1 Introduction

1.1 Cosmology

The speculations about the universe are as old as man himself. That is why, cosmology, the science of the universe, attracts and fascinates us all. Cosmologists have long been dedicated a large amount of time and efforts to reveal the true nature of the universe. The ultimate aim of cosmologists is to understand the universe in which we live today. A full picture should comprise both an inventory of the present matter content (including its spatial distribution) and an understanding of the dynamics governing the past and future evolution of the universe as a whole.

The task in cosmology is to explain the structure of the universe as it is observed. Contemporary cosmologists carry out this task by means of two, to some extent contradictory, approaches. One wants to explain the structure of the universe observed through what the universe was at the very beginning, the other, on the other hand, tries to show its present structure as an inevitable consequence of past physical and chemical processes, irrespective of what was at the beginning. A permissible possibility here is that the universe is so constructed that no observable traces of its quantum origin remain. The theory of the Big Bang in its classical, purely relativistic interpretation, is an example of the former approach. However, its inflationary development which refers to the physics of high energies and to quantum mechanics is the fullest expression of the latter approach. Irrespective of how the universe began (and whether it began at all) that which we observe at present on a cosmological scale is the result of an extremely short period of inflation in which, due to the unparalleled intensity of extension at an exponential rate, it was transformed from a microobject (with respect to its spatial dimensions) into a macro object.

Obviously to solve the problem of the origin of the world, of fundamental importance is the former approach which assumes that further evolution of the universe did not preclude the significance of its initial states for what we observe now on a large scale. It does not mean that the prevalence of inflationary models in modern relativist and quantum cosmology renders cosmology incapable of dealing with questions of the origin of the universe. It is only in recent years that scientists have concentrated on the former approach. There has been a search for fundamental laws which would determine the initial conditions of the universe.

The first key idea dates to 1916 when Einstein developed his "General Theory of Relativity" which he proposed as a new theory of gravity. His theory generalizes Isaac Newton's original theory of gravity , in that it is supposed to be valid for bodies in motion as well as bodies at rest. Newton's gravity is only valid for bodies at rest or moving very slowly compared to the speed of light (usually not too restrictive an assumption!). A key concept of General Relativity is that gravity is no longer described by a gravitational "field" but rather it is supposed to be a distortion of space and time itself. Physicist John Wheeler put it well when he said "Matter tells space how to curve, and space tells matter how to move." Originally, the theory was able to account for peculiarities in the orbit of Mercury and the bending of light by the Sun, both unexplained in Isaac Newton's theory of gravity.

1.2 General Theory of Relativity

Einstein's theory of relativity is based on the fundamental idea of relativity of all kinds of motion. The special theory of relativity formulated by Einstein (1905) makes a restricted use of this general idea since it merely assumes the relativity of uniform translatory motion in a region of free space where gravitational effects can be neglected. As such it fails to study relative motion in accelerated frame of reference and is not applicable to all kinds of motion. Taking into account these limitations, Einstein (1915) generalized the special theory of relativity and put forth Einstein's general theory of gravitation. Einstein arrived at a novel concept which says that gravitation has a basic relationship with the space-time in which it is always present. The theory of general relativity is a more accurate and comprehensive description of gravitation than Newtonian theory. This theory of gravitation has great formal beauty and mathematical elegance and is found to lead to a complete theory of gravitational action.

In the development of general relativity, Einstein was mainly guided by three basic principles:

1. Principle of covariance which helps us to write the physical laws in covariant form so that their form remains unaltered in all systems of coordinates. This implies that the physical laws should be expressed in tensor form.

2. The principle of equivalence which states that at any point of space-time we can find a locally inertial system in which the law of special theory of relativity is valid.

3. The Mach's Principle which is based on the Machian idea that inertia as well as gravitation depends upon mutual action between bodies. This principle can be used to determine the geometry of space-time and thereby the inertial properties of test particle from the information of density and mass energy distribution in its surroundings.

Thus, Einstein's general theory of relativity describe the gravitational phenomenon successfully which served as the basis for the model universe. In this theory the Riemannian metric describes the geometry of space-time, which is given by

$$ds^2 = g_{ij} dx^i dx^j, \qquad i, j = 1, 2, 3, 4. \tag{1.1}$$

Here coordinates are labeled as $x_1 = x$, $x_2 = y$, $x_3 = z$ and $x_4 = t$. In this case the fundamental metric tensor g_{ij} represents gravitational potential and gravitational field is the manifestation of the curvature of space-time. In general relativity the field equations are

$$R_{ij} - \frac{1}{2} g_{ij} R = -\frac{8\pi G}{c^4} T_{ij}, \tag{1.2}$$

where G is gravitational constant, c is speed of light, T_{ij} is stress energy tensor of the matter field, $R = g^{ij} R_{ij}$ is the Ricci scalar corresponding to the space-time metric g_{ij} of signature $(+, -, -, -)$ and R_{ij} is the Ricci tensor given by

$$R_{ij} = \frac{\partial^2}{\partial x^i \partial x^j} \log \sqrt{-g} - \frac{\partial}{\partial x^l} \Gamma^l_{ij} + \Gamma^m_{il} \Gamma^l_{mj} - \Gamma^m_{ij} \frac{\partial}{\partial x^m} \log \sqrt{-g} \tag{1.3}$$

with

$$\Gamma^l_{ij} = \frac{1}{2} g^{lh} \left(\frac{\partial g_{ih}}{\partial x^j} + \frac{\partial g_{jh}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^h} \right), \tag{1.4}$$

the Christofell's symbols of second kind. The tensor G_{ij} is defined by

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R \tag{1.5}$$

is called Einstein tensor. As a result of the symmetry of G_{ij} and T_{ij}, actual number of the field equations reduces to ten, although there are additional differential identities (the Bianchi identities), given by

$$G^{ij}_{;j} = 0 \tag{1.6}$$

where a semicolon denotes the covariant differentiation. So, covariant divergence of the Einstein tensor vanishes, which in turn implies

$$T^{ij}_{;j} = 0, \tag{1.7}$$

known as energy conservation equations.

Equations (1.2) lead to non-linear complicated differential equations, which can, in general, be solved if one makes certain simplifying assumptions.

The energy-momentum tensor or stress energy tensor T_{ij} describes the source of matter in space-time. It plays an important role during different epochs in the history of the universe. In order to obtain physical conclusions from the equations of general relativity, we must of course apply it to some particular kind of physical medium for which we actually know the dependance of the energy momentum tensor on observational properties of the medium. Hence, we desire explicit expressions for the energy momentum tensor T_{ij} in terms of quantities which can be determined by ordinary methods of measurements. Following are some specific forms of the energy momentum tensor in different physical medium.

1.2.1 Perfect Fluid Distribution

A perfect fluid is a mechanical medium incapable of exerting transverse stresses. The form of the energy momentum tensor for the perfect fluid is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij}. \tag{1.8}$$

Here ρ is the matter density, p is the the thermodynamical pressure and u^i is the fluid 4-velocity vector satisfying $u^i u_i = 1$. In comoving coordinate system, we have $u^i = (0, 0, 0, 1)$.

1.2.2 Perfect Fluid Distribution with Heat Conduction

The matter content of the universe is satisfactorily described by a perfect fluid. As the matter is not expected to attain thermal equilibrium in the early stages of the evolution of the universe, it is evident that there would be heat flow in the universe. The energy momentum tensor of a perfect fluid with heat conduction is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} + h_i u_j + h_j u_i. \tag{1.9}$$

where h_i is the heat conduction vector satisfying

$$h^i u_i = 0 \tag{1.10}$$

1.2.3 Viscous Fluid Distribution

The adequacy of cosmological models with perfect fluid is no basis for expecting that it is equally suitable for describing its early stages of the evolution of the universe. At the early stages of evolution of the universe, when radiation in the form of photon as well as neutrino decoupled, the matter behaved like a viscous fluid. The energy momentum tensor of a viscous fluid is given by

$$T_{ij} = (\rho + \bar{p})u_i u_j - \bar{p} g_{ij} + \eta \delta_j^\beta [u_{i;\beta} + u_{\beta;i} - u_i u^\alpha u_{\beta;\alpha} - u_\beta u^\alpha u_{i;\alpha}] \tag{1.11}$$

where

$$\bar{p} = p - \left(\xi - \frac{2}{3}\eta\right)\theta. \tag{1.12}$$

$\bar{p}$ is the effective pressure and $\xi > 0$ and $\eta > 0$ are the coefficients of the bulk and shear viscosities respectively, θ is the expansion scalar.

1.3 Homogeneous and Isotropic Universe

The modern era of cosmology had begun when Hubble (1929) was able to announce his famous law: All galaxies recede from us at velocities proportional to their distances from us, he developed the ways of estimating the distance of the remote galaxies and studied their distribution in space. Hubble's finding was the first observational evidences for the first cosmological models. The assumption of a large scale homogeneity together with isotropy is called the "Cosmological Principle" (CP). According to CP, the universe should be homogeneous on large scale. However, on smaller scales, inhomogeneities exist. The assumption concerning homogeneity and isotropy of the universe helps in sense that all spatial directions are equivalent and no part of the universe can distinguished from any other. The distribution of galaxies in sky along their apparent magnitudes and redshifts, the distribution of radio sources, cosmic X-ray background, cosmic microwave background all offer at least some circumstantial evidences that distribution of these materials on large scale exhibit to be isotropy.

1.4 Cosmological Models

A cosmological model is a mathematical description of the universe, which tries to explain the reason of its current aspects and to describe evolution with time. Of course, it must account for the observations and be able to make predictions that later observation will be able to check. The origin of modern cosmology is the Einstein's general theory of relativity which opens new avenues of approach to the solution of problems related to the properties of the universe on cosmic scale. Einstein (1917) himself constructed the static cosmological models filled with a contineous distribution of perfect fluid. However, the model is unsatisfactory for several reasons. It contradicts the actual universe where, according to Hubble and Humason (1934), a definite redshift is observed in the light from the nebulae which (redshift) increases at least very closely in linear propagation with distance. Very shortly after the presentation of Einstein's static model, de-Sitter (1917) described the Einstein static universe including a pressure term and gave the field equations for this case. The de-Sitter universe is completely empty containing neither matter nor radiation. In the de-Sitter universe we get an explanation of the of the actual redshift observed by Hubble and Humason (1934). Thus, we see that the de-Sitter universe is completely empty, yet it predicts the observed recession of nebulae. On the otherhand, Einstein universe is full of matter, but it does not predict the observed recession of nebulae. Thus, neither Einstein's universe nor de-Sitter universe represents a true model of the actual universe. In order to construct a model in which advantages of two static models of Einstein and de-Sitter are combined, one has to take recourse to non-static models in which the metric tensor is intrinsically time-dependent.

Further, attempts were made by Friedmann (1922, 1924), Lemaitre (1927) and Robertson (1935, 1936) to investigate the most general quadratic line element which would describe non-static but isotropic and homogeneous universe. Tolman (1031,1934) described a non-static model containing only black body radiation. Meanwhile, de-Sitter (1930,1931) obtained analytic solution for open, flat and closed models both with and without cosmological constant. The most satisfactory non-static cosmological model was given by Robertson (1935,1936) and Walker (1935,1936). The assumptions of Robertson and walker model are:

(i) There exists a cosmic time which is orthogonal to the spatial geometry

$$ds^2 = dt^2 - g_{ij}dx^i dx^j, \quad (i, j = 1, 2, 3) \tag{1.13}$$

(ii) The three dimensional spatial surfaces belonging to $t = constant$ are locally isotropic and homogeneous.

Robertson and Walker proved that any Reimannian space-time, which is isotropic at every point, is spatially homogeneous and that the metric can be expressed by

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{1.14}$$

with the coordinate (r, θ, ϕ, t). The constant K being the curvature parameter which can take the values 1, 0, -1. Corresponding to K=1, 0, -1, the spatial sections are, respectively, three sphere (constant positive curvature), flat (Zero-curvature) and three-hyperboloid (constant negative curvature). Here $a(t)$ is the cosmic scale factor which is determined from the field equations. This metric can also be expressed into other convenient forms, but for our purpose, we use the line element as given in equation (1.14).

The dynamics of the Robertson-Walker geometry depends on the scale factor $a(t)$ which tells us how the spatial geometry changes with time. This function is determined, for a particular value of K by the field equations and the equations of state relating pressure and density of the cosmic fluid. Thus, for different choice of these factors, different models are obtained.

1.5 Homogeneous and Anisotropic Universe

In relativistic and observational cosmology, even often use a simplified space-time model having restricted dynamical degree of freedom. The well-known models are Friedmann-Roberson-Walker models in which the spatial sections are assumed to be homogeneous and isotropic. These models are the simplest one's that explain the large amount of astronomical data (Parker (1971), Isham (1974), Ford(1976), Audretsch and Schafer (1978), Villalba and Percoco (1990)).

Although the universe seems homogeneous and isotropic, there are no observational data guaranteeing the isotropy at the early stage of its evolution. In fact, there are theoretical arguments to the existence of an anisotropic phase that approaches an isotropic one. Bianchi I-IX spaces are very useful tools for constructing the homogeneous and anisotropic models of the universe.

Bianchi types cosmological models are anisotropic generalization of the homogeneous FRW-cosmologies, and orthonormal frame methods have been very useful in the study of them because of the close connection between the orthonormal frame variables and structure constants of Lie algebra of the killing vector fields of the isometry group (Bianchi (1918)). Bianchi spaces have a three parameter isometry group acting on spatial slices, which are described dynamically by the dynamical system whose state space variables are independent of spatial coordinates. The different group types correspond to different invariant of the state space and arrange the different Bianchi types into a hierarchy of increasing complexity. It is possible to take the surface of symmetry Σ to be given by $t = constant$, and choose a frame of vectors e_a with component E_a^i dependent only on spatial variables the line element of such space time is given by

$$ds^2 = dt^2 - g_{ij}(t)(E^i_\mu dx^\mu)(E^j_\nu dx^\nu). \tag{1.15}$$

Here E^i_μ is the matrix inverse of E^μ_i. The evolution of the universe is then represented by the time dependence of the six independent frame components g_{ij} of the metric. The basis vector of E_i of the Bianchi models satisfy $[E_i, E_j] = C^k_{ij} E_k$, in which C^k_{ij} are the structure constants of the relevant symmetry group.

The Bianchi models have been studied extensively since the late 60's as example of exact solutions (Stephani et al. 2003), and from a dynamical systems perspective at least since 1971 by Collins (1971), and developed further by Bogoyavlensky (1985), Rosquit and Jantzen (1988) and others. In connection with data from the Wilkinson Microwave Probe (Jafe et al. (2005), Hinshaw et al. (2009)), it has been discovered that the standard cosmological model requires positive and dynamic cosmological parameters, a case which resembles Bianchi morphology. According to this result, the universe should achieve the following features: (i) a slightly anisotropic geometry in spite of inflation, and (ii) a non trivial isotropization history of the universe due to the presence of an anisotropic energy source. The advantage of these anisotropic models are that they have significant role in the description of evolution of early phase of the universe and they help in finding more general cosmological models than the isotropic FRW models.

1.6 Lyra's Geometry

The Lyra's geometry is very close to the spirit of Einstein's principle of geometrization since both scalar and tensor fields have more or less a geometrical significance. Einstein while discussing cosmological solutions had to introduce the cosmological constant Λ into the field equations because the large scale recession of the galaxies i.e., the expansion of the universe had not been discovered at the time, this was discovered later by Hubble. The theory has been successful in describing not only the gravitational phenomena but has served as a basis for cosmological models of the universe. Gravitation, however, is not the only force described by classical physics, Electromagnetic forces are also important and they are not explained by general relativity as a geometric phenomena. Subsequently, there have been many attempts to unify electromagnetism and gravitation. Weyl (1918) proposed a more general theory in which electromagnetism is also described geometrically. Lyra (1951) suggested a modification of the Riemannian geometry, which may also be considered as a modification of Weyl geometry, by introducing a gauge function into the structureless manifold as a result of which a displacement field arises naturally. Halford (1972) pointed out that the constant displacement vector field in Lyra's geometry plays the role of cosmological constant in the normal general relativistic treatment. He has also shown that the scalar-tensor treatment based on Lyra's geometry predicts the same effects, within observational limits, as Einstein's theory. Sen (1957), Sen and Dunn (1971) proposed a scalar-tensor theory of gravitation and constructed analogue of Einstein field equations based on Lyra's geometry. The field equations, in normal gauge, for Lyra's manifold are

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}(\phi_i\phi_j - \frac{1}{2}g_{ij}\phi_\alpha\phi^\alpha) = -8\pi G T_{ij} \tag{1.16}$$

where ϕ is a time-like displacement field vector defined by $\phi_i = (0, 0, 0, \beta(t))$, and other symbols have their usual meaning as in Riemannian geometry. The function β may be time-dependent or time-independent.

Beesham (1988) has investigated Friedmann-Robertson Walker(FRW) cosmological models in Lyra's geometry with time-dependent displacement field vector. Pradhan and Vishwakarma (2004) have obtained LRS Bianchi type-I cosmological models by considering a time-dependent displacement field for constant deceleration parameter models of the universe. A lot of works has been carried out by many authors in different physical contexts within the framework of Lyra's geometry.

1.7 Saez-Ballester Theory

It is held that the long-range forces in the universe are produced by scalar fields. Scalar-tensor theories of gravitation have become a focal point of interest in many ares of gravitational physics and cosmology since the Einstein's theory of general relativity does not seem to resolve some of important problems in cosmology such as dark matter (Brans and Dicke (1961), Bergmann (1968), Nordvedt (1070)). These alternative theories of gravitation provide the most natural generalizations of the general relativity and thus provide a convenient set of representations for the observational limits on possible deviations from general relativity. The most widely accepted and possibly the best motivated theory is that of Brans and Dicke (1961), in which the scalar field shares the stage of gravitation. Saez and Ballester (1985) have developed a theory in which the metric is coupled with a dimensionless scalar field ϕ in a simple manner. The ϕ coupling in the field equations gives a satisfactory description of the weak fields in which antigravity regime appears inspite of the dimensionless character of the scalar field. This theory suggests a possible way to solve the missing matter problem in non-flat FRW cosmologies. Saez (1985) has also discussed the initial singularity and inflationary universes in this theory. He has derived a non-singular zero curvature FRW model and has shown that there is an antigravity regime which could act either at the beginning or inflationary epoch or before. Several authors have investigated cosmological models within the framework of Saez-Ballester theory of gravitation given by the field equations

$$G_{ij} - \omega\phi^r \left(\phi_{,i}\, \phi_j - \frac{1}{2} g_{ij} \phi_{,k} \phi^{\prime k} \right) = -T_{ij} \tag{1.17}$$

in proper units $8\pi G = c = 1$. The scalar field ϕ satisfies the equation

$$2\phi^r \phi'^i_{;i} + r\phi^{r-1}\phi_{,k}\phi'^k = 0 \tag{1.18}$$

Here r is an arbitrary constant and ω is a dimensionless coupling constant.

1.8 Big- Bang Cosmology

The observed expansion of the universe (Slipher (1915), Lundmark (1924) and Hubble and Humason (1931)) is a natural (almost inevitable) result of any homogeneous and isotropic cosmological model based on general relativity. However, by itself, the Hubble expansion does not provide sufficient evidence for what we generally refer to as the Big-Bang model of cosmology. While general relativity is in principle capable of describing the cosmology of any given distribution of matter, it is extremely fortunate that our universe appears to be homogeneous and isotropic on large scales.

The formulation of the Big-Bang model began in the 1940s with the work of George Gamow and his collaborators Alpher and Herman. In order to account for the possibility the early universe which was once very hot and dense (enough so as to allow for the nucleosynthesis processing of hydrogen), and has expanded and cooled to its present state (Gamow (1946) and Alpher et al.(1948)). Alpher and Herman (1948, 1949) predicted that direct consequences of this model is the presence of a relic background radiation with a temperature of few Kelvin. Of course this radiation was observed 16 years later as the the microwave background radiation by Penzias and Wilson (1965). Indeed, it was observation of the 3K background radiation that singled out the Big-Bang as the prime candidate to describe our universe.

Subsequent works on Big-Bang nucleosynthesis further confirmed the necessity of our hot and dense past.

1.9 Topological Defects: Cosmic String

Provided our understanding about unification of forces and big bang cosmology are correct, it is natural to expect that topological defects, appearing as solutions to many particle physics models of matter, could have formed naturally during phase transitions followed by spontaneously broken symmetries, in the early stages of the evolution of the universe. Certain types of topological defects like local monopoles and local domain walls may lead to disastrous consequences for cosmology, hence being undesired, while cosmic strings may play a useful role.

Cosmic strings are one-dimensional (that is line-like) objects which form when an axial or cylindrical symmetry is broken. They can be associated with grand unified particle physics models, or they can form at the electroweak scale. They are very thin and may stretch across the visible universe. A typical GUT string has a thickness that is less than a trillion times smaller that the radius of a Hydrogen atom. Still, a 10 km length of one such string will weigh as much as the earth itself.

The general relativistic treatment of strings was initiated by Letelier (1979, 1983) and Stachel (1980). Letelier (1979) has obtained the solution to Einstein's field equations for a cloud of strings with spherical, plane and cylindrical symmetry. Further in 1983, he solved Einstein's field equations for a cloud of massive strings and obtained cosmological models for Bianchi type -I and Kantowaski-Sachs space-times. Benerjee et al. (1990) have investigated an axially symmetric Bianchi type-I string dust cosmological model in presence and absence of magnetic field. String cosmological models with magnetic field are also discussed by Chakroborty (1991). Tikekar and Patel (1992) have discussed some Bianchi type-III cosmological models with and without source free magnetic field. Tikekar et al (1994) have studied a new class of specific inhomogeneous string cosmological solutions for cylindrically symmetrical space-times. Ram and Singh (1992) have obtained an exact solution of the field equations in general relativity for Bianchi type-II spacetime filled with string dust. Roy and Banerjee (1995) have discussed cosmological solutions for a cloud of geometrical strings and also for massive strings in LRS Bianchi type-II space-time. Kilinc and Yavuz (1996) have solved Einstein's field equations for inhomogeneous cylindrically symmetric space with string source in the presence of magnetic field. Mahanta and Mukherjee (2001) have presented some Bianchi type string cosmological models in Lyra geometry. Bhattacharjee and Baruah (2001) have studied the problem of cosmic strings taking Bianchi type cosmologies with a self interacting scalar field. Reddy (2003) has discussed an exact Bianchi type-I string cosmological model in a scalar tensor theory proposed by Saez and Ballester (1985). Bali and Singh (2003) have investigated an LRS Bianchi type-V bulk viscous fluid string dust cosmological model in general relativity. Wang (2005) has presented string cosmological models with bulk viscosity in Kantowski-Sachs models. Pradhan and Mathur (2008), Pradhan et al.(2008, 2010) and Tripathi et al. (2009, 2010) have studied string cosmological models in different context.

1.10 Dark Matter and Dark Energy

As the name implies, dark matter acts like regular matter gravitationally, but does not emit any electromagnetic radiation that can be observed on earth. Dark matter plays an important role in the formation of structure in the early universe. The structure of the universe that we observe, galaxies, stars, and other large scale objects evolved from small fluctuations in the plasma of the early universe that underwent gravitational collapse over the eons. Without dark matter, structure can only be formed by ordinary baryonic matter. But up to the recombination era, ordinary matter is coupled to the photons in the universe. This coupling results in a restoring force that acts to prevent further collapse, the result is acoustic oscillations and inhibition of structure formation. Such a picture would not be able to produce the amount of structure that is observed. The addition of dark matter (assuming it is still 'dark' at those energies, i.e. it is decoupled from the photons) changes the picture since dark matter is free to collapse gravitationally without resulting in a restoring force. This helps the formation of structure around local concentrations of dark matter. Current results from the WMAP experiment support the existence of dark matter in the early universe in amounts comparable to those today, indicating that dark matter is a long-lived species. Though the theory of dark matter remains the most widely accepted theory to explain the anomalies in observed galactic rotation, some alternative theoretical approaches have been developed which broadly fall into the categories of modified gravitational laws and quantum gravitational laws (Kroupa et al. (2010)).

Dark energy is one of the mysteries of modern science. It is unlike any known form of matter or energy and has been detected so far only by its gravitational effect of repulsion. Owing to its effects being discernible only at very very large distance scales, dark energy was only detected at the turn of the last century when technology had advanced enough to observe a greater part of the universe in finer detail. The astronomical observations of type Ia supernovae, galaxy redshift surveys cosmic background radiation data and large scale structure convincingly suggest that the observable universe is undergoing an accelerated expansion. Observations also suggest that there had been a transition of the universe from the earlier deceleration phase to recent acceleration phase. The cause of this sudden transition and source of the accelerated expansion are still unknown. Measurements of cosmic microwave background (CMB) anisotropies, most recently by the WMAP setellite, indicate that the universe is very close to being flat. For a flat universe, its energy density must be equal to a certain density, which requires a huge contribution from some unknown energy source. Thus, observational effects like cosmic acceleration, sudden transition, flatness of the universe and many more, need explanation. It is generally believed that some sort of dark energy (DE) is pervading the whole universe. This is a hypothetical form of energy that permeates all of space and tends to increase the rate of expansion of the universe. Thus DE is a prime candidate to explaining the recent cosmic observations. The most recent WMAP observations indicate that DE accounts for 74% of the total mass energy of the universe.

The study of anisotropic universes has a long history. For a realistic cosmological model, one should consider spatially homogeneous and anisotropies space-times and then observe the amount of homogeneity and isotropy. In such models, when both the the metric and the EoS parameter of the fluid are allowed to exhibit anisotropy, the universe can exhibit non-trivial isotropization histories, and it can be examined whether the metric and the EoS parameter of the fluid evolve toward isotropy. We may take about two main classes of such models according to whether this isotropization occurs at an early time or at late time of the universe. The former class can be related to the inflation field that drives inflation, while the latter one can be related DE that drives the late-time acceleration of the universe. An anisotropic DE can drive an anisotropic late-time acceleration and can break the isotropy achieved during inflation. DE has been conventionally characterized by EoS parameter $\omega = \frac{p}{\rho}$, where p is the fluid pressure and ρ is the energy density of the matter. By now, methods allowing for restoration of the quantity $\omega(t)$ from exponential data have been developed (Sahni and Strarobinsky (2006)) and an analysis of experimental data has been conducted to determine to this EoS parameter as a function of cosmological time (Sahni et al. 2008). The cosmological constant Λ or vacuum energy is the most efficient and simplest candidate for explaining the observed accelerated background expansion with equation of state parameter $\omega = -1$. However, the Λ term requires to be fine tuned to have the currently observed very small value. For this reason, different forms of dynamically changing DE with an EoS were proposed. The other conventional alternatives, which can be discussed with minimally coupled scalar fields are quintessence ($\omega > -1$) and phantom energy ($\omega < -1$). Rodrigues (2008) has investigated a Bianchi type-I Λ-CDM model with a non dynamical component of DE. Koivisto and Mota (2008) have investigated a cosmological model where the accelerated expansion of the universe is driven by a fluid with an anisotropic EoS by introducing two skewness parameter $\delta(t)$ and $\gamma(t)$ to quantify the deviation of pressure from isotropy.

1.11 Cosmologies with Varying G and Λ

The Einstein's field equations (1.2) with cosmological term Λ are given as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R + 2\Lambda) = -8\pi G T_{\mu\nu}. \tag{1.19}$$

The Newtonian constant G plays the role of the coupling constant between geometry and matter in Einstein's field equations. In an evolving universe, it appears natural to look this constant as a function of time. Numerous suggestions based on different arguments have been proposed in the past decades in which G varies with time (Wesson (1980)). Variation of gravitational constant G was originally suggested by Dirac (1938) on the basis of his large hypothesis. Many other extensions of Einstein's theory with time-dependent G has also been proposed in order to achieve a possible unification of gravitation and elementary physics or incorporate Mach's principle in general relativity (Hoyle and Narlikar (1962,1971), Brans and Dicke (1961), Alfonso-Faus(1986), Uehara and Kim (1982), Enginol (1989)). Among these theories, the most widely accepted scalar-tensor theory of gravitation is that of Brans-Dicke (1961) which was formulated to incorporate Mach Principle in general relativity. So far a number of cosmological models based on this theory have been presented. Canuto et al.(1977) suggested that G is indeed a time-dependent quantity. In most of the varible-G models, it has been shown that G is a decreasing function of time. But Levitt (1980), Massa (1995) and Abdel-Rahamann (1990) have studied the possibility of increasing G.

Recent measurements from type Ia supernovae (SNe) at intermediate and redshifts (Riess et al.(1998), Perlmutter et al.(1999)) indicate that the bulk of the energy in the universe is repulsive and appears like a quintessences component, that is, an unknown form of dark energy (in addition to the ordinary CDM matter) probably of primordial origin (Tunner 2000). Together with the observations of CMB anisotropies (de-Bernardis et al.(2002)) such results seem to provide an important piece of information connecting an early inflationary universe stage with the astronomical observations. This state of affairs has stimulated the result in more general models containing an extra component describing this dark energy, and simultaneously accounting for the present accelerated stage of the universe. A possible list of old and new candidates for quintessence include a time-varying cosmological term Λ (Overduin and Copper-stock (1998), Weinberg (1989)). The basic reason is the widespread belief that the early universe evolved through some phase transitions, thereby yielding a vacuum energy density which is at present is at least 118 orders of magnitude smaller than the plank time. Such a discrepancy between theoretical expectation (from the modern microscopic theory of particle and gravity) and empirical observations constitutes a fundamental problem in the interface uniting astrophysics, particle physics and cosmology, which is often called "the cosmological constant problem" (Cunha et al.(2002)). The Λ term has also been interpreted in terms of Higgs scalar field (Wagoner (1970)). Linde (1974) has proposed that Λ is a function of temperature and related in to the process of broken symmetries. Gasperini (1988) has argued that the Λ term can be interpreted as a measure of temperature of the vacuum which should decrease like the radiation temperature with cosmic expansion. Drietlein (1974) has suggested that the mass of the Higgs boson is connected with Λ as well as G. By taking into account the conservation of the energy-momentum tensor of matter and vacuum taken together, many researchers have invoked the idea of a decreasing vacuum energy and hence a varying cosmological constant Λ with cosmic expansion in the framework of Einstein's gravity.

1.12 $f(R)$ and $f(R,T)$ Theories of Gravitation

The late time accelerated expansion of the universe has attracted much attention in the recent years. Direct evidences of cosmic acceleration comes from high redshift supernova experiments (Riess et al.(2004)). Some other observations, such as cosmic microwave background fluctuations (Spergel et al.(2003)) and large scale structure (Tegmark et al.(2004)), provide an indirect evidence. These observations seem to change the entire picture of our matter filled universe. It is now believed that most part of the universe contains dark matter and dark energy. The modifications of general relativity seem attractive to explain late time acceleration and dark energy. Among the various modifications of general relativity, the $f(R)$ theory of gravity is treated most seriously during the last decade. It provides a natural gravitational alternative to dark energy. It has been suggested that cosmic acceleration can be achieved by replacing the Einstein-Hilbert action of general relativity with a general function of Ricci scalar $f(R)$. The $f(R)$ theory of gravity has been shown equivalent to scalar-tensor theory of gravity that is incompatible with solar system tests of general relativity, as long as the scalar field propagates over solar system scales (Chiba et al.(2007)). The explanation of cosmic acceleration is obtained just by introducing the term $1/R$, which is essential at small curvatures. Capozziello et al.(2008) have shown that dust matter and dark energy phases can be achieved by the exact solutions derived from a power law $f(R)$ cosmological models. These models consist of higher order curvature invariants as functions of the Ricci scalar. Viable $f(R)$ gravity models (Nojiri and Odintsov (2008)) have been proposed which show the unification of early time inflation and late time acceleration. The problem of dark matter can also be addressed by using viable $f(R)$ gravity models. These models can be used for the explanation of hierarchy problem in high energy physics. It also describes the transition phase of the universe from deceleration to acceleration. The conditions of the existence of viable cosmological models have been found by many researchers (Capozziello et al (2006), Koivisto (2007), Carloni et al.(2008), Ananda et al.(2008), Guarnizo et al.(2011)). Severe weak field constraints obtained from the classical tests of general relativity for the solar system regime seem to rule out most of the models proposed so far (Chiba et al.(2007), Nojiri and Odintsov (2008), Capozziello et al. (2008)).

Bertolami et al.(2007) proposed a generalization of $f(R)$ theory of gravity by including in the theory an explicit coupling of an arbitrary function of Ricci scalar R with matter Lagrangian density L_m. As a result of the coupling, motion of the massive particle is non geodesic and an extra force orthogonal to four velocity vector arises. The connections with modified Newtonian dynamics (MOND) and the pioneer anomaly were also explored. Harko (2008) extended this model to the case of arbitrary couplings in both geometry and matter.

Harko et al. (2011) developed another extension of standard general relativity, which is known as $f(R,T)$ modified theory of gravity, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace T of the stress-energy tensor. They have obtained the gravitational field equations in the metric formalism, as well as, the equations of motion for test particles, which follow from the covariant divergence of the stress-energy tensor. The $f(R,T)$ gravity model depends on a source term, representing the variation of the matter stress energy tensor with respect to the metric. A general expression for this source term is obtained as a function of the matter Lagrangian L_m, so that each choice of L_m would generate a specific set of field equations. They have studied particular models corresponding to specific choices of the function $f(R,T)$.

In $f(R,T)$ gravity, the field equations are obtained from the Hilbert-Einstein type variational principle. The action principle for this modified theory of gravity is given by

$$S = \frac{1}{16\pi G}\int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x. \tag{1.20}$$

The energy momentum tensor of the matter is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g})L_m}{\delta g^{ij}}. \tag{1.21}$$

If we assume that L_m of matter depends only on the metric tensor g_{ij}, and not on its derivatives, one obtains

$$T_{ij} = g_{ij}L_m - 2\frac{\partial L_m}{\partial g^{ij}}. \tag{1.22}$$

By varying the action S of the gravitational field with respect to the metric tensor g_{ij}, we obtain

$$f_R(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}+(g_{ij}\Box-\nabla_i\nabla_j)f_R(R,T)=8\pi T_{ij}-f_T(R,T)T_{ij}-f_T(R,T)\theta_{ij} \tag{1.23}$$

where

$$\theta_{ij}=-2T_{ij}+g_{ij}L_m-2g^{\alpha\beta}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{\alpha\beta}}, \tag{1.24}$$

$\Box=\nabla^i\nabla_i$, $f_R(R,T)=\frac{\partial f(R,T)}{\partial R}$, $f_T(R,T)=\frac{\partial f(R,T)}{\partial T}$ and ∇_i denotes the covariant derivative.

A contraction of (1.23) gives

$$f_R(R,T)R+3\Box f_R(R,T)-2f(R,T)=(8\pi-f_T(R,T))T-f_T(R,T)\theta \tag{1.25}$$

where $\theta=\theta_i^i$. Equation (1.25) gives a relationship between R and T. The matter Lagrangian can be taken as $L_m=-p$ since there is no unique definition of the matter Lagrangian. Then the energy momentum tensor of matter can be written as

$$T_{ij}=(\rho+p)u_iu_j-pg_{ij}. \tag{1.26}$$

Here ρ is the energy density and p the pressure of the matter and u^i is the four velocity vector satisfying $u^iu_i=1$. Then with the use of (1.26), we obtain the variation of the stress energy of the perfect fluid, the expression for θ_{ij} is

$$\theta_{ij}=-2T_{ij}-pg_{ij}. \tag{1.27}$$

Generally, the field equations also depend through the tensor θ_{ij} on the physical nature of the matter field. Hence, several theoretical models corresponding to different matter sources in $f(R,T)$ gravity can be obtained. Harko et al.(2011) suggested three possible forms of the function $f(R,T)$ as follows:

$$f(R,T) = \begin{cases} R + 2f(T), \\ f_1(R) + f_2(T), \\ f_1(R) + f_2(R)f_3(T). \end{cases} \tag{1.28}$$

The astrophysical and cosmological implication of $f(R,T)$ gravity have been extensively investigated by several authors so far.

1.13 Phases of the Universe

Today's universe (redshift z=0) is dominated by DE, but it did undergo three transitions, from inflationary phase to radiation-dominated, radiation-dominated to matter-dominated and from matter-dominated to dark energy dominated. These different phase of the universe can be described by an EoS parameter, where pressure and energy density are related via

$$p = (\gamma - 1)\rho, 0 \leq \gamma \leq 2 \tag{1.29}$$

Different values of γ give rise to different physical situations, e.g., the values $\gamma = 1, 4/3, 2, 2/3, 0$ correspond to dust, radiation phase, stiff matter, inflation and DE respectively.

1.13.1 Inflationary Phase

The most convincing explanation, for the flatness, isotropy and homogeneity of the observed universe, is the inflationary scenario. The standard cosmological models suffer a number of long standing problems such as the horizon problem and the flatness problems. In order to overcome these difficulties, Guth (1981) has proposed an inflationary model. The inflationary phase can be described by an equation of state relating the pressure p and the density ρ known as "gamma-law" equation of state, given by equation (1.29) provided the parameter $\gamma = \frac{2}{3}$.

Linde (1982) has proposed the idea of "slow-rollover" or new inflation which solved many problems of the early phases of the universe. According to "slow-rollover" idea, the inflationary expansion is driven by the potential energy of a scalar field ϕ while the field slowly rolls down its potential V(ϕ). When ϕ reaches the minimum of the potential, the vacuum energy thermalizes and inflation is followed by the usual radiation-dominated expansions. All inflationary models are based upon the idea of "slow-rollover".

Although the inflationary universe scenarios have gone a longway towards explaining many fundamental features of our currently observed universe such as its near flatness, its large entropy content and its large size, it has not been successful in explaining the large-scale structure and the the formation of galaxies. This has prompted several workers to venture into exotic models based on Brans-Dicke rather than Einstein's gravity. Most of the analysis of Brans-Dicke inflation or extended inflation assumes a flat R-W model at the outset. Duncan and Jensen (1990) have considered the cases of curved spatial hypersurfaces (K=$\pm$1) and found a new solutions for extended inflationary phase. Elst et al. (1995) have examined the degree for restrictiveness on the dynamical relationship between the cosmological scale factor and the inflation driving self-interaction potentially of a minimally coupled scalar field.

1.13.2 Radiation-Dominated Phase

A non-static distribution would radiate energy and so it would be surrounded by an ever-expanding zone of radiation. according to the modern story of Genesis, our universe began in a very hot and dense state. Then, as it expanded and cooled down, the universe went through several phase transitions, with each transition corresponding to breaking of some symmetry between the elementary particles and their interactions. The early universe has an undifferentiated soup of matter and radiation in a state of thermal equilibrium. The radiation dominated phase of the evolution of universe corresponds to $\gamma = \frac{4}{3}$, and is therefore characterized by the EoS

$$p = \frac{\rho}{3}, \tag{1.30}$$

1.13.3 Matter-dominated Phase

The matter-dominated phase can be modeled after a dust that uniformly fills the space. Since the temperature of the universe has fallen to around 3000k, most of the particles have non-relativistic valocity ($v << c$). This corresponds to a negligible pressure. At $z \approx 5000$, about 70,000 years after the Big-Bang, we have matter-radiation equality, and the universe becomes matter-dominated. Thus, the matter-dominated universe corresponds to $\gamma = 1$, and is therefore expressed by

$$p = 0, (\rho \neq 0). \tag{1.31}$$

The present universe is believed to be matter-dominated. There are some intermediate phase such as Zel'dovich or stiff matter phase ($\gamma = 2, p = \rho$) and vacuum universe ($p = 0, \rho = 0$), which describe several important phenomena of evolution of the universe.

1.13.4 Vacuum Energy Dominated Phase

An important breakthrough in cosmology occurred in late 90's with the measurement of SNe Ia by high-z Supernova team (Riess et al.(1998)) and Supernova Cosmology Project (Perlmutter et al.(1999). The data from these surveys as well as from the complementary probes provide strong evidence that the universe has recently entered in a phase of accelerated expansion. In order to explain the accelerated expansion, cosmologist introduce a new fluid, which possesses negative pressure, called dark energy. A number of models have been proposed to describe the universe with DE, partly such as scalar fields: Quintessence (Steinhardt et al.(1999)) and phantom (Caldwell (2002)), exotic EoS: Chaplygin gas (Kamenshchick et al. (2001)) and generalized Chaplygin gas (Bento et al.(2002)), a linear EoS (Babichev et al.(2005)). The observational constraints indicate that the current EoS parameter $\gamma = \frac{p}{\rho} + 1$ is around 0, which shows the dominance of vacuum energy.

1.14 Cosmological Parameters

In this section, we discuss some physical parameters which are useful in studying the physical behavior of the cosmological models.

1.14.1 Hubble parameter

One of the most important parameters in cosmology is the Hubble parameter. It is denoted by H and is defined as

$$H = \frac{\dot{a}}{a} \tag{1.32}$$

where a is the average scale factor. It describes the present expansion rate of the universe.

As the universe expands, the distance l between galaxies increases proportional to the scale factor, i.e, $l(t) = l_0 a(t)$, where l_0 is the initial distance. The recession velocity $v(t)$ of the observers, relative to each other, due to the expansion of the universes, is given by

$$v(t) = \frac{dl}{dt} = l_0 \dot{a}(t) = \frac{\dot{a}(t)}{a(t)} l(t) = H(t) l(t), \tag{1.33}$$

which is the Hubble's law proposed by Edwin Hubble in 1927.

From this derivation of the Hubble's law, it becomes manifest that the Hubble parameter is not constant but can be a function of time. The time-varying Hubble parameter $H(t) = \frac{\dot{a}}{a}$, measures the rate of change of the scale factor $a(t)$ and offers a link the observations with a proposed model using the scale factor. The Hubble parameter parameterizes the expansion rate of the universe. Its value for the present-day universe is referred to as the Hubble constant H_0.

1.14.2 Deceleration Parameter

The deceleration parameter (DP), denoted by q, is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{1.34}$$

This parameter measures the rate at which the expansion of the universe is changing with time and its sign characterizes accelerating or decelerating nature of the universe. The positive sign of q corresponds to decelerating models whereas the negative sign indicates acceleration. The case $q = 0$ corresponds to expansion of the universe with constant velocity.

1.14.3 Critical Density

It is given by the relation

$$\rho_c = \frac{3H^2}{8\pi G}. \tag{1.35}$$

The critical density is the average density of matter required for the universe to just halt its expansion, but only after an infinite time. A universe with the critical density is said to be flat. In theory of general relativity, Einstein demonstrated that the gravitational effect of matter is to curve the surrounding space. In a universe filled with matter, its overall geometry and its fate are controlled by the density of the matter within it.

1.14.4 Density parameter

The expression for the density parameter Ω is given by the ratio of the actual (or observed) density ρ to the critical density ρ_c i.e,

$$\Omega = \frac{\rho}{\rho_c}. \tag{1.36}$$

The matter distribution determines the spatial geometry of our universe. For, the universe is closed, flat and open according as $\Omega > 1$, $\Omega = 1$ and $\Omega < 1$

respectively. This is true regardless of what type of matter we have in the universe. Therefore, Ω determines the fate of the universe. Observations have shown that the current universe is very close to a spatially flat geometry ($\Omega \simeq 1$). This is actually a natural outcome of inflation in the early universe.

1.15 Kinematical Parameters

The streamlines of the motion of a cosmic fluid are characterized kinematically by various parameters defined as follows:

1.15.1 Expansion Scalar

The expression for the expansion scalar (θ), which deals with the expansion of the universe, is given in tensor form as,

$$\theta = u^i_{;i}. \tag{1.37}$$

1.15.2 Shear

The shear plays an important role in general relativistic and stellar cosmological models. The shear tensor σ_{ij} is defined as

$$\sigma_{ij} = \frac{1}{2}(u_{i;\alpha}P^{\alpha}_{j} + u_{j;\alpha}P^{\alpha}_{i}) - \frac{1}{3}\theta P_{ij}. \tag{1.38}$$

Here the projection tensor P_{ij} has the form

$$P_{ij} = g_{ij} - u_i u_j. \tag{1.39}$$

The shear tensor aries in the decomposition of 4-velocity vector of the fluid. It describes the rate of distortion of matter flow.

The shear scalar σ is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}. \tag{1.40}$$

1.15.3 Anisotropy Parameter

The expression for the anisotropy parameter, which explains the isotropy of the evolution of the universe, is given by

$$A_m = \frac{1}{3}\left[\left(\frac{H_x - H}{H}\right)^2 + \left(\frac{H_y - H}{H}\right)^2 + \left(\frac{H_z - H}{H}\right)^2\right], \tag{1.41}$$

where $H_x = \frac{\dot{A}}{A}$, $H_y = \frac{\dot{B}}{B}$ and $H_z = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions of x, y, z axes respectively.