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It is certified that the work contained in the thesis titled "Thermoelastic Damping Analysis in Micro and Nanomechanical Systems under Generalized Thermoelasticity and Non-Classical Continuum Theories" by **Harendra Kumar** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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Dedicated

to

My Grandmother

Mrs. Kailashi Devi

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CONTENTS

3 THERMOELASTIC DAMPING ANALYSIS IN MICRO AND NANO-BEAM RESONATORS CONSIDERING MOORE-GIBSON-THOMPSON THERMOELASTICITY THEORY 59

LIST OF FIGURES

LIST OF SYMBOLS

σ_{ij}	Components of stress tensor
u_i	Components of displacement vector
ϵ_{ij}	Components of strain tensor
q	Heat flux vector
T_0	Reference temperature
θ	Temperature increment from the reference temperature
T	Temperature
t	Time
L	Length
b	Width
h	Thickness
s	Entropy per unit mass
ρ	Mass density
k	Thermal conductivity
k^*	Thermal conductivity rate
w	Deflection
λ, μ	Lame's constants of material
α_T	Thermal expansion coefficient
C_v	Specific heat at constant volume
E	Young's modulus
Δ_E	Relaxation strength of Young's modulus
ν	Poisson's ratio

τ_q	Phase-lag of heat-flux vector
τ_T	Phase-lag of temperature gradient
τ_v	Phase-lag of thermal displacement
A	Cross-section area
∇	Gradient operator
∇^2	Laplacian operator
χ	Thermal diffusivity of the material
δ_{ij}	Kronecker delta
U	Strain energy
K	Kinetic energy
M_T	Thermal moment
I	Moment of inertia
m_{ij}	Deviatoric part of the couple stress tensor
χ_{ij}	Symmetric curvature tensor
ω	Frequency of vibration
Q	Quality factor
$\boldsymbol{r}$	Position vector

Note: The bold notation is used for vector or tensor quantities. Subscripts i, j, k take the values x, y, z and summation is implied by index repetition.

ABBREVIATIONS

TED	**T**hermoelastic **D**amping
LS	**L**ord-**S**hulman Thermoelasticity Theory
DPL	**D**ual-**P**hase-**L**ag Thermoelasticity Theory
GN	**G**reen-**N**aghdi Thermoelasticity Theory
TPL	**T**hree-**P**hase-**L**ag Thermoelasticity Theory
MGT	**M**oore-**G**ibson-**T**hompson Thermoelasticity Theory
MCST	**M**odified **C**ouple **S**tress **T**heory
MNCS	**M**odified **N**onlocal **C**ouple **S**tress Theory
MEMS/NEMS	**M**icro/**N**ano-**E**lectromechanical **S**ystems

PREFACE

Based on the fast growth of current scientific and engineering technologies, micro and nano-electromechanical systems (MEMS/NEMS) have received a great interest of researchers because of a wide range of promising applications of such systems in mechanical signal processing, ultra sensitive mass detection, probe microscopes scanning etc. MEMS/NEMS are the devices that are integrated with the electrical and mechanical functions at the micro and nanoscales. They consist of miniaturized electrical and mechanical apparatuses such as actuators, beams, sensors, pumps, resonators, and motors. These components convert one form of energy into another, which can be quickly and conveniently measured. One of the important applications of MEMS is micromechanical resonators for their high sensitivity and fast response. For resonators, it is possible to construct and design systems with very little loss of energy dissipation during vibration.

The performance of MEMS/NEMS systems is generally measured by the quality factor. The quality factor is a dimensionless quantity defined as the ratio of stored energy in the system and dissipated energy by the system per cycle of vibration. A high value of the quality factor indicates the low rate of energy dissipation in the vibrating system. For designing the MEMS/NEMS with high performance, the prediction of the high quality factor is important. The less energy dissipation is usually beneficial in technical applications because it makes a device more effective and responsive. In order to apply them suitably, a thorough understanding of the energy dissipation mechanisms in such systems is therefore essential. Several energy loss factors in MEMS/NEMS, such as air damping, anchor loss, thermoelastic damping, etc have been identified.

Among these energy loss factors, thermoelastic damping (TED) is one of the important energy loss factor in vibrating structures at micron and submicron scales. The TED appears in the vibrating systems due to coupling between temperature and elastic fields. Thermoelastic damping can be minimized by constructing system with high quality factor. The TED analysis in micro/nano resonator system is therefore a topic of active research in recent time.

It is worth to be mentioned that thermoelastic damping phenomena was first observed by Zener (1937; 1938). Further, Lifshitz and Roukes (2000) extended the Zener's work and derived a closed-form solution of TED in terms of the inverse quality factor of microbeam based on the classical Fourier heat conduction theory. Several experimental observations (Fleck et al., 1994; Stolken and Evans, 1998; Faris et al., 2002; McFarland and Colton, 2005) indicate the small scale effect in the vibrating structures at micron and submicron scales. But, due to the lack of internal material length-scale parameter, the classical (local) continuum theory cannot appropriately predict the mechanical behaviour of such structures. Therefore, several nonclassical continuum theories in this direction have been developed over the years to address this shortcoming/limitations in classical theory. The couple stress theory (Toupin, 1964), nonlocal elasticity theory (Eringen, 1983; Eringen and Edelen, 1972), and strain gradient theory (Mindlin and Eshel, 1968; Lam et al., 2003), modified couple stress theory (Yang et al., 2002) are some popular nonclassical theories which are used to predict the size effect in the structures at micron and submicron scales. Among all the nonclassical theories, modified couple stress theory and nonlocal elasticity theory are generally used to capture the size effect in the systems due to the involvement of only one material length-scale parameter at micron and submicron scales, respectively.

The present thesis is concerned with the analysis of thermoelastic vibration of micro and nanomechanical systems considering the effects of classical and non-classical continuum theories in the framework of non-Fourier heat conduction theories.

Mathematical formulation is carried out to derive basic governing equations for micro/nanobeam and plate resonators. The recently proposed generalized thermoelastic models that are considered to be more realistic as compared to the classical model, are employed to consider the coupling effects of mechanical strain and thermal fields. The expressions of the quality factor for analyzing TED in beams and plates are found out analytically by following various methods including frequency generation approach as well as entropy generation approach. Attempts are made to investigate the impact of TED on the responses of deflection, temperature, and thermal moment of microbeams in detailed manner. Moreover, the effect of phase-lag times associated with non-Fourier heat conduction equations are also investigated. Analytical results are further illustrated by carrying out numerical work for suitable examples and the effects of various parameters on quality factor and TED are highlighted. The analysis of the present investigation is intended to be useful in the design of mechanical resonators at micron and submicron scales.

The work carried out in the thesis is divided into seven chapters and outlines of various chapters are as follows:

Chapter 1 begins with an introduction related to the MEMS/NEMS systems and their applications, including the brief account of the energy dissipation mechanisms in such systems and a history on the development of various modifications to the classical thermoelasticity and classical elasticity theories, and then moves on to a comprehensive literature review on the work relevant to the current thesis. The chapter concludes with a summary of the objective of the thesis.

Chapter 2 discusses TED in micro and nanobeam resonators with the help of the expression of inverse quality factor in the context of thermoelastic heat conduction model with a single delay term (Quintanilla, 2011). This chapter is divided into **two different subchapters**. **Subchapter 2.1** attempts to investigate TED in microbeam resonators by deriving the formula for the inverse quality factor. We consider the heat conduction model with a single delay term, and derive an expression of the inverse quality factor by applying complex frequency approach. The variation of TED versus normalized frequency and thickness of Silicon microbeam resonator for different aspect ratios have been studied. We compare the results of present model with the corresponding results of thermoelasticity theories of type Green-Naghdi (GN-III), three-phase-lag (TPL), and Lord-Shulman (LS) models. **Subchapter 2.2** demonstrates TED in micro and nanobeam resonators utilizing the heat conduction model with a single delay term. Here the expression of the quality factor for TED is derived by adopting the entropy generation approach method. With the help of numerical results, the influences of TED as functions of normalized frequency as well as beam thickness are presented. Further, the effects of the time delay parameter and the several other material constants on TED have been discussed in detail. The results of the present model are compared with those obtained for GN-III model.

Chapter 3 aims at the investigation of TED in microbeam resonators under the classical and nonclassical continuum theories within the framework of recently proposed Moore-Gibson-Thompson (MGT) thermoelasticity theory. This chapter contains **three different subchapters**. **Subchapter 3.1** presents the analysis of TED in microbeam resonators under the classical continuum theory and MGT thermoelastic equations. This sub chapter establishes the expressions of deflection and thermal moment for analyzing TED in microbeam resonators. The finite Fourier sine transform and Laplace transform methods are used to solve the coupled thermoelastic equations. In microbeams with simply supported and isothermal boundary conditions, the vibration responses of deflection and thermal moment are investigated. By comparing the findings obtained under the MGT model with the corresponding results obtained under the LS and GN-III models, the responses of deflection and thermal moment of beams are validated. **Subchapter 3.2** investigates the dynamic behaviour of microbeam by taking the impact of MGT thermoelastic equations. In order to capture the small-scale effect on thermoelastic vibration of microbeams, modified couple stress theory (MCST) is taken into consideration. The coupled thermoelastic equations of motion are derived by using the Hamilton's principle. The solutions for deflection and thermal moment of microbeams are obtained with the help of integral transform methods. The obtained results under MCST are compared with the existing results for the classical continuum theory. **Subchapter 3.3** describes the investigation on the dynamic and mechanical behaviors of nanobeams utilizing nonlocal elasticity theory and the MGT heat conduction equation while accounting for surface energy effects. In the context of nonlocal elasticity and surface elasticity theories, the governing equations of motion for an Euler-Bernoulli nano beam are firstly derived when the beam is simply supported at both ends and has a uniform load on the top surface. Thereafter, the solution for deflection and temperature distribution along the thickness direction of the nanobeam is obtained by using the finite Fourier sine transform and Laplace transform techniques. The influences of nonlocal parameter, phase-lag time, residual surface tension, surface elastic modulus, thickness, and length of the nanobeam on deflection and temperature over time are systematically analyzed in depth. Some important points are highlighted regarding the size dependency and surface effects on vibration of nanobeams in the present context.

Chapter 4 intends to examine TED in nanobeam resonators employing modified nonlocal couple stress (MNCS) theory that uses the MCST and Eringen's nonlocal elasticity theory simultaneously. Here the study is performed within the context of MGT thermoelasticity theory. The frequency relation between natural and eigen frequencies is first derived by considering the isothermal case of nanobeams. Thereafter, a closed-form expression of size-dependent TED is obtained by following frequency approach method reported by Lifshitz and Roukes (2000). With the help of numerical results for a suitable material, the influences of nonlocal parameter and material length-scale parameter on TED are analyzed in a detailed manner. In addition, the effects of phase-lag time on TED associated with the MGT model is demonstrated. Furthermore, the obtained results under MNCS theory are compared with the results under classical, MCST, and nonlocal elasticity theories.

Chapter 5 deals with the study of the dynamic behaviour of Timoshenko microbeams by combining the effects of MCST and dual-phase-lag (DPL) thermoelastic model. The explicit expressions for normalized mid-span deflection and thermal moment of a Timoshenko microbeam considering MCST and DPL heat conduction model are derived. The constructed mathematical model is capable of predicting the size-effects in microbeam due to the presence of the internal material length-scale parameter. The governing equations and related boundary conditions of microbeam are constructed by employing the Hamilton's principle. The obtained results in the present context are compared with the corresponding results obtained in the contexts of other existing models including the classical Fourier heat conduction model. Analytical results are illustrated by carrying out computational work for a Timoshenko microbeam made of Silicon material at the constant reference temperature.

Chapter 6 demonstrates the analysis of TED in microplate resonators by using three-phase-lag (TPL) heat conduction model. In order to capture the size-effect in microplate, MCST is taken into consideration for present analysis. As a microplate resonator, a well-known Kirchhoff's plate model is used. The expression of the quality factor for TED is obtained following complex frequency approach method. The variations of TED as functions of the normalized frequency, microplate thickness, and length-scale parameter have been investigated. The effect of phase-lag parameters on TED has also been discussed. The results of the present model are compared to the existing results for the classical continuum theory.

Chapter 7 provides a synopsis of the current work and the scope for further research in the related topics.