Chapter 6

Summary and Future Prospects

6.1 Summary

In this chapter, we provide a brief summary of the work presented in this thesis. **Chapter 1** began by outlining the historical foundations of equilibrium and nonequilibrium statistical physics, positioning active matter as a distinct subclass of nonequilibrium systems. We discussed the key differences between active matter and both equilibrium and nonequilibrium systems, followed by a classification of active particles based on their symmetry and ordering properties. Additionally, we introduced various numerical and analytical tools used to model active matter systems. The chapter also addressed the role of inhomogeneities in active matter and the key observables used to characterize the phases and phase transitions within these systems.

In **Chapter 2**, we have studied a collection of self-propelled polar particles on a two-dimensional substrate with birth and death. We introduced a minimal lattice model for the system using active Ising spins, where each particle could have two possible orientations. The activity was modeled as a biased movement of the particle along its direction of orientation. The particles also aligned with their nearest neighbors using the Metropolis Monte Carlo algorithm. The system exhibited a disorder-to-order transition by tuning the temperature of the system. Additionally, the birth and death of the particles were introduced through a birth and death parameter γ. We studied the system near the disorder-to-order transition and observed that the nature of the transition showed a crossover from first-order, discontinuous to continuous type as γ was tuned from zero to finite values. We also wrote the effective free energy of the local order parameter using perturbative calculations, confirming the dependence of the nature of the phase transition on the birth and death parameter.

Chapter 3 explored the effects of quenched disorder on the characteristics of self-propelled particles in one dimension. This study investigated the influence of disorder, modeled as directional cues, on the emergence of ordering and clustering among self-propelled particles. Microscopic models were introduced, along with corresponding coarse-grained equations of motion for the local density and orientation of the particles. The results showed that disorder strongly affected macroscopic ordering, with the size of ordered clusters decaying algebraically with disorder. Additionally, clustering was influenced by the presence of disorder, as large macroscopic clusters broke into smaller clusters, leading to particle localization and higher densities around the disorder sites.

Chapter 4 investigated the dynamics and steady-state behavior of Active Brownian Particles (ABPs) moving on a substrate with space-dependent activity, inspired by recent studies showing that microorganisms can sense and respond to substrate characteristics. ABPs have proven effective in modeling these moving microorganisms. In our study, certain regions of the substrate were marked as high in activity, while others allowed particles to behave as passive Brownian particles. The system was analyzed in two dimensions with different activity profiles, including step, sigmoid, Gaussian, and cone-shaped distributions. The interface between the active and passive regions was symmetrically divided, inducing a flow of particles from the high-activity region to the passive region. The resulting steady-state particle density, polarization, and flux closely followed the structure of the inhomogeneous activity. We observed lower density in the high-activity regions, a peak at the interface, and nearly constant density in the passive regions. Additionally, the steady-state density profiles were studied for various shapes and designs of activity distributions on two-dimensional substrates, demonstrating how a collection of ABPs can effectively mimic substrate inhomogeneities.
Lastly, **Chapter 5** focus on the effects of polar impurities in a liquid crystal system by tuning the coupling strength γ and the average density of polar particles ρ_{p0}. We adopted a coarse-grained approach by simulating the time-dependent Ginzburg-Landau (TDGL) equations for the order parameters $\vec{P}$ and Q. When $\gamma = 0$, no string formation was observed. However, as γ increased beyond a certain optimal value, strings began to form between defect pairs. The width of these strings decreased with increasing γ. These strings were visible in the order parameter fields of both polar and nematic particles, and for polar particles, the magnitude of the order parameter approached zero along the string, effectively acting as a domain wall that separated oppositely oriented domains. Interestingly, strings connected two defects regardless of their charge, meaning the defect pairs could be of the same type (e.g., +1/2, +1/2 or -1/2, -1/2) or opposite (e.g., +1/2, -1/2). Higher polar particle densities and stronger coupling led to narrower strings, underscoring the critical role of

these parameters in determining the structural properties of the system. Further investigation into the trajectory and dynamics of defects was conducted by introducing artificial defects into the system. The evolution of the system from random and polar-ordered initial conditions revealed that an increase in γ enhanced the dynamics of the defects, leading to faster movement. This chapter provided a thorough analysis of the behavior of topological defects and string formation in systems coupling polar and nematic particles.

6.2 Future scope

Now, we write some key future scope of the problems discussed in this thesis. The first project in the thesis, disucssed about the phase transitions in polar flock with birth and death. In the present study, once the particle is dead it gets disappear from the system, maintaining the mean density constant. It is possible a dead particle can act like disorder or food for the nearby particles and how it affects the steady state and phase transition in the system. We also found that quenched disorder breaks the LRO state in one-dimensional polar flock and disorder lead the trapping of particles. We are planning to study the system in quasi-one dimensional system where we can study how giving some freedom in another direction affects the ordering and clustering of particles. Further the inhomogeneity introduced in next chapter is quenched in time, it will be interesting to look the response of the ABPs where pattern of the substrate evolves with time. The last chapter, the effect of polar particles on liquid crystal can be expanded for the case, where polar particles are active in nature, which will be like bacteria in liquid crystal environment. How the dynamics of bacteria get affected by the background liquid crystal medium?

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