward till the inclination angle is 90° increases continuously. However, its analysis was restricted to laminar flow only. Guha et al. [46] studied laminar free convection over inclined surfaces whose inclination varying from horizontal to vertical. They formulated a unified integral theory for specified variation in surface temperature and surface heat flux. The temperature and velocity profiles for the integral analysis of free convection were optimized in their work.

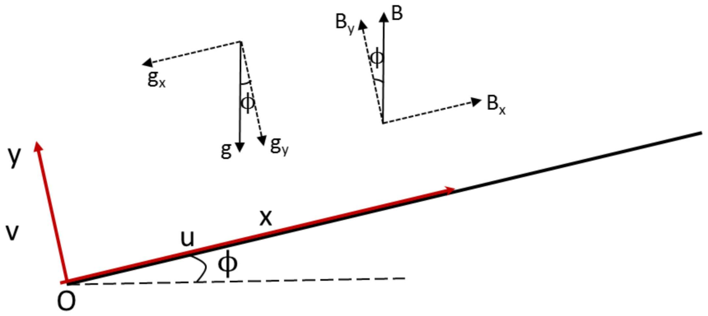

Figure 2.2: Force components in the case of flow over an upward facing heated inclined plate

These reviews of the literature indicate that previous works did not fully describe the variation of hydrodynamic parameters like inner VBL thickness, shear stress and onset of velocity transition. Since, in many engineering applications, such as solar collectors and roof of the buildings, the surface exchanging energy by convection is in an inclined orientation. Hence, an investigation of flow over inclined plates was additionally conducted, and the results are presented in chapter 5.

2.1.3 Flow over horizontal and slightly inclined flat plates

Many real-world scenarios can be approximately modeled as free convective heat transfer from a heated flat horizontal or nearly horizontal surface. It occurs in various applications, like the cooling of electronic components, the heating of spaces, the solidification process in casting, and the heat dissipation of exposed surface of electric transformers. The phenomenon of a FCBL along a heated VFP has been comprehensively examined in previous works. Due to its simple flow mechanism, significant success has been achieved in understanding its flow physics and obtaining an analytical solution for the flow. However, the mechanism of free convection above a horizontal or slightly inclined plate (figure 2.3) is more complex. For the case of horizontal plate, the tangential component of buoyancy force (B_x) turns to zero, and only the normal component (B_y) is a source of motion. The magnitude of B_x along the plate increases when inclination angle of the plate from horizontal increases. Figure 2.3 below depicts the general shape of the flow over a heated surface in such circumstances. The flow is inward from the sides of the surface and then upward, as depicted in figure 2.3. Therefore, flow over a plate with a higher inclination occurs due to buoyancy force, while it occurs due to a pressure gradient (generated due formation of buoyant plume) along the plate length for a lower inclination plate. Flow along a horizontal or slightly inclined surface is hence referred to as indirect free convection [47]. These types of flows are intrinsically unstable because there is a higher-density fluid at the top and a lower-density fluid at the bottom. When fluid flows over a slightly inclined plate, distinct flow characteristics such as flow attached to the wall, transition regime and buoyant plume can be noticed. Therefore, the experimental and analytical examination have presented substantial challenges due to these flow characteristics.

The previous experimental studies on FCBL over a flat surface have focused on temperature measurement using techniques such as Schlieren [48], thermocouples [49,50], liquid crystals [51] and Mach-Zender interferometer [52]. There are limited studies on measurement of hydrodynamic parameters. Fujii et al. [14] carried out both heat transfer measurements and flow visualizations over a wide range of inclination angles. In their experiments, water flows over a slightly inclined plate were visualized with aluminium powder. However, their main concerns were directed to study of heat transfer characteristics. Pretot et al. [53] analysed the flow patterns, temperature field and heat convection coefficient in free convection flow above an upward facing horizontal plate, located in a semi-infinite medium. Near the sides of the plate, they observed existence of two regions with high heat convection coefficients. These regions break away in the middle of the heated surface and form a thermal plume. Studies for visualizing the flow of water overheated rectangular flat plates and methods to obtain correlations for the Nusselt number (mean and local) as a function of the Rayleigh number based on the width of the plate were performed by Kitamura et al. [49]. They used liquid crystal thermometry to visualize the temperature on the plate surface. Lewandowski et al. [12] performed an experimental and theoretical analysis of free convection from horizontal plates having different aspect ratios. By injecting dye through perforations along the plate edge, the flow structure was made visible in their experiment. They presented two models, the solutions to which were given as Nusselt-Rayleigh relations. These visualization methods give only a qualitative estimation of the flow patterns but for a better understanding of the flow physics, a quantitative measurement of the entire flow field is required [9,15,16].

Apart from experimental methods, CFD simulation were used in several studies [45,54,55] to explore the free convection flows above a heated flat plate. Guha et al. [54] investigated the influence of the finite size of horizontal plates on the free convection flow. They demonstrated that the entrainment effect of the central plume limits the similarity theory from being applied at any position over the plate. For a range of Grashof numbers, they developed a theoretical correlation for the Nusselt number. In another study, the semi-infinite assumption in the theories of free convection for the determination of Nusselt number was examined by Guha et al. [55]. They found that utilising similarity or integral theories to determine the average Nusselt number of a finite-size plate would result in significant errors. In order to account for these inaccuracies, they offered correction factors. Guha et al. [45] studied the free convection above the flat plates of various inclination. The behaviour of the lift-off point, which is the point at which the buoyant plume emerges from the FCBL, was presented. Nevertheless, all of these investigations were restricted to laminar flow and also, do not provide any quantifiable data on parameters like the width of the buoyant plume, the length of the transition region, the location of the start of the transition, etc.

A plume may form from the heated plate which is horizontal and slightly inclined due to buoyancy. The buoyant plume is important to investigate because of its numerous environmental and industrial applications [56,57]. Hattori et al. [58] investigated the unsteady behaviour in the near-field of a planar thermal plume from a finite-area source Using numerical simulation. To validate their numerical findings, they used PIV measurement and shadowgraph techniques. They observed that the near-field unsteady behaviour is characterized by bulge and puff structures. Fan et al. [59] investigate free convection from a square horizontal heated surface in a stably stratified environment. They used TIV and PIV to identify shifting, expanding, and twisting eddy structures. In another study, Fan et al. [60] used various background buoyancy frequencies to determine the flow patterns over a horizontal flat plate. They observed plume-like and dome-like flow patterns and provided the critical Froude number for the plume-to-dome transition. Many other investigations [61–65] of buoyant plumes over horizontal surfaces were also carried out. Nevertheless, these results are limited to horizontal surfaces, with no discussion of slightly inclined plates.

As per the literature review, the flow physics in FCBL over a horizontal and slightly inclined surface is defined based on either flow visualization techniques or on temperature measurement. To the best of the authors' knowledge, very little information is available in literature on the quantitative estimation of the velocity field in such a geometry. Therefore, in chapter 6, the complete flow pattern around the slightly inclined plate with the help of PIV technique has been discussed for various imposed heat flux conditions and at different inclination angles.

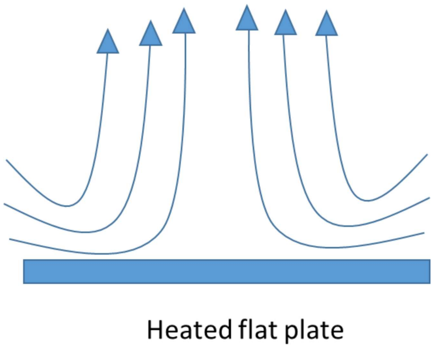

Figure 2.3: Free convection flow over a horizontal plate

Summary

As per literature review, the flow physics of flow over inclined surfaces is identified based on either flow visualization technique or on temperature measurement. To the best of the authors' knowledge, very little information is available in literature on the quantitative estimation of the velocity field in such a geometry. However, when designing various thermal systems, it is essential to investigate the behavior of hydrodynamic parameters at various inclinations of the plate under different heating conditions. Therefore, it is expected that the results obtained in the present work would be helpful in obtaining valuable information that may be incorporated for enhanced fundamental knowledge of free convection.