Fractal Approximation on the Sierpiński Gasket and Fractal Surfaces

The thesis submitted in partial fulfilment

for the Award of Degree

DOCTOR OF PHILOSOPHY

by

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It is certified that the work contained in this thesis titled "***Fractal Approximation on the Sierpiński Gasket and Fractal Surfaces***" by ***Vishal Agrawal*** has been carried out under my supervision and this work has not been submitted elsewhere for a degree.

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I, ***Vishal Agrawal***, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of ***Prof. Tanmoy Som*** from ***January, 2019*** to ***June, 2023*** at the ***Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi***. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

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Vishal Agrawal
22/06/23

Vishal Agrawal

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Symbols

$\forall$	for all
$=$	equal to
$\in$	belongs to
$\notin$	does not belong
$\subset$	subset or equal
$\cup, \cap$	union, intersection
$\|x\|$	absolute value of x
SG	Sierpiński gasket
$\mathbb{N}$	the set of natural numbers
$\mathbb{R}$	the real line
$\mathcal{P}(X)$	the power set of a set X
$\mathcal{C}(X)$	the space of all real-valued continuous functions on X
$\mathcal{C}(X, Y)$	the space of all Y-valued continuous functions on X
$\mathcal{B}(A)$	the space of all the bounded below and integrable functions on A
$\mathcal{L}^p(x)$	the space of all real-valued p-integrable functions on X
$\mathcal{H}^\sigma(X)$	the space of all continuous Hölder functions with exponent on X
$\|.\|$	norm
$\|.\|_\infty$	the uniform norm
Id	Identity map
$\mathcal{H}^s(A)$	s-dimensional Hausdorff measure of a A

$\dim_H(A)$	Hausdorff dimension of set A
$\dim_B(A)$	Box dimension of set A
$\underline{\dim}_B A$	lower Box dimension of A
$\overline{\dim}_B A$	upper Box dimension of A
μ_p	self-similar measure
f^α	α-fractal function corresponding to f
$d(t, A)$	distance between point t and set A
$\square$	end of a proof