Appendix-I

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HYDRAULICS OF FLUID FLOW IN A SINGLE COHESIVE CRACK: A REVIEW OF SOME BASIC CONCEPTS

By

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ABSTRACT

Present review discusses the scope and limitations of basic equations of continuity and momentum used in the study of flow behavior in single crack. The continuity equation for liquid flow used in the literature does not account for source and sink which occurs during crack wall leakages. In two-phase continuity equation, inter-phase transfer terms have been identified to be used as source or sink terms. Navier-Stokes momentum equation and its simplifications under various assumptions, leading to derivations of Stokes and Reynolds equations, and their limitations along with computational difficulty in crack flow problems is discussed. Emergence of Local Cubic Law (LCL) equation and its limitations has been indicated. Validity of LCL equation has been tested in the literatures under the influence of various factors. The relative importance of inertial and viscous forces show that LCL equation is valid for Reynolds number of 10 and a non-linear relation between pressure gradient and volumetric flow rate need to be used when Reynolds number is between 10 and 100. The use of scaling law used in the fracture mechanics shows that in smooth walled fracture, the volumetric flow rate vary with fifth power of crack aperture and so this law is termed as 'quintic' law. There are few published works in literature to study the effect of slip boundary condition in deriving the correction factor in hydraulic conductivity of fracture using Beaver-Joseph slip boundary conditions in case of smooth permeable fracture walls. Some works have shown that formulation of momentum equation for different flow regimes of two phase flow in cracks is a difficult task.

Keywords: Crack, Momentum Equation, LCL Equation, Crack Wall Geometry, Self-affine, Quintic law, Hydraulic Conductivity, Two-Phase Flow.

INTRODUCTION

Fractured porous media consist of interconnected fracture and pores networks. Fractures, generally considered as fast transport pathways(Zimmerman and Bodvarsson, 1996), are large pores and crevices (10 -4 m to 10 -2 m) (Tsang and Tsang, 1987; Fischer et al. 1998).The study of transport processes in fractured porous media is of capital importance to understand the single and multi-phase flow of groundwater; to predict the fate of pollutants in aquifer contaminated by industrial, (Sung-Hoon et al. 2003), agricultural and radioactive wastes; to study the transient uplift pressure due to seismicity (Slowik and Saouma, 2000; Saouma et al. 1991)or time varying reservoir level in concrete gravity dams; leakage and stability of dam foundations(Terzhagi, 1936; Bazant, 1975), mine drainages, slope stability and hydraulic fracturing (Crisstianovich and Zheltov, 1955), (Cleary 1980; Bagherian et al. 2010;Bahrami and Mortazavi 2008; Soliman et.al. 2004; Sarris and Papanastasiou, 2012; Jeffery et al. 2001; Murdoch and Slack 2002; Logan et al. 2000).

Present study reviews the applications and limitations of basic physical laws employed in the literatures to study the flow behavior in a single cohesive crack. Review is limited to basic flow equations and does not consider the mechanical behavior and theoretical and numerical flow modeling.

1. Basic Equations of Flow in Single Crack

Flow within the fractures are governed by (1) mass conservation law and (2) Local cubic law (LCL) derived from lubrication theory for flow between two parallel plates. When employing these basic equations to study flow in cracks, various assumptions are made. So each are

REVIEW PAPER

discussed separately to have a clear picture of the forms and applicability of these equations to crack flow problems.

1.1 Mass Conservation Law

Based on the principle of conservation of mass, which requires that the rate of increase or decrease of fluid mass in a finite elemental control volume situated in the flow field be equal to the net rates of inflow or outflow, a 2-D continuity equation can be written as (Illangasekare et.al, 1992).

$$\frac{\partial(\rho b v_x)}{\partial x} + \frac{\partial(\rho b v_y)}{\partial y} = \frac{\partial(\rho b)}{\partial t} \qquad (1)$$

Where, v_x, v_y =velocity of fluid in x and y directions respectively; b= crack aperture; ρ=density of the fluid (Figure1).

In the derivation of equation (1), the absence of sink or source is assumed. The above equation can be modified in several forms by considering any one or combinations of the following assumptions: (1) flow is steady (2) flow is incompressible (3) fluid density changes with time but is independent of space (4) crack aperture is varying linearly with fluid pressure p as $p= K_n b$, where K_n =normal stiffness of crack wall.

1-D continuity equations widely used in the literature (Slowik and Saouma, 2000;Sarris and Papanastasiou, 2012), are written with the assumption that fluid leakage through walls of the crack are negligible i.e walls of the crack are impermeable. For any cross sectional area A, normal to flow direction x, the 1-D continuity equation can be written as

$$\frac{\partial(\rho Q)}{\partial x} = \frac{\partial(\rho A)}{\partial t} \qquad (2)$$

Where, Q is the discharge through the crack.

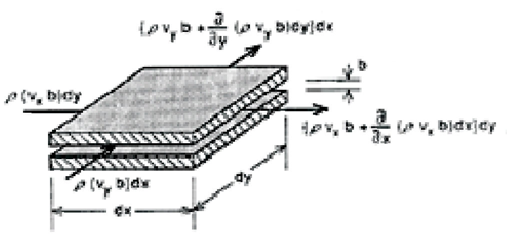

Figure 1. Infinitesimal element used in the derivation of continuity equation (Illangasekare et.al (1992))

Javanmardi et al. (2004, 2005) and Pekauand and Zhu (2008) derived the expression for discharge in cracks of concrete gravity dams after assuming that the crack walls are rigid and impermeable.

Continuity equation for two phase flow in cracks are written and supplemented with additional conditions. Following equation of continuity, wetting fluid 'w' (water) and a non-wetting fluid 'nw' (gas) can be derived (Bear, 1979;Reichenberger et al., 2005;Bastian,1999;Helmig, 1997).

$$\frac{\partial(\rho_\alpha n S_\alpha)}{\partial t} + \nabla.(\rho_\alpha v_\alpha) = \rho_\alpha q_\alpha \qquad (3)$$

with the condition

$$S_w + S_{nw} \qquad (4)$$

Where, α is w or nw; S_α, ρ_α, q_α, v_α are saturation, density, source term and velocity respectively of phase α, and n is the effective porosity. The set of equations (3) and (4) are completed with Brooks-Corey constitutive relations(Brooks and Corey, 1964).

$$p_c(S_w) = p_d S_e^{-1/\lambda} \qquad (5)$$

Which employs the effective saturation S_e of the wetting phase

$$S_e = \frac{S_w - S_{wr}}{1 - S_{wr}}, \qquad S_{wr} \le S_w \le 1 \qquad (6)$$

Where, p_c, p_d are capillary and entry pressure respectively, S_{wr} is residual water saturation and λ is a constant that depends on effective porosity and usually is in range 0.2 ≤ λ ≤ 3.

1.2 Momentum Equation and Darcy Law

The most general form of Navier-Stokes equations for fluid flow in a single fracture can be written as (White, 1994).

$$\frac{\partial v}{\partial t} + (v.\nabla)v = F - \frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2 v \qquad (7)$$

Where, $v = (v_x, v_y, v_z)$ is the velocity vector, F= body force per unit mass, μ = dynamic fluid viscosity and p is the pressure. In most of the cases, the body force is gravity, in which case F= $-ge_z$, where, e_z is the unit vector in upward vertical direction and g is acceleration due to gravity. Assuming that fluid density is uniform and replacing p by p+ρgz, the steady state Navier-Stokes equation can be written as

$$\mu\nabla^2 v - \rho(v.\nabla)v = \nabla p \qquad (8)$$

The Navier-Stokes equation (8) is non-linear due to the

REVIEW PAPER

presence of advective acceleration term ($\upsilon . \nabla$) υ and so, it is very difficult to solve this equation. Consequently, the Navier-Stokes equations are further simplified to Stokes or Reynolds equations.

The Stokes equations are derived from the Navier-Stokes equation after neglecting the advective acceleration terms. To ensure the negligible effect of advective acceleration term in Navier-Stokes equation, the reduced Reynolds number $R_e^* << 1.R_e^*$ was derived, based on the prior estimates of the order of magnitude of the various terms in the steady state Navier-Stokes equation and is written as the product of traditional Reynolds number and a geometric parameter b/D.

$$R_e^* = \frac{\rho U b^2}{\mu D} \quad (9)$$

Where, U and D are characteristic velocity and length respectively. Under this condition, the Navier- Stokes reduces to Stokes equation

$$\mu \nabla^2 v = \nabla p \quad (10)$$

For few simulated fracture profiles, the Stokes equation (10) has been solved numerically (Brown et al., 1995; Mourzenko et al., 1995), but use of Stokes equation has limited application due to computational difficulty.

Stokes equation is reduced to Reynolds lubrication equation under the assumption: (1) $(b/D)^2 << 1$, (2) velocity profile is parabolic at every location in fracture plane and (3) velocity satisfies the no-slip boundary condition at fracture walls. Under these assumptions, the 2D Reynolds lubrication equation is written as

$$\frac{\partial}{\partial x}\left(b^3 \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(b^3 \frac{\partial p}{\partial y}\right) = 0 \quad (11)$$

On the basis of some studies (Brown et al., 1995; Mourzenko et al., 1995; Yeo et al., 1998; Nicholl, et al., 1999), a consensus emerged that the Reynolds equation overestimates the flow rate as much as 100%. Later, the Reynolds equation was improved after including fracture tortuosity in the definition of fracture geometry (Ge, 1997; Waite et al., 1999; Yeo and Ge, 2005) which discussed the issue of range of applicability of Reynolds equation. They found that the Reynolds equation yields accurate transmissivity predictions when (b/D) (a/D) <0.01, where 'a' is the fracture roughness amplitude and 'b' is the uniform aperture.

Most widely used conceptual model for fracture is that of two smooth, parallel walls (Boussinesq, 1868) separated by a uniform aperture 'b'. For one dimensional flow, the Navier-Stokes equation can be solved (Krantz et al., 1979; Tsang and Witherspoon, 1981) exactly for this geometry because nonlinear term vanishes identically. Solution yields a parabolic velocity profile with no-slip condition at walls. Total velocity flux, after integrating the velocity profile (Zimmerman and Bodvarsson, 1996) is,

$$Q = -\frac{wb^3}{12\mu}\frac{\partial p}{\partial x} \quad (12)$$

Where 'w' is the depth of fracture. The term $T = wb^3 / 12$ is known as fracture transmissivity. As transmissivity is propotional to the cube of the aperture, this result is known as Local Cubic Law (LCL). Also, equation (12) is known as Darcy law for fracture due its similarity to Darcy equation, used in porous media. This model, usually applied in 1D flow in fractures having wide channel and smooth parallel walls, are found to be valid (Sausse and Genter, 2005). However, many laboratory experiments (Raven and Gale, 1985; Pyrak-Nolte et al., 1987; Durham and Bonner, 1994) and some field studies (Rasmuson and Neretnieks, 1986; Novakowski et al.1985,Novakowski et al. 1995; Raven et al. 1988) indicate that the parallel plate concept for flow in fractures are not adequate.

In addition to parallel plate model, approximate solution can also be obtained by perturbation methods for sinusoidal fracture walls (Basha and El-Asmar, 2003; Brush and Thompson, 2003; Sisavath et.al.,2003).

However, in real cracks, crack wall geometry is neither parallel nor smooth. Also crack walls may have leakages. Therefore to simulate the flow in real cracks, equation (12) has been widely discussed (Zimmermann and Bodvarsson, 1996; Kim et.al., 2003) and modified in the literature to address some of the following issues: (1) how the crack wall geometry is to be represented (2) how the crack aperture is to be measured (3) is LCL or Darcy law valid for convergent or divergent crack profile (4) What is the range of validity of Darcy law (5) how Darcy law is to be modified for rough crack walls (6) what is the modification in the Darcy law if no-slip boundary condition is violated etc. The above said things and their related issues follow next.

2. Crack Wall Geometry

Real fracture surfaces are very rough and unevenly distributed and are in contact at some locations. Past research (Thompson and Brown, 1991) suggests that, roughness significantly contributes to the hydro-mechanical behavior of fractures and mean mechanical aperture of a fracture is greater than hydraulic aperture used in LCL equation (Zimmermann and Bodvarsson, 1996). To measure the fracture aperture in x-y plane, two surface height functions $z_1(x,y)$ and $z_2(x,y)$, are used. These functions measure the distance of surface points from two parallel reference planes located above and below the crack walls in the matrix as shown in Figure 2.

So aperture measured perpendicular to the two reference planes is defined as

$$b(x,y) = d - z_1(x,y) - z_2(x,y) \quad (13)$$

The information about surface height functions are not known. Hence aperture 'b' in equation (13) is treated as random variable and fractures are characterized in terms of small number of statistical parameters. For stationary stochastic process (Box and Jenekis, 1976), the autocovariance γ_k and autocorrelation ρ_k functions at lag k are written as

$$\gamma_k = E\left((z_x - \mu)(z_{x+k} - \mu)\right) \quad (14)$$

and

$$\rho_k = \frac{E\left((z_x - \mu)(z_{x+k} - \mu)\right)}{\sigma_z^2} \quad (15)$$

Where, μ and σ_z are respectively mean and standard deviation for random variable z(x) with probability distribution p (z), defined as

$$\mu = E[z_x] = \int_{-\infty}^{\infty} z p(z) dz \quad (16)$$

And

$$\sigma_z^2 = E[(z_x - \mu)^2] = \int_{-\infty}^{\infty} (z - \mu)^2 p(z) dz \quad (17)$$

Auto covariance function can also be represented into its Fourier cosine transform, called power spectrum P (f) (Box

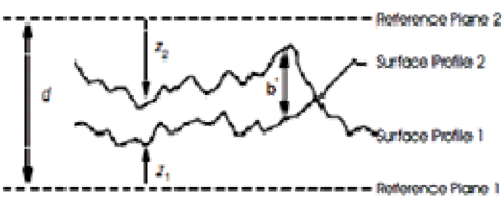

Figure 2. Two rough fracture surface profile separated by distance b along with two reference planes separated by distance d (Zimmerman, R. W., and Main (2004)).

and Jenekis, 1976) where f is wave number $2\pi/\lambda$ and λ is wavelength. Two commonly used models of power spectra are exponential and Gaussian models.

$$\text{Exponential: } P(f) = \frac{\sigma_z^2}{\pi} \frac{1/z_0}{\left(\frac{1}{z_0}\right)^2 + f^2} \quad (18)$$

$$\text{Gaussian: } P(f) = \frac{z_0 \sigma_z^2}{2\sqrt{2}} \exp\left(-\frac{f^2 z_0^2}{4}\right) \quad (19)$$

Where z_0 is a parameter which gives an indication of correlation length. For exponential model, the surface is effectively uncorrelated for $z > 4 z_0$, whereas for Gaussian model, the correlation is negligible for $z > z_0$. Lognormal and Gamma probability distribution functions are commonly used to fit the aperture distributions (Tsang and Tsang,1987; Renshaw, 1985; Unger et al., 1993; Zhan, 1998; Fetter, 1999).For reasons that zero-aperture cannot be represented by Lognormal or Gamma distribution, these PDFs are not sufficient. Oronand Brian (1998) compared the numerical fracture sample to the measured histograms of Brown et al. (1986) and Hakamiand Larsson (1996). They concluded that the skewed form of aperture distribution is the result of contact and deformation and it can be shown that the surface may quite accurately be described by "shifted" Gaussian model.

A profile z(x) is said to be self-affine if $z(\lambda x) = \lambda^H z(x)$, where H is a constant called Hurst exponent (Zimmerman, R. W., and Main (2004)).

A self-affine profile has a power spectrum as

$$P(f) = Cf^{-\alpha} \quad (20)$$

Where $\alpha = 2H + 1$ (Adler and Thovert, 1999). Self-affine power spectrum has been observed in various study of crack wall geometry (Brown and Scholz, 1985; Mandelbrot, 1982; Power and Tullis, 1991). Generally, rough fracture surfaces possess self-affine fractal properties with Hurst exponent H=0.8 (Brownand Scholz, 1985; Poon et al., 1992;Schmittbuhl et al., 1995). However, in practice, it is not possible to detect a dominant wavelength for self-affine curve (Oron and Brian, 1998).

2.1 Measurement of Crack Aperture

In LCL equation (12), it is not clear how to measure aperture 'b'. Equation (13) is used to measure'b' vertically. However there may be regions where both the surfaces are significantly inclined to global fracture plane. In these

REVIEW PAPER

regions, the vertical measurement may be erroneous. Ge (1997) suggested measuring the aperture, normal to local orientation of centerline. Mourzenko et al. (1995) suggested drawing a sphere around each point on centerline and increasing the sphere until it touches both the walls. Oron and Brian (1998) addressed this issue through non-dimensional analysis of Navier-Stokes equation, with Poiseuille flow as a leading order approximation, for the solution of flow between parallel plates. On basis of their analysis, they concluded that aperture for parallel oriented fracture, should not be measured on point-by-point basis, but rather as an average over certain characteristic length. They also showed that, for unmated (diagonal) walls, above conditions hold, provided that mean half angle of the channel is limited to 0.5 radians.

Figure 3 shows the various methods of aperture measurements. The two rock surfaces are indicated by thick wavy lines. Aperture (a) is the distance between the two rock surfaces measured normal to the nominal macroscopic fracture plane, aperture (b) is the distance between rock surfaces measured to the local normal fracture plane, aperture (c) is the distance between two smoothed-out versions of the fracture surfaces, and aperture (d) is the diameter of the largest sphere that can fit between the rock surfaces (Zimmerman and Main, 2004).

In LCL equation, aperture is termed as hydraulic aperture and denoted here as 'b_h' as against the mechanical aperture 'b' represented by equation (13). Elrod (1979) used Fourier transform to solve the Reynolds equation for a fracture with aperture having sinusoidal ripples and showed that, for isotropic case

$$b_H^3 = \langle b \rangle^3 \left(1 - \frac{3}{2}\frac{\sigma_b^2}{\langle b \rangle^2} + \cdots\right) \quad (21)$$

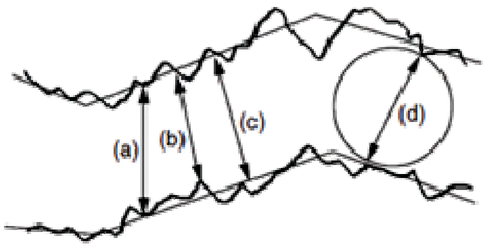

Figure 3. Various definitions for fracture aperture 'b' (Zimmerman, R. W., and Main (2004)).

Where σ_b is the standard deviation and <b> is the mean value of mechanical aperture b.

Zimmerman et al. (1991) showed that the above equation is valid up to second order for both sinusoidal and saw tooth profiles. Renshaw (1995) derived an alternative expression which includes the second-order terms of equation (21)

$$b_H^3 = \langle b \rangle^3 \left(1 - \frac{\sigma_b^2}{\langle b \rangle^2}\right)^{-3/2} \quad (22)$$

Dagan (1993) gave the following expression in series form and showed that terms after fourth term vanish, indicating that geometric mean is a very good approximation for hydraulic aperture in lognormal case.

$$b_H^3 = b_G^3(1 + a_2\sigma_Y^2 + a_4\sigma_Y^4 + \cdots) \quad (23)$$

Where, Y = lnb, σ_Y is the standard deviation of lnb and b_G = exp (<lnb>) is the geometric mean of aperture distribution. Equation (21) is in good agreement with several numerical simulations and some laboratory data (Zimmerman and Main, 2004).

Patir and Cheng (1978) solved the Reynolds equation for Gaussian surface using finite difference technique.

3. Limitations of LCL Equation

The LCL equation was derived for laminar flow in smooth walled fracture. Various investigators (Oron and Brian, 1998; Zimmerman and Yeo, 2000; Skjetne et. al., 1999; Hasegawa and Izuchi, 1983), using different approaches, concluded that LCL is valid for Reynolds numbers less than 10.

Retaining the same assumptions used for the derivation of LCL equation, Christian et al. (2010) modified the LCL equation for penny shaped cracks after taking into account, the displacement-length scaling laws for opening mode (Vermilye and Scholz, 1995; Olson, 2003; Schultz et al., 2008) and derived the following equation.

$$Q = -\frac{4g}{3\nu(\pi\gamma)^2} b^5 \nabla h \quad (24)$$

Where γ is the function of critical stress intensity factor, Poisson ratio and Young's modulus of the material, ν is the kinematic viscosity of the fluid and ∇h is the hydraulic gradient. This equation shows high degree of non-linearity with crack aperture than LCL and is termed as "quintic" law.

At higher Reynolds number, the relationship between pressure gradient and flux is nonlinear and for 1D flow, it can

REVIEW PAPER

be written as

$$\frac{\partial p}{\partial x} = \alpha Q + \beta Q^2 \qquad (25)$$

Where α and β are constants. This nonlinearity is not necessarily due to turbulence, which occurs only at much higher Reynolds numbers. For Reynolds number between 10 to 100, the observed nonlinearity is merely due to the effects of curvature of streamlines (Phillips, 1991) and occurs in the laminar flow regime. Non-Darcian flow is more likely in single fracture (Sharp and Maini, 1972;Quien et.al., 2005, Quien et.al.,2007, Quien et.al.,2010, Wen et.al., 2005) due to high flow velocity and hence high Reynolds number.

To describe flow regime in rough walled fracture, it is important to estimate the boundary layer thickness near the fracture wall (Quinet.al., 2011). Figure 4 shows the schematic diagram of the boundary layer near one fracture wall. When the thickness of the boundary layer is greater than the roughness of the fracture, then the surface is assumed to be hydraulically smooth. On the other hand, if the thickness of the boundary layer is smaller than the roughness of the fractures, then the surface is treated as hydraulically rough. Under hydraulically rough condition, recirculation zones inside the roughness cavities rather than the boundary layer would be the controlling parameter for flow behavior.

The earliest comprehensive work on parallel plate flow of water was conducted by Lomize (1951) who studied the effect of aperture, surface roughness, and aperture variability. Similar experimental studies were later performed by Louis (1969) with nearly identical results. Flow regime chart (Figure 5) which identified five distinct flow regions was formulated from these results.

For this chart, Reynolds number and relative roughness is defined respectively as

$$R_e = \frac{D_h v}{\nu} \qquad (26)$$

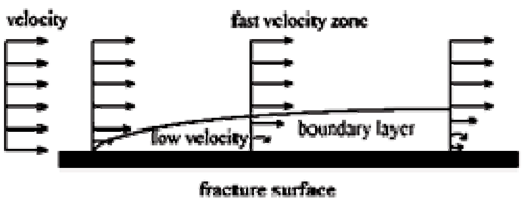

Figure 4. Schematic diagram showing the boundary layer near a fracture wall (Quien et al. 2011)

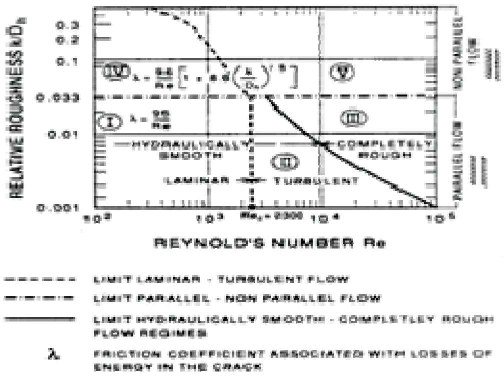

Figure 5. Flow regime chart flow in cracks with varying degree of roughness (Louis (1969)).

And

$$Relative\ Roughness = \frac{\epsilon}{D_h} \qquad (27)$$

Where ∈ is average asperity height or absolute roughness and D_h is hydraulic radius of the crack which is equal to 2b. For hydraulically smooth condition, the relative roughness is less than 0.033 and transition from laminar to turbulent flow occurs when Reynolds number is approximately equal to 2300.Louis(1969) also developed relationships between velocity v and hydraulic gradient J for these regimes in the form

$$v = -KJ^n \qquad (28)$$

With values of hydraulic conductivity K and exponent n given in Table 1.

For zone IV, nearly same equation was obtained by (Witherspoon et al., 1980). These equations have been

Hydraulic Zone	Hydraulic Conductivity, K	Exponent, n	Flow Condition
I	$\frac{gb^2}{12\nu}$	1.0	Laminar
II	$\frac{1}{b}\left[\frac{g}{0.079}\left(\frac{2}{\nu}\right)^{0.25} b^3\right]^{4/7}$	$\frac{4}{7}$	Turbulent
III	$4\sqrt{g}\log\left[\frac{3.7}{\epsilon/D_h}\right]\sqrt{b}$	0.5	Turbulent
IV	$\frac{gb^2}{12\nu[1+8.8(\epsilon/D_h)^{1.5}]}$	1.0	Laminar
V	$4\sqrt{g}\log\left[\frac{1.9}{\epsilon/D_h}\right]\sqrt{b}$	0.5	Turbulent

Table 1. Values Hydraulic Conductivity K and exponent n (Louis 1969).

REVIEW PAPER

discussed by Wittke (1990). Illangaskare .T, Amadei. B and Chinnaswamy. C (1992) developed a finite element computer programme, called CRFLOOD, to compute uplift pressure distribution along the cracks of the concrete gravity dams for all types of flow.

When crack walls contact each other at discrete points, then fluid will flow through a few preferred paths (channels) which are tortuous and having variable aperture along their length and may or may not intersect with each other. Under these conditions, LCL equation does not provide the correct result (Gangi, 1978; Brown and Scholz, 1985; Brown, 1987; Brown, 1995) at low velocity due to formation of tortuous path at contact points (Iwai, 1976; Brown, 1987; Sisavath et al., 2003) and aperture regions of recirculation due to streamline separation in flow field (Brush and Thompson, 2003; Bout et.al., 2006). If an equivalent hydraulic aperture can be found for the open regions of the fracture, the effect of asperity regions can be modeled by assuming that the fracture consists of regions of aperture $b=b_0$. Many researchers have found the following equivalent hydraulic aperture to be generally valid (Liu, 2005).

$$b_H^3 = b_0^3 \frac{1-c}{1+c} \qquad (29)$$

Where $c=1-a/a_0$ is the fraction of the fracture plane occupied by the asperity region, 'a' is the half length of cracks and a_0 is half spacing between cracks. This expression has been validated for asperity concentration up to 0.25 (Zimmermann et al., 1992). Also, the contact regions are modeled as randomly oriented ellipses of aspect ratio $\alpha \leq 1$ by the following expression (Liu, 2005).

$$b_H^3 = b_0^3 \frac{1-\beta_0 c}{1+\beta_0 c} \qquad (30)$$

Where $\beta_0 = \frac{(1+\alpha)^2}{4\alpha}$ Although contact areas are not perfectly elliptical, the above equation can be used if asperity shapes are replaced by equivalent ellipses having same perimeter-area ratios.

When walls of fractures contain cracks and fissures, then no-slip boundary condition is violated and LCL equation does not provide the correct result for volumetric flow. Berman (1953) was the first to investigate the effects of wall

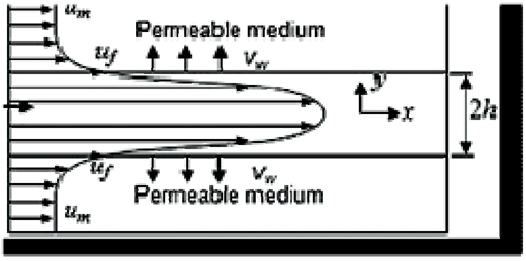

Figure 6. Velocity profile for coupled parallel flows within a channel and bounding porous medium according to the slip flow hypothesis of Beavers and Joseph (Mohais et.al. (2012))

porosity. Berkowitz (1989) studied the permeable walls incorporating Brinkman slip boundary conditions in 1D modeling of fractures having permeable walls and concluded that, no-slip underestimates the volumetric flow by as much as 19%. Similar deviations were also noticed by Crandall et al. (2010). Flow experiments have shown that in such situations, slip boundary condition should be applied (Beavers et al.,1970) and therefore the LCL equation is modified after incorporating slip boundary condition. Study of Tilton and Cortelezzi (2006) have shown that wall permeability can significantly destabilize flows in channels with permeable walls compared to those with impermeable walls. Chang et.al. (2006) and Liu et.al. (2006) studied the instability of the flow and used a two-layer approach by coupling governing equations in the separate Newtonian fluid and Darcy/Brinkman porous region by appropriate interfacial boundary conditions. Beavers and Joseph (1967) hypothesized that the tangential component u_f of free fluid velocity at the boundary of the permeable wall is considerably higher than mean seepage velocity u_m within the permeable body (Figure 6) and can be defined as

$$\frac{du_f}{dy} = \frac{\delta}{\sqrt{k}}(u_f - u_m) \qquad (31)$$

Where k is the wall permeability and δ is a dimensionless quantity that depend on the structure of the permeable material within the boundary region, flow direction at the interface and Reynolds number (Sahraoui and Kaviany, 1992).

This Beaver-Joseph slip flow hypothesis is valid for common viscous fluids at low Reynolds number (Neale and Nader, 1974). Mohais et al. (2011) present an analytical solution,

REVIEW PAPER

incorporating Beaver-Joseph slip boundary conditions, using perturbation expansion to determine flow. Mohais et al. (2012) extended this study further and concluded that the hydraulic conductivities are modified by a factor of (-3/C). Expression for C is

$$C = \frac{-3}{1+3\sigma} + R_{e_w}\left(\frac{9}{140(1+3\sigma)^3}(7\sigma+1) + \left(\frac{3+6\sigma}{2+6\sigma}\right)^2\right) \quad (32)$$

Where $\sigma = \frac{\sqrt{k}}{\delta h}$ is slip coefficient and $R_{e_w} = \frac{V_w h}{\nu}$ is wall Reynolds number. Thus the transmissivity in LCL equation can be written as

$$T = \frac{wb^3}{12}\left(\frac{-3}{C}\right) \quad (33)$$

Figure 7 shows the variation of -3/C with Φ for $\delta = 0.1$.

In two-phase flow, the determination of volumetric flow rate is a challenging problem. The applicability of cubic law for

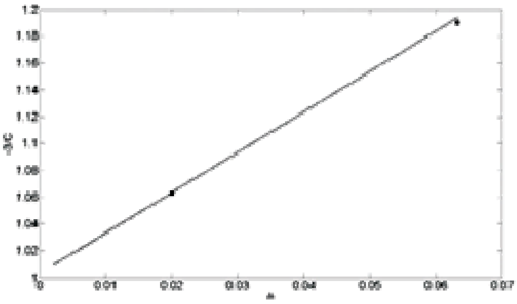

Figure 7. The variation of the correction factor, -3/c, with ∅, for permeability values between $10^{[illegible]} m^2$ and $10^{[illegible]} m^2$, h = 0.00001m, Rew = 0.004.

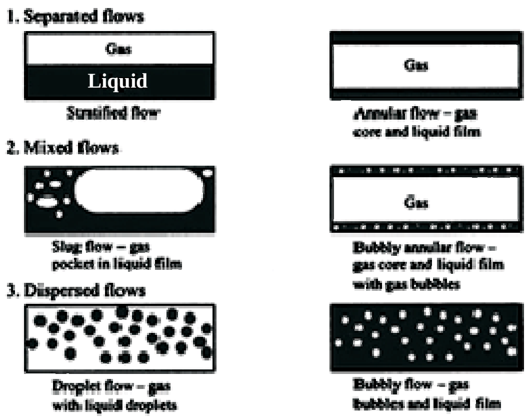

Figure 8. Different modes of two-phase flow (Ishii(1975), Fourar and Bories(1995))

flow rate calculations were earlier investigated by some researchers (Iwai, 1976; Tsang and Witherspoon, 1981; Tsang and Witherspoon, 1983; Brown, 1987). Fourar et al. (1993) observed different flow pattern in their parallel plate model experiments. The two-phase air-water flow undergoes different modes of flow under the influences of pressures and different flow conditions. The classification of possible flow pattern by Ishii (1975) and by Fourar and Bories (1995) is given in Figure 8.

In two-phase-stratified flow, the air-water interface depends on aperture and pressure variations and therefore there is a need of identifying this interface (Nichole and Glass, 2001). Rasmussen (1991) applied the capillary pressure criteria for determining the air-water interface to study the fracture flow under the conditions of partial fluid saturation. Pruess and Tsang (1990) conducted a numerical analysis based on relative permeability of two-phase flow subject to pressure difference between air and water. The interrelation of multiphase flows, relative permeability, and pressure variation through rock fractures has not been properly identified (Indraratna et al., 2003). Indraratna et al. (2003), after following the idea of Keller et al. (2000), argued the existence of two-phase stratified flow in cracks of size greater than 50 micron and thus neglecting the effect of capillary pressure. To compute the flow rate, LC Lequation was applied separately to each zone of fracture aperture occupied by air and water. To determine these zones, Indraratna et al. (2003), derived the expression for locating the air-water interface after taking into account the effects of solubility of air in water, compressibility and density of air and water and fracture deformation, in the equation of two-phase momentum conservation equation by Wallis (1969).

Conclusions

A large number of phenomenons affect flow behavior in a crack. Various investigators used the continuity equation for single phase assuming the absence of source and sink terms which must be accounted especially for the cases where fracture walls have appreciable amount of fissures and cracks. Source and sink terms are included in mass balance equations used for two-phase flow in cracks to account for inter-phase mass transfer and still does not

REVIEW PAPER

account for fluid leakages through fracture walls.

Momentum equation is widely discussed in the literature. Starting from the Navier-Stokes equations, the momentum equation has been simplified under various assumptions to derive Stokes equation and then Reynolds lubrication equation. Applicability of these equations to solve flow problems in cracks are under severe conditions and in most of the cases, they are computationally complex. So momentum equation is solved on analogy of flow in smooth parallel plate with no-slip boundary condition on the plate boundary and resulting equation was termed as Local Cubic Law (LCL). Validity of LCL has been tested by various investigators using laboratory or field data. Deviations of laboratory or field data from LCL equation were attributed to various factors like wall roughness and its statistics, representations of crack aperture, non-linearity of LCL equations, pervious fracture walls, fracture wall contacts etc.

Shifted Gaussian model were shown to be best fit for the description of aperture distribution of rough fractures. Also it is found that, in general, rough walled fractures possess a self-affine fractal property with Hurst exponent of 0.8.

Various methods of measurement of mechanical aperture are reported in the literatures. Relations between mechanical apertures with hydraulic aperture (used in LCL equation) have been obtained by various researchers using the parameters of aperture statistics.

The LCL equation is found to be invalid when Reynolds number is greater than 10 and it is shown that when Reynolds number is between 10 and100, a non-linear relation exists between pressure gradient and volumetric flow rate. For rough walled fractures, Louis (1969) equations for flow of liquids in wide open fractures are widely used, but still these equations does not consider the permeability of fracture walls and its validity for flow of gases in the fractures is open to question. A limited attempt in literature is available to estimate the hydraulic aperture when walls of the fracture contact each other. Because of limited application of hydraulic aperture in case of contact walls of the fracture, there is need for its general representation.

There are few published works in literature to study the effect of slip boundary condition in case of permeable fracture walls. Work of Mohais et al. (2012) is important in deriving the correction factor in hydraulic conductivity of fracture using Beaver-Joseph slip boundary conditions. But this correction is applied for smooth walled fractures and does not consider the effect of roughness of fractured walls.

Works of Ishii (1975) and Fourar et al. (1993) show the different flow regimes of two phase flow in cracks and formulation of momentum equation for each regime is a difficult task. Indraratna et al. (2003) attempted to calculate the volumetric flow rate of each flow for separated flow regimes.

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RESEARCH PAPERS

FORMULATION OF TRANSIENT UPLIFT PRESSURE IN CRACKS OF CONCRETE GRAVITY DAMS

By

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ABSTRACT

Safety guidelines recommend the presence or absence of an assumed static uplift pressure in cracks at dam-foundation interface during seismic activity. In recent past, some research work has been done to develop a mathematical model for transient uplift pressure in smooth walled cracks under laminar flow conditions. In this paper, a mathematical model for transient uplift pressures in real cracks under both laminar and turbulent flow regimes are developed. One dimensional continuity and momentum equations have been coupled to derive the governing integral equations of pressure as a function of crack wall motion history and flow regimes (i.e. laminar/turbulent). These equations are supplemented with required number of boundary conditions based on the understanding of hydraulic phenomenon described in the literatures. Model is used to solve the transient uplift pressure in wedge shaped crack of constant length and is validated using the experimental data from literature. A difference (less than 5%)between computed and measured value of uplift pressure occurs at beginning and end of the opening-closing cycle of the crack in comparison to other points of time. These differences may be attributed to violation of some assumptions used in the problem formulation (e.g. vapour pressure at point of saturation may not be zero and two-phase flow may occur in unsaturated portion of the crack) or the possibility of cavitation in unsaturated portion of the crack. However, overall computed pressure variation obtained using present formulations are in good agreement with experimental data from the literature.

Keywords: Uplift Pressure, Concrete, Crack, Fluid.

INTRODUCTION

Concrete gravity dams are structures that rely on their own weight for resistance against sliding and overturning to maintain stability. With the development of design and analytical expertise, as well as of construction techniques and equipment, dams have become ever larger with regard to both height and volume. If a dam on this scale were to fail and collapse, this could lead to probably the greatest disaster in human history. Therefore, the safety of huge structures such as concrete gravity dams is of utmost concern to the engineers involved in the design, construction and post-built safety evaluation of dams.

Many concrete gravity dams have experienced cracking problems to various extents. Crack formation and propagation in concrete gravity dams could influence their structural stability and endanger the safety of the dams. Normally, the huge size of a concrete dam excludes direct experimental tests on the structural cracking behaviour under various loading conditions. Therefore, evaluation of possible cracking trajectory in concrete dams by means of an accurate constitutive model, in order to simulate the cracking response of the concrete effectively, becomes vital and would be a useful tool for practising engineers to ensure the safety of dam structures.

1. Literature review

In past decades (Bhattacharjee & Léger, 1994; Saouma et al., 1991), study of concrete gravity dams were mainly focused to address the fracture failure of the materials and its influence on dynamic response of the dam without much concern for safety considerations. Also, study of the concrete gravity dams with penetrated cracks attracted much attention in the past. (Pekau & Zhu, 2006).

A static triangular uplift pressure at the base of the dam with no-drainage condition is assumed in safety guidelines (ICOLD, 2014; USACE, 1992; USBR, 1987). However, there are no unified considerations for uplift pressure developed during dynamic pressure variations in the reservoir when cracks develop either in the body or at the dam-foundation interface of the concrete gravity dams. Chávez and Fenves (1995) and Chopra and Zhang (1991) studied the safety of concrete gravity dam under seismic condition after assuming a constant triangular uplift pressure at the base of the dam with crack.

To consider the transient nature of the uplift pressure, Hall (1998) assumed that pressure at the mouth of the crack varies in accordance with hydro dynamic pressure due to dam reservoir interaction and uplift pressure remains constant with triangular variation. Zhang and Ohmachi (1998)used this model for seismic crack analysis without crack wall motion to validate the experimental results of Ohmachi et al. (1998). Yi et al. (1997) developed a one dimensional fluid pressure relation for the growing crack in brittle materials under hydrodynamic pressure conditions and combined their results with fracture mechanics model to compute crack tip stress intensity. However, this model was developed for small crack length that is not applicable for the case of cracks in concrete gravity dams where crack lengths are much longer.

There are only few experimental and theoretical studies for uplift pressure distribution in cracks during hydrodynamic loading. Slowik and Souma (1994) conducted dynamic wedge-splitting (WS) test to measure the water pressure during propagating cracks for slow and rapid opening velocities of crack walls. During slow opening, the water front and crack front velocities were almost equal, while during rapid wall movement, the same are quite different. They also reported the results for sudden crack closing and cyclic opening and closing of existing cracks. The result indicates that in sudden crack closure case, the water is trapped in the crack resulting in a temporary over pressurization. This over pressurization induces additional stresses causing failure of the specimen. During cyclic opening and closing mode, pressure variations along the crack were different with maximum pressure (which was more than initial hydrostatic pressure) reached during closing of crack walls. Slowik and Saouma (2000) proposed a theoretical model for opening mode considering only laminar flow in the cracks. Laminar flow based model is used extensively by different researchers for hydro mechanical coupling analysis of rock masses (Jing et al., 2001). Tinawi and Guizani (1994) developed a theoretical model for hydrodynamic model for pre-existing cracks in concrete gravity dams using laminar flow in cracks. These methods are not applicable for uplift pressure computations in cracks of concrete dams, because turbulent and unsteady flows occur in cracks during hydrodynamic condition. Independent experimental works by Louis (1969) and Lomize (1951) provided almost identical results for relationships between velocity and hydraulic gradient in cracks after taking into account the different flow condition (i.e., turbulent/laminar) and crack wall roughness. Illangasekare et al. (1992) developed a finite element computer programme, called CRFLOOD, to compute uplift pressure distribution along the cracks of the concrete gravity dams for all types of flow characterized by Louis (1969).

Transient pressure variation in single cohesive crack is developed in the present study. Transient nature of pressure is introduced through crack wall motion. The crack wall motion may be caused either by earthquakes, creep phenomenon in fracture process zone (FPZ) or fatigue due reservoir level variations.

2. Hydrodynamic Description of Water Flow

Based on the test results for temporal and spatial pressure variation along the crack length of existing and new cracks, Javanmardi et al. (2005a) developed the hydraulics of one dimensional water flow in a single crack under moving impermeable walls (Figure1).

Due to pushing of the water by existing reservoir water pressure at crack mouth, water flows from crack mouth towards the crack tip during its first opening cycle in both existing and new cracks. Water fills the voids developed due to crack opening and saturates completely the existing crack while propagating crack is saturated partially with saturation length L_{sp}. The magnitude of pressure varies along full length of the existing crack, but in propagating crack, pressure decreases from reservoir pressure p_m at crack mouth to void pressure at the end of saturation length. The void is occupied by both water vapour and air. So the void pressure is determined by using certain thermodynamic equilibrium equation representing two-component single phase flow. But when there is no air in the concrete, the pressure decreases to water vapour pressure (cavitation) and water vapour fills the voids.

During crack closing, volume of the voids decreases due to squeezing out of water from existing crack and propagating crack as shown in Figure 1 ((b), (d) & (f)). On increasing the crack closing velocity, the existing water is pressurized and now it flows in two opposite directions from a stagnation point along the saturation part of the crack in propagating crack, while same phenomenon is found in existing crack after first opening –closing cycle as shown in Figure 1 ((c) & (d)). The pressure at stagnation point is maximum and it decreases from this point towards

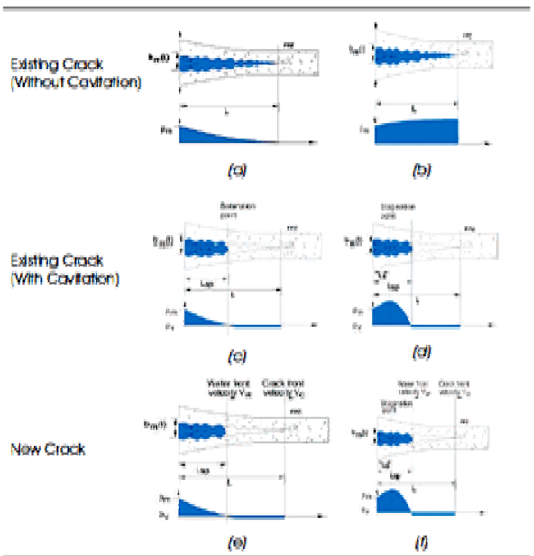

Figure 1. Water flow and pressure distribution in existing and new crack with moving walls(Javanmardi et al., 2005b).

the saturation region boundaries to become equal to the pressure at these two points.

In subsequent opening cycles, new voids are developing along propagating crack with water flow along crack from mouth towards crack tip during opening cycle. Water flow can only fill the voids close to the crack mouth and saturation length decreases. The only difference between the first opening and subsequent opening cycles is the existence of some water in residual opening that was already filled due to crack closing. The water flow and corresponding pressure during second and subsequent cycle is therefore similar to that of the first closing cycle except that there is already some water in wetted unsaturated region. Thus, saturation length may be longer at the end of the second closing cycle. The case of propagating crack with saturated and unsaturated part is basically similar to an existing crack when cavitation occurs.

3. Theoretical Modelling

Javanmardi et al. (2005b) developed a theoretical modelling using the momentum equations proposed by Louis (1969) and discussed by Wittke (1990) for un-penetrated wedge shaped existing cracks under the following assumptions (Figure 2 and Figure 3):

- Walls of the crack are impermeable and rigid. This assumption can be introduced, as the fluid permeability of cracks is significantly higher than that of un-cracked concrete.
- Flow is one-dimensional along straight length of the wedge crack.
- Water compressibility is negligible.

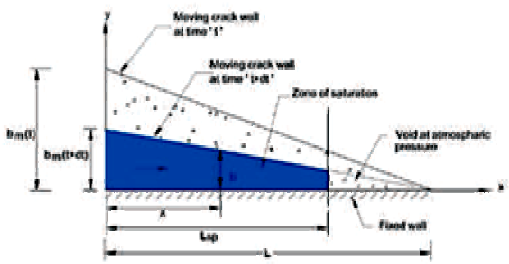

Figure 2. Assumed crack in opening mode

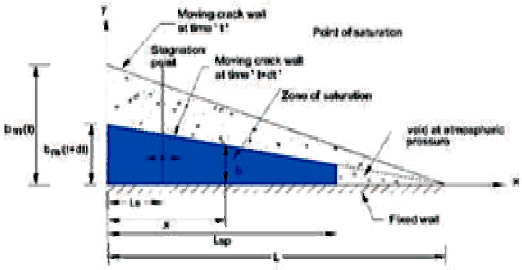

Figure 3. Assumed crack in closing mode

- The flow in unsaturated part is insignificant.
- Water vapor pressure is zero.
- Density ρ is independent of space.
- There is no sink or source.
- Lower walls of the wedge shaped crack are fixed and only upper wall moves.
- Stiffness is constant.
- Crack width is of unit dimension.
- Crack length is fixed.
- Velocity head gradient is neglected in both laminar and turbulent flow.
- Flow in crack is a single phase flow and there is no interaction between air and water.

3.1 Continuity Equation

3.1.1 Opening mode

1-D continuity equation, widely used in the literature (Sarris & Papanastasiou, 2012; Slowik & Saouma, 2000) is written with the assumption that fluid leakage through walls of the crack are negligible i.e., walls of the crack are impermeable. For any cross sectional area A normal to flow direction x, the 1-D continuity equation can be written as

$$\frac{\partial(\rho Q)}{\partial x} = \frac{\partial(\rho A)}{\partial t} \qquad (1)$$

Where, Q is the discharge through the crack

Considering unit thickness, equation (1) can be modified as:

$$\rho\frac{\partial Q}{\partial x} + \frac{\partial(\rho b)}{\partial t} = 0 \qquad (2)$$

Where b is the crack opening at any distance x from the crack mouth and time t.

Neglecting the water compressibility, equation (2) can be written as

$$\frac{\partial Q}{\partial x} + \frac{\partial b}{\partial t} = 0 \qquad (3)$$

For rigid crack wall,

$$b = b_m\left(1 - \frac{x}{L}\right) \qquad (4)$$

Where b_m is the crack mouth opening distance (CMOD) and L is the crack length. Using value of b in equation (4) and using boundary condition, that at x = L_{sp} (Saturation length), Q=0; Equation (3) can be integrated as

$$Q = \frac{\dot{b}_m}{L}\left[\left[L(Lsp - x) - \frac{1}{2}\left(Lsp^2 - x^2\right)\right]\right] \qquad (5)$$

where $\dot{b}_m$ is the crack mouth opening displacement (CMOD) rate. Equation (5) gives the discharge equation for flow in wedge shaped rigid crack in a concrete gravity dam during crack opening mode.

3.1.2 Closing mode

During the closing phase, there occurs a point along the crack where discharge becomes zero. This point in the literature, is called the stagnation point. Water moves in both the directions from the stagnation point during the closing process of crack mouth. The distance of the point from the crack mouth is termed as stagnation length and denoted as L_s as shown in Figure 3.

The discharge formula given by equation (5) is modified for closing mode after using the boundary condition that at x = L_s, Q = 0.

$$Q = \frac{\dot{b}_m}{L}\left[\left[L(Ls - x) - \frac{1}{2}\left(Ls^2 - x^2\right)\right]\right] \qquad (6)$$

Equation (6) gives the discharge equation for flow in wedge shaped rigid crack in a concrete gravity dam during crack closing mode.

3.2 Momentum equation

(Louis 1969) developed relationships between velocity v and hydraulic gradient J for these regimes in the form

$$v = -KJ^n \qquad (7)$$

Where K is hydraulic conductivity and n is an exponent.

Work of (Louis (1969)) has been utilized for present formulation. Javanmardi et al. (2005a) have shown that for concrete cracks, the relative roughness $\frac{\varepsilon}{D_h}$=0.5 is a good approximation. Therefore for this relative roughness value, the crack is assumed to be hydraulically rough $\left(\frac{\varepsilon}{D_h} > 0.033\right)$. Louis (1969) flow diagram shows that hydraulic zone IV (laminar flow) and hydraulic zone V (turbulent flow) are applicable for hydraulically rough crack. Flow changes from laminar to turbulent at Reynolds number $\left(R_e = \frac{2Q}{\nu}\right)$ nearly 2300. Using the Louis (1969) flow table, the momentum equations are written for laminar and turbulent flow separately.

For Laminar Flow n = 1 and

$$K = \frac{gb^2}{12\nu\left[1 + 8.8(\varepsilon / D_h)^{1.5}\right]}$$

Where ν is the kinematic viscosity of water and K is the hydraulic conductivity. Therefore, for laminar flow, Louis (1969) momentum equation becomes

$$Q = -KbJ \qquad (8)$$

where $J = \frac{\partial H_T}{\partial x}$ hydraulic gradient and H_T is the total head given by

$$H_T = z + \frac{p}{\gamma} + \frac{v^2}{2g} \qquad (9)$$

Here z is the location of crack which is fixed. On assumption that velocity head is negligible,

$$J = \frac{\partial H}{\partial x} = \frac{1}{\gamma}\frac{\partial p}{\partial x} \quad \left(\frac{\partial z}{\partial x} = 0\right) \qquad (10)$$

Where H is now pressure head. Therefore Equation (8) simplifies to

$$Q = -C_2 b^3 \frac{\partial p}{\partial x} \qquad (11)$$

Where

$$C_2 = \frac{g}{12\nu\gamma\left[1 + 8.8(\varepsilon / D_h)^{1.5}\right]}$$

For Turbulent Flow, n = 0.5, and

$$K = 4\sqrt{g}\log\left[\frac{1.9}{\varepsilon / D_h}\right]\sqrt{b}$$

Therefore for turbulent flow (Louis, 1969), momentum equation becomes

$$Q = -KbJ^{0.5} \qquad (12)$$

After inserting the values of K and J in Equation (12), the resulting equation is

$$Q^2 = C_3 b^3 \frac{\partial p}{\partial x} \qquad (13)$$

Where,

$$C_3 = (16g/\gamma)\log\left[1.9/(\varepsilon / D_h)\right]^2$$

3.3 Transient Pressure Calculation

For the sake of clarity, $\left(\frac{dp}{dx}\right)_l$ and $\left(\frac{dp}{dx}\right)_t$ are used respectively for laminar and turbulent pressure gradients in subsequent discussions. Pressure calculation starts from the determination of opening $b_m \geq 0$ and closing $b_m \leq 0$ mode of the crack. The respective continuity and momentum equations for laminar and turbulent flows are coupled together when solving the differential equation for pressure. The laminar and turbulent flow regions are separated by assuming transition lengths. It is assumed that when Reynolds number $R_e \left(= \frac{2Q}{\nu}\right) \leq 2300$, then flow is laminar, otherwise flow is turbulent. These criteria can be used to find the transition lengths or it can be calculated iteratively. Pressure in crack varies from hydrostatic pressure at mouth to vapour pressure (taken as zero) at the tip of crack. Considering these factors in mind, the integral equations of pressure are written separately for opening and closing mode.

3.3.1 Opening Mode

In opening mode, laminar flow occurs near the crack mouth. The position where from laminar flow changes to turbulent flow is denoted by L_t. L_t is calculated using expressions for Reynolds number and continuity equation for opening mode. The integral equation (5) for variation of pressure along the crack is written as

$$P(x,t) = \begin{cases} p_m + \int_0^x \left(\frac{dp}{dx}\right)_l dx & \{0 \leq x \leq L_t\} \\ p_m + \int_0^{L_t} \left(\frac{dp}{dx}\right)_l dx + \int_{L_t}^x \left(\frac{dp}{dx}\right)_t dx & \{L_t \leq x \leq L_{sp}\} \end{cases} \qquad (14)$$

Here L_{sp} is the saturation length of crack where the pressure at any given time is assumed to be zero. Using this condition, L_{sp} is calculated as

$$p_m + \int_0^{L_t} \left(\frac{dp}{dx}\right)_l dx + \int_{L_t}^{L_{sp}} \left(\frac{dp}{dx}\right)_t dx = 0 \qquad (15)$$

3.3.2 Closing Mode

In closing mode, flow starts from stagnation point in both upstream and downstream directions. Flow is laminar near the stagnation point. With the help of Reynolds expression and continuity equation (6) for closing mode, two transition lengths L_{tu} and L_{td} can be calculated where change of flow from laminar to turbulent occurs. Denoting the stagnation length and stagnation pressure by L_s and p_s respectively, the pressure variation for upstream ($x \le L_s$) and downstream ($x > L_s$) can be written as follows.

Pressure in upstream section ($x \le L_s$)

$$P(x,t)=\begin{cases} p_m+\int_0^x\left(\frac{dp}{dx}\right)_t dx & \{0\le x\le L_{tu}\} \\ p_m+\int_0^{L_{tu}}\left(\frac{dp}{dx}\right)_t dx+\int_{L_{tu}}^x\left(\frac{dp}{dx}\right)_t dx & \{L_t\le x\le L_{sp}\} \end{cases} \quad (16)$$

Pressure in downstream section ($x > L_s$)

$$P(x,t)=\begin{cases} p_m+\int_0^{L_{tu}}\left(\frac{dp}{dx}\right)_t dx+\int_{L_{tu}}^{L_s}\left(\frac{dp}{dx}\right)_t dx+\int_{L_s}^x\left(\frac{dp}{dx}\right)_t dx & \{L_s\le x\le L_{td}\} \\ p_m+\int_0^{L_{tu}}\left(\frac{dp}{dx}\right)_t dx+\int_{L_{tu}}^{L_s}\left(\frac{dp}{dx}\right)_t dx+\int_{L_s}^{L_{td}}\left(\frac{dp}{dx}\right)_t dx+\int_{L_{td}}^x\left(\frac{dp}{dx}\right)_t dx & \{L_{td}\le x\le L_{sp}\} \end{cases} \quad (17)$$

In the above set of equations, L_s, P_s and L_{sp} are unknown parameters and can be determined by using two boundary conditions and Reynolds transport theorem for mass conservation as follows.

3.3.3 Boundary Conditions

First boundary condition:

When $x = L_s$ then $p = p_s$ therefore

$$p_m+\int_0^{L_{tu}}\left(\frac{dp}{dx}\right)_t dx+\int_{L_{tu}}^{L_s}\left(\frac{dp}{dx}\right)_t dx = p_s \quad (18)$$

Second boundary condition:

When $x = L_{sp}$ then $p = 0$ and hence

$$p_m+\int_0^{L_{tu}}\left(\frac{dp}{dx}\right)_t dx+\int_{L_{tu}}^{L_s}\left(\frac{dp}{dx}\right)_t dx+\int_{L_s}^{L_{td}}\left(\frac{dp}{dx}\right)_t dx+\int_{L_{td}}^{L_{sp}}\left(\frac{dp}{dx}\right)_t=0 \quad (19)$$

Third boundary condition:

Reynolds transport equation for mass conservation (White, 1990) for elemental control volume can be written as

$$\left(\frac{dm}{dt}\right)=\frac{d}{dt}\iiint \rho dV+\iint \rho(v.n)dA=0 \quad (20)$$

Where m and ρ are mass and density of water; V is the water velocity vector and n is the outward unit normal vector on surface dA of control volume dV.

For the present case, a control volume lying between crack mouth and stagnation point as shown in Figure 4 is assumed. Applying the above equation and noting that at stagnation point, velocity of water is zero, equation (20)

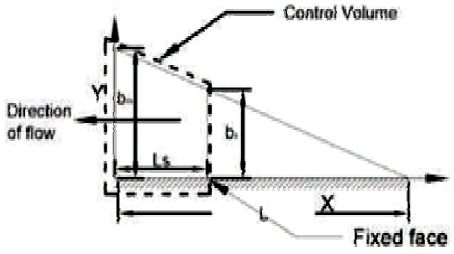

Figure 4. Control volume for calculation of stagnation point

yields

$$\frac{d}{dt}\left\{\frac{1}{2}L_s b_m\left(2-\frac{L_s}{L}\right)\right\}+Q_m=0 \quad (21)$$

where Q_m is the discharge at crack mouth.

$$Q_m=-\frac{\dot{b}_m}{2L}(2LL_s-L_s^2) \quad (22)$$

Equation (21) is first differentiated w.r.t. time and then solved by method of separation of variables with the condition that $L_s = L$ when $b_m = b_{mr}$. The resulting expression is

$$L_s=L-L\sqrt{\left(1-\frac{b_{mr}}{b_m}\right)} \quad (23)$$

With these known values, the set of equations (14), (16) and (17) are solved to get the pressure distribution along the crack. Uplift force in the crack is calculated by integrating P(x,t) for both opening and closing modes.

4. Results and Discussion

Experimental data for transient uplift pressure in Javanmardi et al. (2005b) for existing 4 m long wedge shaped crack is adopted to verify the present model. Crack mouth subjected to a constant pressure of 500kPa, was imparted to a sinusoidal motion of 2 Hz with residual and maximum crack mouth opening of 0.5mm and 1.0 mm respectively.

Thus CMOD function and CMOD-rate (also called Crack Mouth Opening Velocity (CMOV)) is represented respectively as

$$b_m=1.0+0.5\sin(1.5+4t)\pi \ (mm) \quad (24)$$

and

$$\dot{b}_m=2\pi\cos(1.5+4t)\pi \ \left(\frac{mm}{s}\right) \quad (25)$$

where, t is the time in seconds. Above functions are

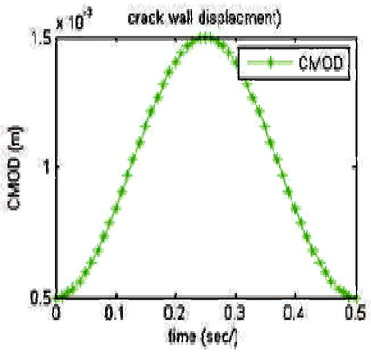

Figure 5. CMOD Vs. time plot of Equation [24]

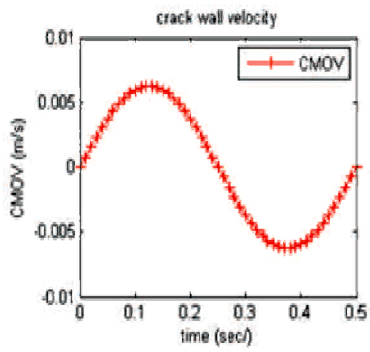

Figure 6. CMOV Vs. time plot of Equation [25]

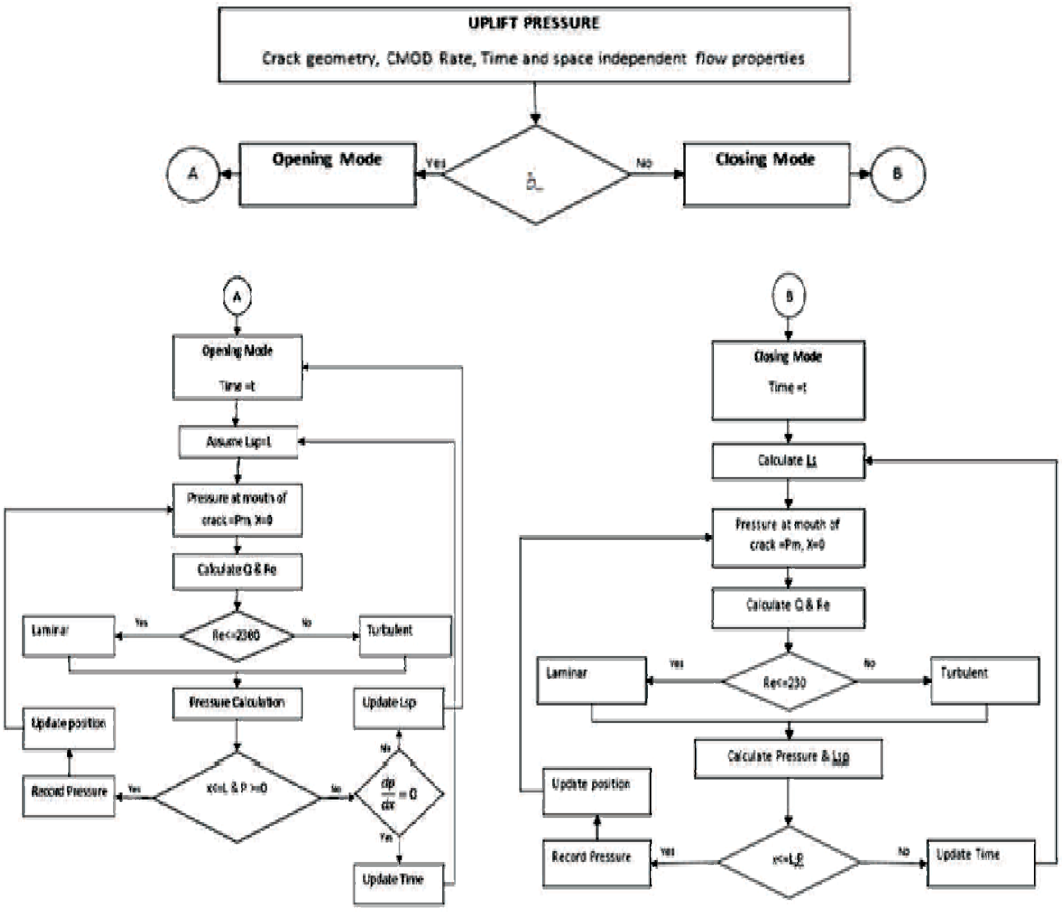

Figure 7. A composite flow chart of uplift pressure calculation for opening and closing mode of the crack

represented graphically in Figure 5 and Figure 6.

At the beginning, uplift pressure in the crack is assumed to be uniform and equal to crack mouth pressure i.e., 500 kPa. From time t=0 to t=0.25 s, crack mouth opens at residual opening of 0.5 mm and reaches to its maximum value of 1.5mm. At the end of one cycle, i.e. at time t=0.5 s, crack mouth acquires its initial crack mouth of 0.5 mm and uniform pressure of 500 kPa. Time independent constants in pressure formulation for water at 25°C are given in Table 1.

A general flow chart for calculating the transient uplift pressure for present and subsequent formulation is given in Figure 7. This flow chart has been translated in MATLAB programming.

Result of present formulation and few of the experimental data in Javanmardi et al. (2005b) are compared and shown in Figure 8 and Figure 9.

A difference between computed and measured value of uplift pressure seems a little bit more (less than 5%) at the beginning (t=0.001 s) and end (t=0.43s) of the cycle in comparison to other point of time. These differences may

Time Independent Constants	SI Unit	Value
Unit weight,	$[ML^{-2}T^{-2}]$	10000
Kinematic Viscosity	$[L^2T^{-1}]$	7.24×10^{-7}
Relative Roughness	-	0.5
Transitional Reynolds No	-	2300
C_1	$[L^1T^{-1}]$	2.762×10^{6}
C_2	$[L^1T^{-2}]$	52.7612

Table 1. Time Independent constants used transient uplift pressure formulations

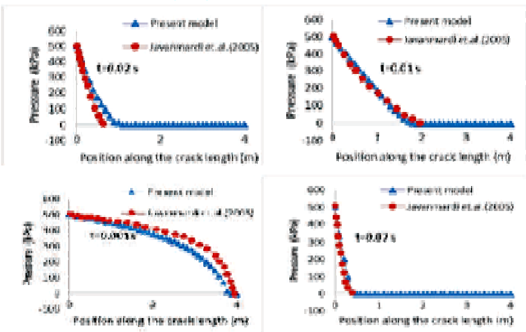

Figure 8. Pressure variation along the crack during opening mode at time t=0.001, 0.01, 0.02 and 0.07 s.

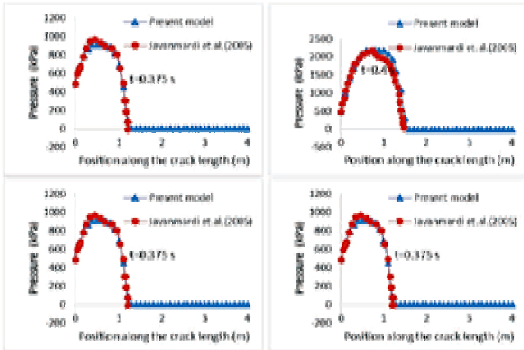

Figure 9. Pressure variation along the crack during closing mode at time t=0.375, 0.4, 0.43 and 0.45 s

be attributed to violation of some of the assumed boundary conditions used in the formulation (e.g. vapour pressure at point of saturation may not be zero and two-phase flow may occur in unsaturated portion of the crack) or occurrence of cavitation phenomenon in the unsaturated portion of the crack. However, overall computed pressure variation obtained using present variation is in good agreement with the measured data by Javanmardi et al. (2005b).

Summary and Conclusion

A theoretical model is developed to analyze the effect of dynamic loading on the stability of concrete gravity dam considering transient uplift pressure in cracks. One dimensional governing equation as a function of crack wall motion history, crack length, crack roughness, and crack mouth pressure is developed to calculate the transient uplift in crack during crack wall movement.

Formulation is validated using experimental data from the literature.

The computed uplift pressures indicate that, during crack opening, water pressure develops near the crack mouth and there is no water pressure along the rest of the crack. Due to the short saturated length during the crack opening mode, the uplift force is relatively small but not equal to zero. A general qualitative description for water pressure during crack opening mode is somewhere between zero (USBR, 1987) and unchanged (USACE, 1992) uplift pressure. During closing of cracks, stagnation point is created within the length of the crack where pressure is maximum. The shape of pressure spatial distribution along an unsaturated crack during its closing mode is like a dome. Its magnitude at the boundaries is equal to pressure magnitude at these points (hydrostatic pressure at crack mouth and vapor pressure at the end of the saturated region), and its maximum magnitude occurs at the location of the stagnation point.

The magnitude of developed uplift force depends on the crack opening characteristics and crack mouth pressure as indicated by the novel water–crack interaction model developed in this paper. The full uplift pressure assumption (ICOLD, 2014)seems very conservative for stability calculation of cracked concrete dams during an earthquake.

Future scope of study

Present formulation for flow in cracks can be modified considering the effect of air and water interaction and designing it as a two phase model. Above model may be improved after taking into account the following: (1) Permeability of crack walls which may act as a source or sink at the time of opening and closing. (2) Elastic/inelastic behavior of crack-wall (3) Water compressibility. (4) Crack propagation and (5) Three dimensional flow in cracks.

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