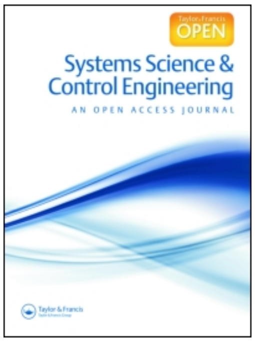

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Novel arrangement of Routh array for order reduction of *z*-domain uncertain system

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ABSTRACT

This paper presents a novel arrangement of Routh table array for deriving an approximate model of a higher order *z*-domain uncertain system. The demand for this computation is to procure a lower order model which is easy to be exercised in comparison to their original large scale systems. Additionally, the derived model should preserve fewer dynamic characteristic of the comprehensive higher order systems. The mentioned new arrangement is achieved from the arena of different combinations of numerator and denominator polynomials. The combinations are validated by their practice over the conventional example from the literature. This precise blend is then applied to a real-time system for its rational acceptability. Both the models play a significant role in establishing the algorithm. Besides this, the limitation encountered during the foundation course of the arrangement is also taken into consideration. The paper also offers a future scope for fellow researchers.

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Discrete-time uncertain systems; bilinear transformation; order reduction; Routh approximation

1. Introduction

Order reduction, an appealing topic of research for control engineers, came into existence in the late 1960s. Its prime concern is to derive an approximate model from the higher order system retaining fewer of the dynamic characteristics. Since then, many reduction methodologies emerged such as aggregation method, balanced truncation method, Pade approximation, continued fraction method, dominant eigenvalue method, Routh approximation (*RA*) and many others. These methods are available in the literature (Bultheel & Barel, 1986; Feng, Zheng, Francis, & Mantooth, 2008; Genesio & Milanese, 1976; Gugercin & Antoulas, 2004; Samuel, Knockaert, & Dhaene, 2014; Sinha & Lastman, 1990). Among the methodologies, *RA* is highly accepted for being computationally effortless and significantly retaining system stability. Various available articles on *RA* (Hwang & Lee, 1997; Narwal & Prasad, 2016; Paraskevopoulos, 1980; Prasad, 2000; Shamash, 1975; Singh, 2005; Singh, Prasad, & Gupta, 2006) prove it.

The rise in the demand of thoroughly analysing the practical system that includes the model dynamics, noise, variation in parameters; disturbances leads to the outcome of uncertain structures. Thus, uncertain or interval systems are systems with constant coefficients, but unknown within a limited range. Since, no competent technique developed for reducing the uncertainty descriptions, investigation moved towards the simplification of the order of uncertain schemes. Algorithm pioneering the approximation of such system is Routh-Pade (Bandyopadhyay, Ismail, & Gorez, 1994), an extension of deterministic systems (Shamash, 1975). It expresses the derivation of a stable reduced model by truncating Routh's table for denominator coefficients and matching time moments for numerator coefficients. The same author proposes a formulation of $\gamma-\delta$ parameters for obtaining reduced-order coefficients (Bandyopadhyay, Upadhye, & Ismail, 1997), resulting in increased computational effort with retention of system stability. These methods pose the limitation of not attaining stable reduced model and is highlighted in the work of Hwang and Yang (1999). This restriction is an outcome of the implementation of interval Routh expansion and inversion algorithms, which confine that algorithm cannot guarantee success in generating a full interval Routh array and the approximant may not be stable even if the original uncertain system is robustly stable. The reason for drawback is the irreversibility of interval arithmetic and is considered by Dolgin and Zeheb (2003), where modification of the method in Bandyopadhyay et al. (1994) assures retention of model stability. Few more queries attempt to sort this limitation through numerical examples (Dolgin, 2005; Yang, 2005). Sastry, Raja Rao, and Mallikarjuna Rao (2000) elaborate an algorithm for 0

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deriving the denominator and numerator coefficients by computing only γ parameters.

With time span, an efficient outcome of digital signals and systems amused the research towards order reduction of discrete-time uncertain systems. Literature available for such systems are few but proficient – remarkably, Pade approximation allowing dominant poles' retention (Ismail, Bandyopadhyay, & Gorez, 1997) and μ-dependent approach (Zhang, Boukas, & Shi, 2009). Hsu and Wang (2000) developed a higher order integrator approach to approximate discrete-time interval model for the continuous-time interval systems sampled by a ZOH. Dolgin and Zeheb (2004) states the demand of assuming the required order polynomial coefficients at an earlier stage, for computation of error between the polygons in the complex plane (original uncertain system) and the point representing the resulted reduced-order fixed-coefficients systems for deriving the reduced-order model. In recent times, algorithms that expressed their advancement from deterministic systems to uncertain systems are work by Choudhary and Nagar (2013a, 2013b, 2015b, 2016), namely direct truncation, gamma–delta approximation and Routh-Pade approximation. Freshly, the article (Choudhary & Nagar, 2015a) offers the application of existing algorithms to power systems components.

From the available and cited literature, *RA* techniques advanced from deterministic systems to uncertain structures. This paper addresses a novel approach for order reduction of the discrete-time uncertain system. The prime motive of this method is to offer a satisfactorily reduced model that retain the significant property of model stability. The realm for attaining novelty is the possible combinations (mentioned as *cases* in the paper) of the numerator and denominator polynomials for computing the Routh table array. The paper is organized in five sections. Section 2 presents the proposed methodology. It also contains the higher and lower order system representation, the proposed algorithm *cases*, validation and stability check procedures. Section 3 includes an example available from literature in support of the proposal. This segment also validates the confronted combination *case* over a real-time test system. A brief discussion of the findings and the limitation through the proposed algorithm is in Section 4. Finally, Section 5 ends with an outcome of the new and promising algorithm.

2. Problem methodology

2.1. System representation

Let the higher order discrete-time uncertain system be

$$T_n(z) = \frac{B_{n-1}z^{n-1} + B_{n-2}z^{n-2} + \cdots + B_0}{A_n z^n + A_{n-1}z^{n-1} + \cdots + A_0} = \frac{B_n(z)}{A_n(z)}, \quad (1)$$

where $B_i = [B_i^-, B_i^+]$ and $A_i = [A_i^-, A_i^+]$ $i = 0, 1, 2, \ldots, n-1, n$.

And its reduced approximate model be (2), where $m < n$

$$R_m(z) = \frac{b_{m-1}z^{m-1} + b_{m-2}z^{m-2} + \cdots + b_0}{a_m z^m + a_{m-1}z^{m-1} + \cdots + a_0} = \frac{B_m(z)}{A_m(z)}, \quad (2)$$

where $b_i = [b_i^-, b_i^+]$ and $a_i = [a_i^-, a_i^+]$ $i = 0, 1, 2, \ldots m-1, m$.

2.2. Proposed algorithm

In the classical control system, *RA* is the favourite tool to check system stability. Similarly, for deriving a stable reduced model, *RA* is taken into consideration. Since *RA* is a continuous-time domain algorithm and the article discusses the discrete-time domain system, an appropriate transformation of latter to its former equivalent is in demand here. This requirement is fulfilled by the use of Bilinear or Tustin transformation $z = (1 + w/1 - w)$ as applied earlier in Hwang and Shin (1982), Hwang and Hsieh (1990) as well as Hwang and Lee (1997), resulting $T_n(z)$ in

$$T_n(w) = \frac{x_n w^n + x_{n-1}w^{n-1} + \cdots + x_0}{y_n w^n + y_{n-1}w^{n-1} + \cdots + y_0} = \frac{X_{n\in k}(w)}{Y_{n\in k}(w)}, \quad (3)$$

where $x_i = [x_i^-, x_i^+]$ and $y_i = [y_i^-, y_i^+]$ $i = 0, 1, 2, \ldots, n$.

Henceforth, the novel arrangement along with the various arrangements (*Cases*) of Routh table array elaborates below. Firstly for a quick review of the usual Routh table array, consider Table 1, with all the entries of interval form as $a_{0,0} = [a_{0,0}^-, a_{0,0}^+]$, in general, $a_{i,j} = [a_{i,j}^-, a_{i,j}^+]$ with $i = 0, 1, 2, \ldots$ and $j = 0, 1, 2, \ldots$.

Equation (3) in two different arrangements offers *four cases* as mentioned below for the entries of first two rows of the respective Routh tables for numerator and denominator polynomials. Once, the above two rows are available for the stated *cases*, the conventional Routh algorithm computes the entries in the tables from the third row as

$$a_{i,j} = a_{i-2,j+1} - \frac{(a_{i-2,0} \times a_{i-1,j+1})}{a_{i-1,0}}, \quad (4)$$

where $a_{i,j} = [a_{i,j}^-, a_{i,j}^+]$ with $i = 2, 3, 4, \ldots, n-1, n$ and $j = 0, 1, 2$.

Table 1. Conventional Routh table.

$a_{0,0}$	$a_{0,1}$	$a_{0,2}$	$a_{0,3}$	$\cdots$
$a_{1,0}$	$a_{1,1}$	$a_{1,2}$	$a_{1,3}$	$\cdots$
$a_{2,0}$	$a_{2,1}$	$a_{2,2}$		
$\cdots$	$\cdots$			
$a_{n-1,0}$	$a_{n-1,1}$			
$a_{n,0}$				

These defined *cases* are then compared within themselves through an example in the next section to present the best among them.

The first two *cases* and their respective entries for first two rows is from Equation (3) as

Case 1

For numerator

1st Row; $a_{i,j} = x_k$ where $i = 0; j = 0, 1, 2, 3, \ldots;$
$k = n, n-2, n-4, \ldots$

2nd Row; $a_{i,j} = x_k$ where $i = 1; j = 0, 1, 2, 3, \ldots;$
$k = n-1, n-3, n-5, \ldots.$

For denominator

1st Row; $a_{i,j} = y_k$ where $i = 0; j = 0, 1, 2, 3, \ldots;$
$k = n, n-2, n-4, \ldots$

2nd Row; $a_{i,j} = y_k$ where $i = 1; j = 0, 1, 2, 3, \ldots;$
$k = n-1, n-3, n-5, \ldots.$

Case 2

For numerator

1st Row; $a_{i,j} = x_k$ where $i = 0; j = 0, 1, 2, 3, \ldots;$
$k = n, n-2, n-4, \ldots$

2nd Row; $a_{i,j} = y_k$ where $i = 1; j = 0, 1, 2, 3, \ldots;$
$k = n-1, n-3, n-5, \ldots.$

For denominator

1st Row; $a_{i,j} = y_k$ where $i = 0; j = 0, 1, 2, 3, \ldots;$
$k = n, n-2, n-4, \ldots$

2nd Row; $a_{i,j} = x_k$ where $i = 1; j = 0, 1, 2, 3, \ldots;$
$k = n-1, n-3, n-5, \ldots.$

For other two *cases*, reciprocate Equations (3) to (5) as

$$\hat{T}_n(w) = \frac{x_0 w^n + x_1 w^{n-1} + \cdots + x_{n-1} + x_n}{y_0 w^n + y_1 w^{n-1} + \cdots + y_{n-1} + y_n} = \frac{X_{n\in k}(w)}{Y_{n\in k}(w)}. \tag{5}$$

Case 3

For numerator

1st Row; $a_{i,j} = x_k$ where $i = 0; j = 0, 1, 2, 3, \ldots\ldots$
$k = 0, 2, 4 \ldots$

2nd Row; $a_{i,j} = x_k$ where $i = 1; j = 0, 1, 2, 3, \ldots\ldots$
$k = 1, 3, 5 \ldots.$

For denominator

1st Row; $a_{i,j} = y_k$ where $i = 0; j = 0, 1, 2, 3, \ldots\ldots$
$k = 0, 2, 4 \ldots$

2nd Row; $a_{i,j} = y_k$ where $i = 1; j = 0, 1, 2, 3, \ldots\ldots$
$k = 1, 3, 5, \ldots.$

Case 4

For numerator

1st Row; $a_{i,j} = x_k$ where $i = 0; j = 0, 1, 2, 3, \ldots\ldots$
$k = 0, 2, 4 \ldots.$

2nd Row; $a_{i,j} = y_k$ where $i = 1; j = 0, 1, 2, 3, \ldots\ldots$
$k = 1, 3, 5, \ldots.$

For denominator

1st Row; $a_{i,j} = y_k$ where $i = 0; j = 0, 1, 2, 3, \ldots\ldots$
$k = 0, 2, 4 \ldots.$

2nd Row; $a_{i,j} = x_k$ where $i = 1; j = 0, 1, 2, 3, \ldots\ldots$
$k = 1, 3, 5 \ldots.$

As stated above, the conventional Routh algorithm completes the Routh tables. The discussion underneath is to derive the desired order reduced transfer function using the entries in the Routh tables.

Let $R_m(w)$, where $m < n$ is the reduced transfer function and is assembled with $(n+1-m)th$ and $(n+2-m)th$ rows of denominator table, along with $(n+1)th$ and $(n+2-m)th$ rows of numerator table, be represented as

$$R_m(w) = \frac{c_{(n+1),0}w^{m-1} + c_{(n+2-m),0}w^{m-2} + c_{(n+1),1}w^{m-3} + \cdots}{d_{(n+1-m),0}w^m + d_{(n+2-m),0}w^{m-1} + d_{(n+1-m),1}w^{m-2} + \cdots} = \frac{X_m(w)}{Y_m(w)} \tag{6}$$

with the respective coefficients in uncertain form.

An appropriate inverse bilinear transformation from $R_m(w)$ to the desired $R_m(z)$ results. The next section discusses the different *cases* through numerical examples.

2.3. Validation tools

Integral Square Error is the convenient tool used here for performance analysis , with a modification towards discrete-time systems. Its adaptation leads to the weighted error sum over a fixed interval of time, which can be expressed as

$$J = \sum_{k=0}^{\infty} [y_n(k) - y_m(k)]^2, \quad (7)$$

where $y_n(k)$ and $y_m(k)$ are the step responses of the original $T_n(z)$ system and reduced $R_m(z)$ model, respectively.

Ideally, minimum J guarantees an approximate model of the higher order system. As the paper considers uncertain system, two individual transfer functions ((i) with only lower limits and (ii) with only upper limits) account for computing the error. Thus, the calculated J under two error columns lower and upper limits make the analysis and comparison easy with the recognized techniques.

Step response of the original system and reduced models also authenticate the proposed algorithm.

2.4. Stability analysis

Ever since the discovery of uncertainty in the system, the worry of its stability exists. This problem, also a key focus of the paper, is checked by applying Kharitonov theorem (Barmish, 1989) as stated below.

Consider the set of real interval polynomials of degree n of the form

$$\lambda(s) = \lambda_0 + \lambda_1 s + \lambda_2 s^2 + \lambda_3 s^3 + \lambda_4 s^4 + \cdots + \lambda_n s^n, \quad (8)$$

where the coefficients lie within given ranges $\lambda_0 \in [x_0, y_0], \lambda_1 \in [x_1, y_1], \ldots \lambda_n \in [x_n, y_n]$.

Then, every polynomial in the family is Hurwitz if and only if the following four extreme polynomials are Hurwitz:

$$\begin{aligned}
K^1(s) &= x_0 + x_1 s + y_2 s^2 + y_3 s^3 + x_4 s^4 + x_5 s^5 + y_6 s^6 + \cdots \\
K^2(s) &= x_0 + y_1 s + y_2 s^2 + x_3 s^3 + x_4 s^4 + y_5 s^5 + y_6 s^6 + \cdots \\
K^3(s) &= y_0 + x_1 s + x_2 s^2 + y_3 s^3 + y_4 s^4 + x_5 s^5 + x_6 s^6 + \cdots \\
K^4(s) &= y_0 + y_1 s + x_2 s^2 + x_3 s^3 + y_4 s^4 + y_5 s^5 + x_6 s^6 + \cdots .
\end{aligned} \quad (9)$$

The Kharitonov theorem involves the standard Routh algorithm to check the stability of above polynomials. Thus, an uncertain family of polynomials $\lambda(s)$ is robustly stable if, and only if, the Kharitonov polynomials are stable. The stated theorem holds true in the discrete-time domain also (Cieslik, 1987; Hollot & Bartlett, 1986).

Graphical interpretation for the stability of derived reduced model is performed by the Bode plot in the frequency domain that includes the original system also.

3. Illustrative examples

An example considered from the literature determines the best *Case* among the above stated agreements. Comparison of error sum performs the assessment between the proposed *cases* with the existing techniques. The step responses also verify the obtained results. In the course of attaining the prime motive for the proposed algorithm (i.e. model stability), a limitation of no importance is discovered. For illustrating the proficiency of the proposed *cases*, the limitation is taken into account and then neglected on a later stage. Additionally, the obtained *case* of the numerator and denominator polynomial is applied to a real-time test system for assuring its possible liability. In both the examples, the sampling time considered is 0.001 s.

Example 1: Consider third-order uncertain system available from literature (Choudhary & Nagar, 2013a, 2013b; Ismail et al., 1997) as

$$T_3(z) = \frac{[1,2]z^2 + [3,4]z + [8,10]}{[6,6]z^3 + [9,9.5]z^2 + [4.9,5]z + [0.8,0.85]}. \quad (10)$$

After bilinear transformation, the above system is represented as

$$T_3(w) = \frac{[-9,-5]w^3 + [17,27]w^2 + [-34,-24]w + [12,16]}{[0.55,1.2]w^3 + [5.9,6.65]w^2 + [19.45,20.2]w + [20.7,21.35]}. \quad (11)$$

First two rows of the Routh table and the reduced models fetched as per the four *cases* are observed below. Consider Equation (11) for first two *cases*

Case 1: See Tables 2 and 3.

$$R_1(z) = \frac{[12,16]z + [12,16]}{[35.8,39.83]z + [2.22,6.25]}, \quad (12)$$

$$R_2(z) = \frac{[-19.77,0.48]z^2 + [24,32]z + [27.52,47.77]}{[41.7,46.48]z^2 + [28.1,30.9]z + [8.12,12.9]}. \quad (13)$$

Case 2: See Tables 4 and 5.

$$R_1(z) = \frac{[20.7,21.35]z + [20.7,21.35]}{[30.32,35.95]z + [-7.95,-2.32]}. \quad (14)$$

$$R_2(z) = \frac{[2.27,29.91]z^2 + [41.4,42.7]z + [12.14,39.78]}{[47.32,62.95]z^2 + [-30,-2]z + [9.05,24.68]}. \quad (15)$$

Table 2. Denominator array.

ω^3	[0.55, 1.2]	[19.45, 20.2]
ω^2	[5.9, 6.65]	[20.7, 21.35]

Table 3. Numerator array.

ω^3	[−9, −5]	[−34, −24]
ω^2	[17, 27]	[12, 16]

Table 4. Denominator array.

ω^3	[0.55, 1.2]	[19.45, 20.2]
ω^2	[17, 27]	[12, 16]

Table 5. Numerator array.

ω^3	[−9, −5]	[−34, −24]
ω^2	[5.9, 6.65]	[20.7, 21.35]

Table 6. Denominator array.

ω^3	[20.7, 21.35]	[5.9, 6.65]
ω^2	[19.45, 20.2]	[0.55, 1.2]

Table 7. Numerator array.

ω^3	[12, 16]	[17, 27]
ω^2	[−34, −24]	[−9, −5]

Table 8. Denominator array.

ω^3	[20.7, 21.35]	[5.9, 6.65]
ω^2	[−34, −24]	[−9, −5]

Table 9. Numerator array.

ω^3	[12, 16]	[17, 27]
ω^2	[19.45, 20.2]	[0.55, 1.2]

For *Case* 3 and *Case* 4, reciprocate $T_3(w)$ to $\hat{T}_3(w)$ and draft the two *cases* as

$$\hat{T}_3(w) = \frac{[12,16]w^3 + [-34,-24]w^2 + [17,27]w + [-9,-5]}{[20.7,21.35]w^3 + [19.45,20.2]w^2 + [5.9,6.65]w + [0.55,1.2]}.$$

Case 3: See Tables 6 and 7.

$$R_1(z) = \frac{[-9,-5]z + [-9,-5]}{[5.13,7.28]z + [3.38,5.53]}, \tag{16}$$

$$R_2(z) = \frac{[2,20.23]z^2 + [22,50.46]z + [16,34.23]}{[24.58,27.48]z^2 + [36.5,39.3]z + [13.92,16.82]}. \tag{17}$$

Case 4: See Tables 8 and 9

$$R_1(z) = \frac{[0.55,1.2]z + [0.55,1.2]}{[-11.1,-1.4]z + [2.9,12.6]}, \tag{18}$$

$$R_2(z) = \frac{[16.56,27.87]z^2 + [32.02,53.34]z + [14.81,26.12]}{[-45.1,-25.4]z^2 + [-58,-30]z + [-46.6,-26.9]}. \tag{19}$$

The computed error *J*, through different *Cases*, is made known in Table 10 offering the support to *Case* 2 for calculating minimum error when compared to the existing techniques. The next section explains the limitation encountered during the error computation.

Another practice to authenticate the algorithm is step response depicted in Figures 1 and 2 for lower and upper limit models of first order. Figures 3 and 4 show the response for lower and upper limit models of second order. The responses displayed are for the models obtained from *Case* 2 and the existing techniques. Limitation explained in the next section can be observed from the demonstrated figures.

Deriving the reduced models confer the fair representation of the higher order system to its lower equivalent. Since the prime focus of the paper is to retain the model stability, it is checked through Kharitonov theorem and displayed in the frequency domain (Bode Plot). Figures 5 and 6 show the Bode plot of first-order reduced model with lower and upper limits and the following Figures 7 and 8 present for second-order model with lower and upper limits. In all the figures, the models have an infinite gain margin and a positive phase margin, clearly indicating the derivation of a stable reduced model through *Case* 2. Thus, the prime motive of the paper is acknowledged.

From the above example, *Case* 2 is sorted to be the satisfactory arrangement of the Routh tables providing a minimum error, an adequate step response and retaining the model stability. To accept the arrangement's rational acceptability, *Case* 2 is applied to the real-time system through the example below.

Table 10. Error for first- and second-order reduced models through the four *cases* and other prevailing techniques.

	Error			
	First-order		Second-order	
Methods	Lower limit	Upper limit	Lower limit	Upper limit
Proposed Case 1	0.3456	0.3271	0.2894	0.1287
Proposed Case 2	*0.3644*	*0.1497*	*0.0211*	*0.0233*
Proposed Case 3	9.4259	1.8765	0.4812	1.9492
Proposed Case 4	0.0801	96.0295	0.7302	6.1975
Pade Appr. and dominant pole (Ismail et al., 1997)	0.1398	0.0195	0.1810	0.0741
Direct truncation (Choudhary & Nagar, 2013a)	2.1491	2.7778	0.0278	0.0077
Gamma–Delta Appr. (Choudhary & Nagar, 2013b)	0.0157	0.0035	0.1292	0.0250

Note: The Proposed Case 2 present an appropriate acceptance as compared to rest of the algorithms.

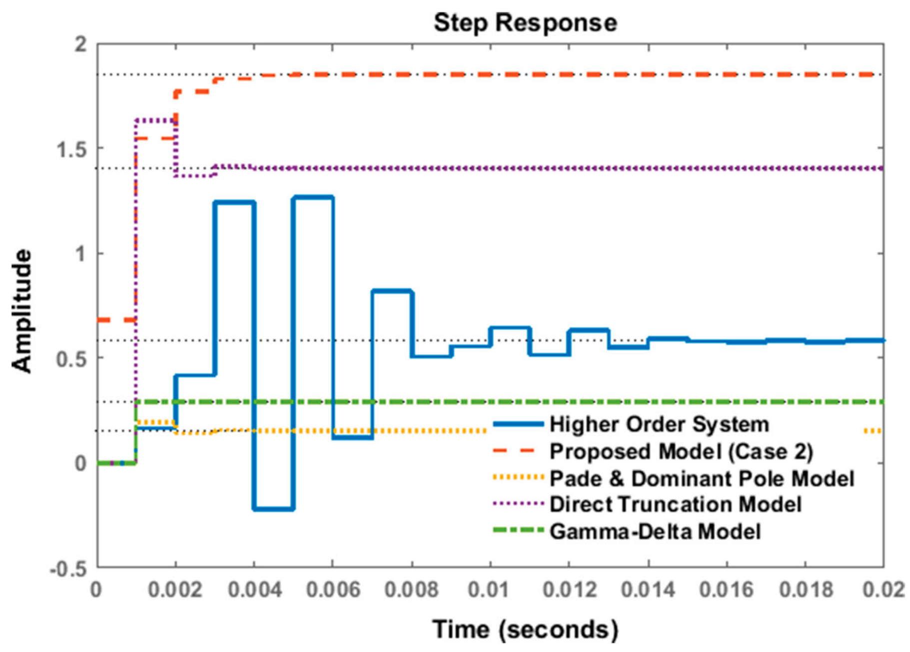

Figure 1. Step response for Example 1 (*lower limit*) first-order model.

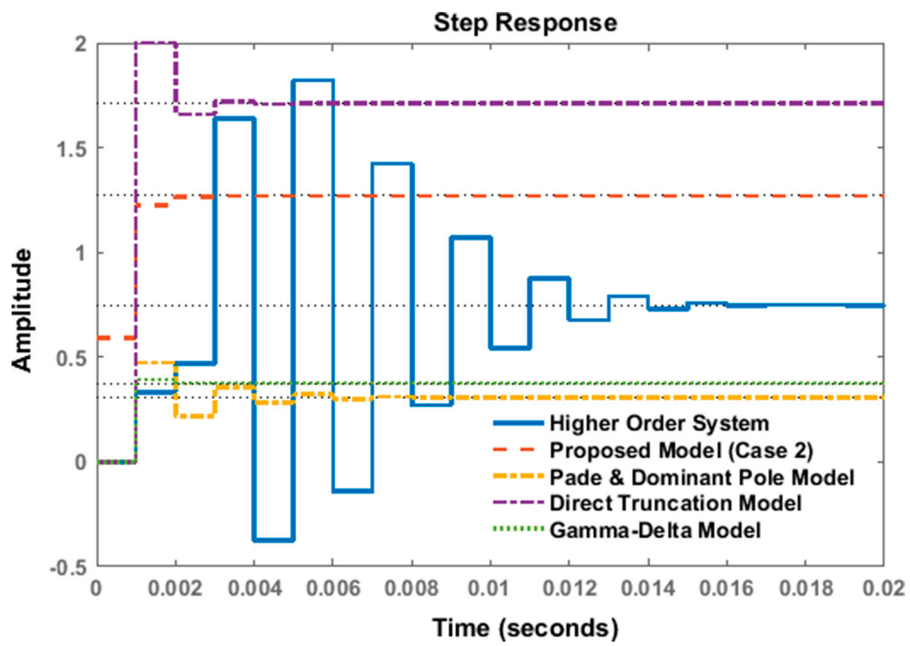

Figure 2. Step response for Example 1 (*upper limit*) first-order model.

Example 2: Consider the real-time digital control system shown in Figure 9, where

$$D(z) = \frac{1.68z^6 - 0.566z^5 + 0.356z^4 - 0.204z^3 - 0.312z^2 + 0.05z - 0.006}{z^6 + 1.159z^5 + 0.76z^4 + 0.466z^3 + 0.096z^2 - 0.016z + 0.003}. \tag{20}$$

With $T = (0.5)^{0.5}$ s, and accepting the robustness of the system into count, the overall transfer function is crafted as

$$T_8(z) = \frac{\begin{array}{l}[1.6484, 1.7156]z^7 + [1.0937, 1.1383]z^6 \\ + [-0.2142, -0.2058]z^5 + [0.1490, 0.1550]z^4 \\ + [-0.5263, -0.5057]z^3 + [-0.2672, -0.2568]z^2 \\ + [0.0431, 0.0449]z + [-0.0061, -0.0059]\end{array}}{\begin{array}{l}[23.52, 24.48]z^8 + [-1.7156, -1.6484]z^7 \\ + [-1.1383, -1.0937]z^6 + [0.2058, 0.2142]z^5 \\ + [-0.1550, -0.1490]z^4 + [0.5057, 0.5263]z^3 \\ + [0.2568, 0.2672]z^2 + [-0.0449, -0.0431]z \\ + [0.0059, 0.0061].\end{array}} \tag{21}$$

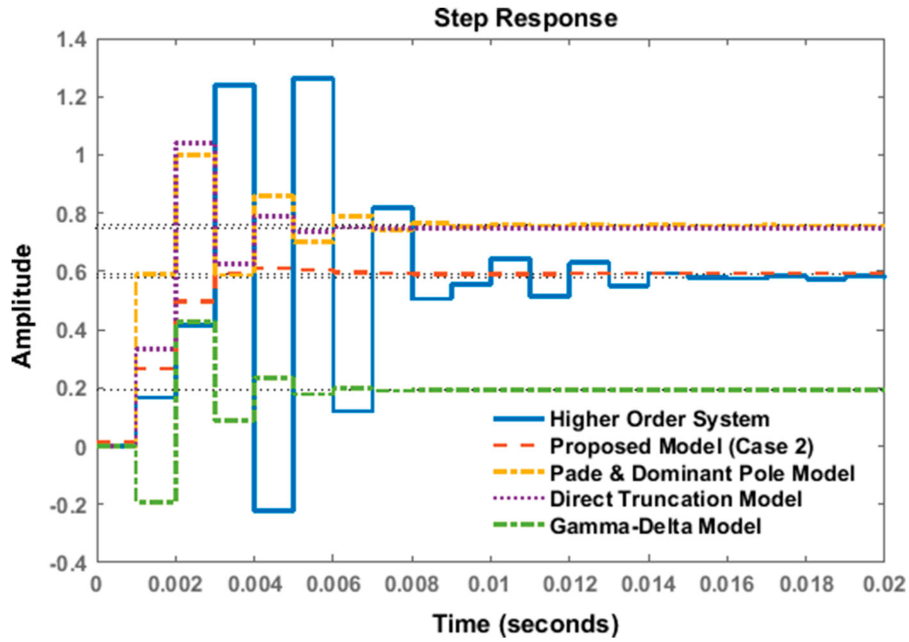

Figure 3. Step response for Example 1 (*lower limit*) second-order model.

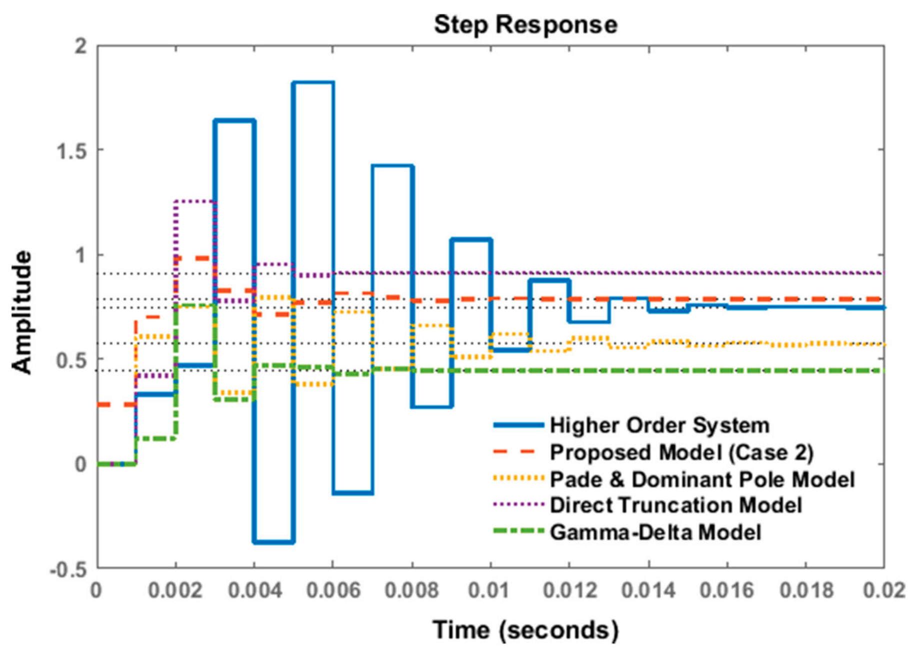

Figure 4. Step response for Example 1 (*upper limit*) second-order model.

By the proposed algorithm *cases*, the reduced-order model is obtained as

Case 1:

$$R_1(z) = \frac{[1.92, 2.07]z + [1.92, 2.07]}{[110.83, 291.57]z + [-247.58, -66.84]}, \quad (22)$$

$$R_2(z) = \frac{[13.21, 21.14]z^2 + [3.84, 4.14]z + [-17.15, -9.22]}{[-49.49, 739.4]z^2 + [-852.78, 365.74]z + [-407.9, 380.99]}. \quad (23)$$

Case 2:

$$R_1(z) = \frac{[1.92, 2.07]z + [1.92, 2.07]}{[36.43, 39.85]z + [4.14, 7.63]}, \quad (24)$$

$$R_2(z) = \frac{[14.94, 182.65]z^2 + [3.84, 4.14]z + [-178.66, -10.95]}{[-1222.74, 886.15]z^2 + [-1649.72, 2563.3]z + [-1254.96, 853.93]}. \quad (25)$$

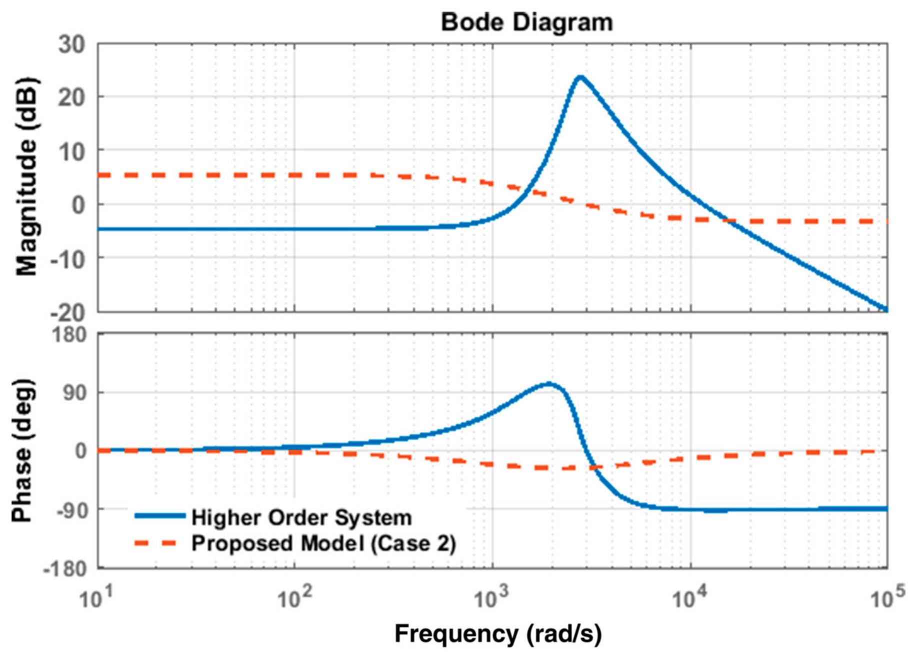

Figure 5. Bode plot for Example 1 (*lower limit*) first-order model.

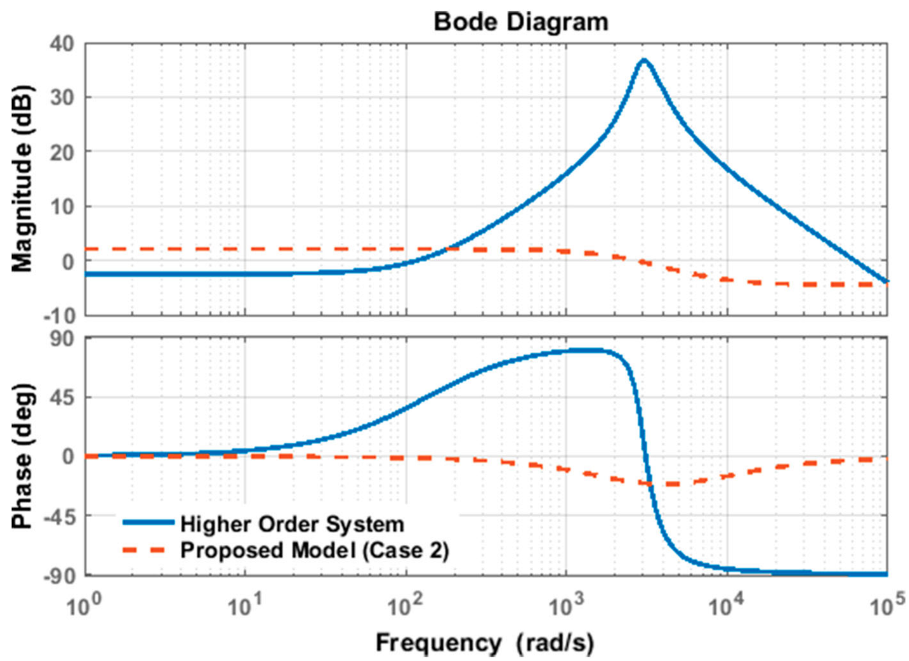

Figure 6. Bode plot for Example 1 (*upper limit*) first-order model.

Case 3:

$$R_1(z) = \frac{[-0.07, 0.07]z + [-0.07, 0.07]}{[129.23, 279.82]z + [81.24, 231.83]}, \quad (26)$$

$$R_2(z) = \frac{[-5.69, -4.03]z^2 + [-11.24, -8.2]z + [-5.69, -4.03]}{[-130.46, 762.05]z^2 + [-568.48, 917.58]z + [-491.52, 400.92]}. \quad (27)$$

Case 4:

$$R_1(z) = \frac{[-0.07, 0.07]z + [-0.07, 0.07]}{[17.24, 22.39]z + [-30.25, -25.6]}, \quad (28)$$

$$R_2(z) = \frac{[181.98, 211.74]z^2 + [364.1, 423.34]z + [181.98, 211.74]}{[-1724.76, 589.39]z^2 + [-3534.1, 1087.12]z + [-1716.9, 597.25]}. \quad (29)$$

Cumulative error J for $R_1(z)$ and $R_2(z)$ of example 2 is presented in Table 11.

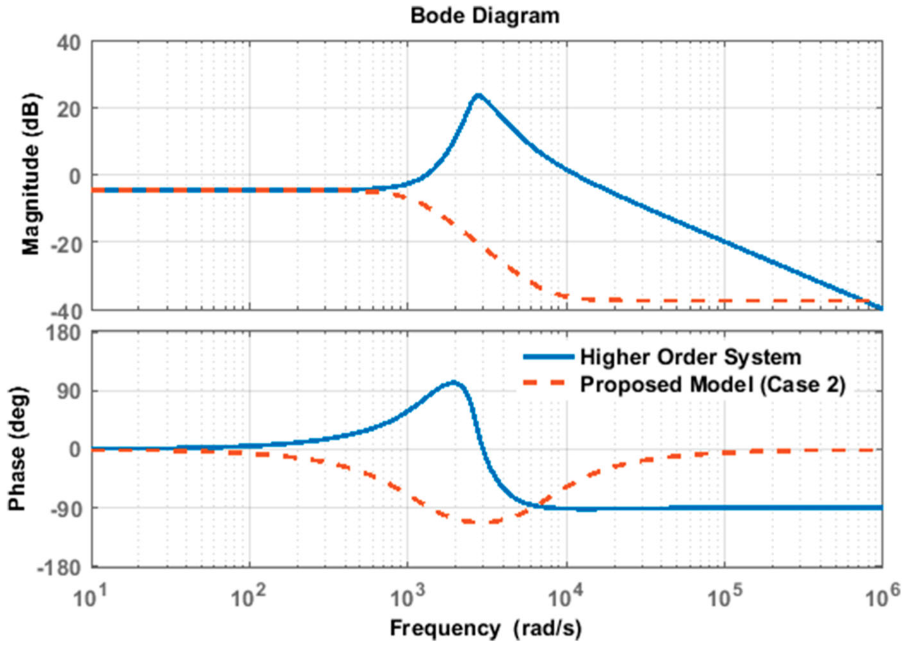

Figure 7. Bode plot for Example 1 (*lower limit*) second-order model.

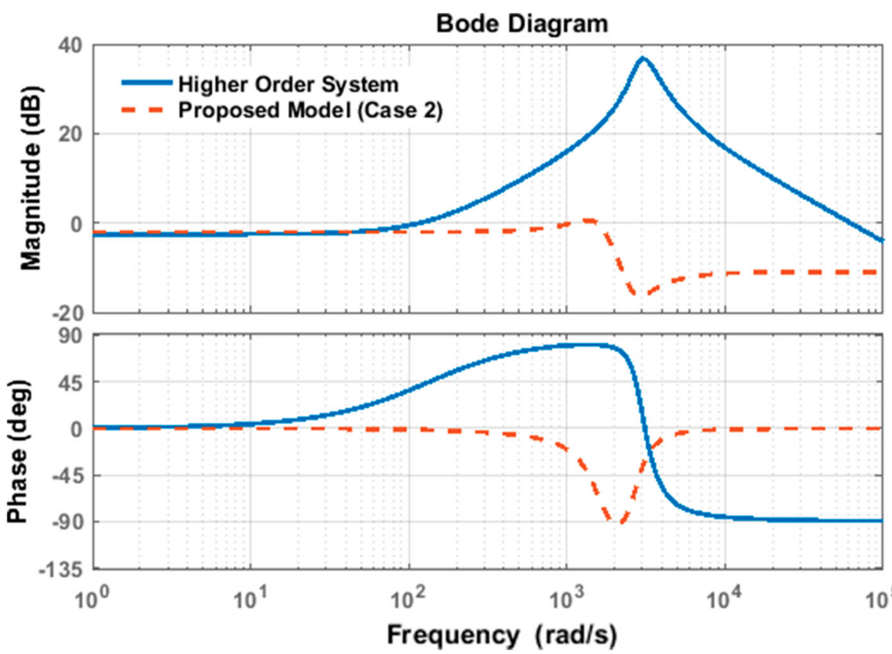

Figure 8. Bode plot for Example 1 (*upper limit*) second-order model.

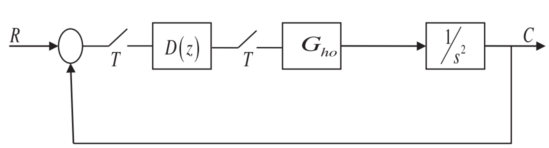

Figure 9. Real-time digital control system.

4. Discussion

Table 10 depicts the error for first- and second-order reduced models through the discussed four *Cases* and other prevailing techniques for Example 1. It offers support towards *Case* 2. Similarly, computation of minimum error for Example 2, real-time test system also support *Case* 2 in Table 11. Both the tables satisfactorily state the acceptance of *Case* 2 for computation of minimum error. Thus, among the four *cases* elaborated, the arrangement for the desired reduced model is *Case* 2. Authors would firmly state that the reduction methodology discussed in this paper may be good for one practical system as considered here and not for other. However, the paper presents an

Table 11. Error for first- and second-order reduced models through the four *cases*.

	Error			
	First-order		Second-order	
Methods	Lower limit	Upper limit	Lower limit	Upper limit
Proposed Case 1	$3.1075*10^{-4}$	0.0030	17.5841	0.0033
Proposed Case 2	*0.0036*	*0.0033*	*0.0049*	*0.0250*
Proposed Case 3	0.0050	0.0040	0.0189	0.0064
Proposed Case 4	0.0073	0.0036	0.0402	0.2480

Note: The Proposed Case 2 present an appropriate acceptance as compared to rest of the algorithms.

explicit arrangement for computing the stable reduced model.

As stated earlier, the limitation discovered is observed in Table 10. It is the computation of higher error by *Case 2* in comparison to other techniques (e.g. *Error of 1st order lower limit is more than the other in the same column*). Precisely, error obtained by *Case* 2 is more than the usual method (Ismail et al., 1997). Similarly, in Table 11, the error in second-order upper limit column by *Case* 2 is more than in *Case* 1 and *Case* 3.

Prime motive of the paper to retain the model stability is fulfilled at the stake of the encountered limitation. Thus, the limitation is taken into consideration for *Case* 2 arrangement of Routh table. Stability of the interval systems and the reduced models is checked with the Kharitonov theorem at the department laboratory using Khariton software package. However, Figures 5–8 demonstrate the frequency response of first- and second-order models with lower and upper limits, respectively. The responses in figures are for the models derived from *Case* 2.

Through the above illustration, it is observed that the tracking of the step responses of the reduced models is not accurate with the original response although it gives an appropriate approximation. The tracking proposes a clear indication towards the future work to a control engineer or a researcher, to design an appropriate controller that tracks the response of the reduced models accurately with the original response.

5. Conclusion

The property of *RA* to yield a stable model is the key focus of discussion in the paper and is successfully achieved. The computational simplicity of *RA* intends this work proposal. The arrangement of Routh table in a varied form confers the methodology. The domain of various *cases* constructed by the different combinations of numerator and denominator polynomials explores the novelty of the algorithm. An example available from the literature is used to get the blueprint of the arrangement and then tested over a real-time system. Both the examples in the paper offer a significant contribution towards the institution of a superior algorithm over the other prevailing techniques based on the performance measure, step response and stability check. The paper also exploits the limitation encountered and provides a future scope to the fellow researchers.

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No potential conflict of interest was reported by the authors.

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