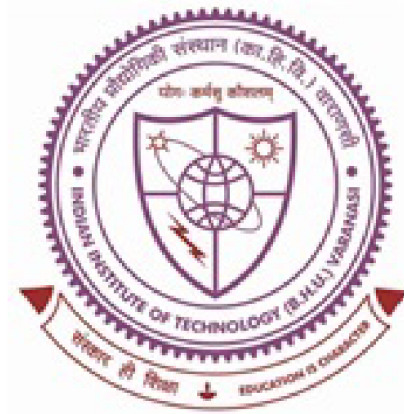


Fractal Approximation on the Sierpiński Gasket and Fractal Surfaces



The thesis submitted in partial fulfilment

for the Award of Degree

DOCTOR OF PHILOSOPHY

by

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It is certified that the work contained in this thesis titled "*Fractal Approximation on the Sierpiński Gasket and Fractal Surfaces*" by *Vishal Agrawal* has been carried out under my supervision and this work has not been submitted elsewhere for a degree.

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Vishal Agrawal

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Symbols

\forall	for all
$=$	equal to
\in	belongs to
\notin	does not belong
\subset	subset or equal
\cup, \cap	union, intersection
$ x $	absolute value of x
SG	Sierpiński gasket
\mathbb{N}	the set of natural numbers
\mathbb{R}	the real line
$\mathcal{P}(X)$	the power set of a set X
$\mathcal{C}(X)$	the space of all real-valued continuous functions on X
$\mathcal{C}(X, Y)$	the space of all Y -valued continuous functions on X
$\mathcal{B}(A)$	the space of all the bounded below and integrable functions on A
$\mathcal{L}^p(x)$	the space of all real-valued p -integrable functions on X
$\mathcal{H}^\sigma(X)$	the space of all continuous Hölder functions with exponent on X
$\ \cdot\ $	norm
$\ \cdot\ _\infty$	the uniform norm
Id	Identity map
$\mathcal{H}^s(A)$	s -dimensional Hausdorff measure of a A

$\dim_H(A)$	Hausdorff dimension of set A
$\dim_B(A)$	Box dimension of set A
$\underline{\dim}_B A$	lower Box dimension of A
$\overline{\dim}_B A$	upper Box dimension of A
μ_p	self-similar measure
f^α	α -fractal function corresponding to f
$d(t, A)$	distance between point t and set A
\square	end of a proof