CHAPTER 1

Introduction

1.1 Introduction

The automated process of improving a system or component depending on an objective function and in accordance with particular design limitations is known as optimization. Aside from factors related to service, safety, aesthetics, and other needs, structures must also take economic issues into account when being designed. Optimized structures are becoming increasingly prevalent in our lives as their adoption rate has significantly increased. The practise of using mathematical models to deal with such problems in a methodical way is known as optimal structural design. The standard method for structural optimization is to create an algorithm that consistently leads to a solution that meets a set of sufficient and required requirements for an optimum. Mathematically a structural optimization problem can be represented as Equation 1.1:

$$\begin{cases} \min x & f(x, y(x)) \\ \text{subject to} & \begin{cases} \text{design constraint on } x \\ \text{state constraint on } y(x) \\ \text{equilibrium constraint} \end{cases} \end{cases} \tag{1.1}$$

The objective function (f), represents an objective that could be minimized/maximized depending on the output requirements. An objective function for an optimized structure is typically related to its mass, stiffness, or volume. Additionally, the structural design domains and state variables associated with the objective function must be defined. The design variable (x) describes the design of the structure, and it may represent the geometry. The state variable (y) represents the structural response which can, for example, be recognized as stress, strain or displacement. Furthermore, the state variables depend on the design variables y(x). The objective function is subjected to the design and state variable constraints to steer the optimization to a sought solution.

The question of what the optimal answer among all optimal solutions arises if more than one solution meets such requirements. It can often be difficult to find the absolute optimal because it does not need to meet any further requirements. Numerous algorithms have been developed as a result of the ambiguity around the definition of the absolute optimal. No one algorithm has been effective in every design scenario; as a result, they are more or less problem-based. Numerous methods for optimizing structures have emerged over time, including topology optimization, size optimization, topography optimization, and shape optimization, among others (Mei and Wang, 2021).

The use of simulation-based design frameworks has increased over the past several decades because they may assess the performance of an alternative design at an early stage of product development or compare the relative benefits of a suggested design. By heavily utilising a variety of numerical techniques, such as the finite element method (FEM), computational fluid dynamics (CFD), boundary element method (BEM), as well as newer methods such as iso-geometric analysis (IGA) and meshless methods (MMs), the conventional product design procedure has been greatly enhanced.

Stochastic and deterministic optimization algorithms are used in conjunction with these numerical approaches to optimize engineering systems and structures. In this situation, structural optimization has shown to be crucial in addressing the constantly growing demands for developing better solutions while taking into account the shape, topology, and size of structures. In 1960, Schmit's seminal article was published, marking the start of an intensive period of research in this area. This study showed how non-linear mathematical programming and other numerical simulation models may be used to produce automated, optimal design capabilities. This approach has proven to be effective in addressing a wider range of problems, expanding the scope of engineering design optimization. Over the years, a number of major firms involved in the manufacture of machines, automobiles, aeroplanes, and other industrial items have invested their research efforts in converting the theoretical advantages of structural optimization into practical advantages. It is crucial to distinguish the scope of the various structural optimization methods, such as size, shape, and topology optimization, at the commencement of the discussion.

a. Size optimization: Size optimization in civil engineering is a technique used to determine the most efficient and cost-effective dimensions for a structure or component. This can include determining the optimal thickness of a beam or the optimal size of a foundation. The goal of size optimization is to minimize the material used while still meeting all of the structural and design requirements. It is a practice often employed in the field of structural engineering that involves finding the optimal dimensions of a structure in order to meet certain design objectives. These objectives may include minimizing the weight of the structure while maintaining its strength, maximizing its stiffness or minimizing its deflection, or a combination of these goals. Size optimization can be applied to various types of structures, including bridges, buildings, aircraft, and automobiles, to enhance the efficiency and performance of such structures. One of the main applications of size optimization in civil engineering is in the design of bridges. Engineers must consider various factors, such as the weight of the bridge, the type of materials used, and the expected load on the bridge. By using size optimization techniques, engineers can determine the optimal dimensions for the various components of the bridge, such as the beams, columns, and foundations. This can help to reduce the overall cost of the bridge while still ensuring its structural integrity. Another application of size optimization in civil engineering is in the design of buildings. Engineers must consider factors such as the weight of the building, the type of materials used, and the expected load on the building. By using size optimization techniques, engineers can determine the optimal dimensions for the various components of the building, such as the walls, columns, and foundations. Size optimization can also be applied to the design of other structures, such as dams, roads, and tunnels. The goal is to estimate the cross-sectional area or thickness of a structural part that will minimize or maximize the selected objective while optimising design performance. The length and location of the structural elements might be known, but the optimal cross-section might need to be determined. This is where the sizing optimization comes into use.

b. Shape optimization: Shape optimization in civil engineering is a method of designing structures that minimizes certain objectives, such as weight, cost, or stress, while meeting certain constraints, such as strength and stability. The goal of shape optimization is to find the optimal shape of a structure that satisfies all the design requirements while being as efficient as possible. It deals with selecting the proper geometric profile when the design topology is fixed while allowing for a gradual change of domain boundaries. The shape or boundary could either be represented by an unknown equation or by a set of points whose locations are unknown. Applications of shape optimization in civil engineering can be found in the design of bridges. Engineers often use shape optimization to minimize the weight of a bridge while ensuring that it can support the expected loads and withstand the forces of nature, such as wind and earthquakes. This can lead to significant cost savings, as lighter bridges require less material and are cheaper to construct. Shape optimization can also be used in the design of offshore structures, such as oil rigs and wind turbines. Engineers can use shape optimization to minimize the weight of these structures while ensuring that they can withstand the forces of the ocean, such as waves and currents. It is also used in the design of buildings to minimize the amount of material needed to construct a building while ensuring that it is stable and can withstand the forces of nature. Shape optimization involves modifying the geometry of a design to meet the desired objective.

c. Topology optimization: Topology optimization is a computational technique used in engineering and architecture to optimize the layout and design of structures and systems. The basic idea behind topology optimization is to use mathematical algorithms to find the optimal layout and configuration of a structure, given certain design constraints and objectives. This can be done by simulating the behaviour of the structure under different loads and conditions and then using optimization algorithms to find the layout that results in the best performance. It operates with a larger flexibility in design and creates the best material distribution through the creation and merging of cavities. For 2D continuum-type structures, topology changes can be achieved by allowing the thickness of a sheet to have values of zero at various locations, which determines the number and shape of cavities or holes. One of the main advantages of topology optimization is that it can be used to find new and innovative solutions to engineering problems. For example, it can be used to design structures that are stronger and more efficient than traditional designs or to find new ways to use existing materials. Another advantage of topology optimization is that it can be used to reduce the weight and cost of structures. This is particularly important in civil engineering, where structures are often built to withstand extreme loads and conditions. Topology optimization in civil engineering has been extensively used in designing truss structures. It changes the overall layout of the structure while creating additional cavities where the material is not required as per the given loading and constraints.

A representation of the three different types of structural optimization is displayed in Figure 1.1. Size optimization focuses on identifying the optimal size parameters, such as width and thickness while maintaining the shape and overall layout of the structure. Shape optimization involves altering the outer boundaries of the structure to achieve the best possible shape. Topology optimization, on the other hand, optimizes the distribution of materials within a specified area and can determine the best layout from various options rather than a specific layout.

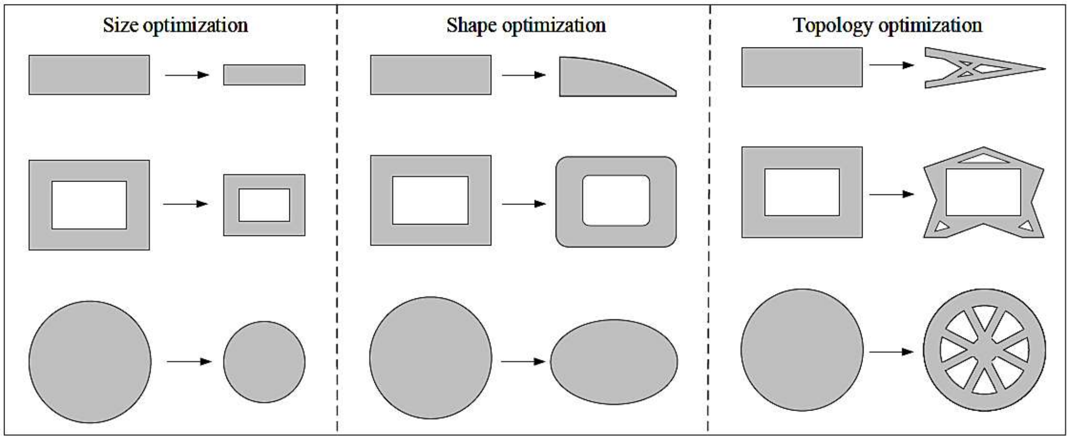
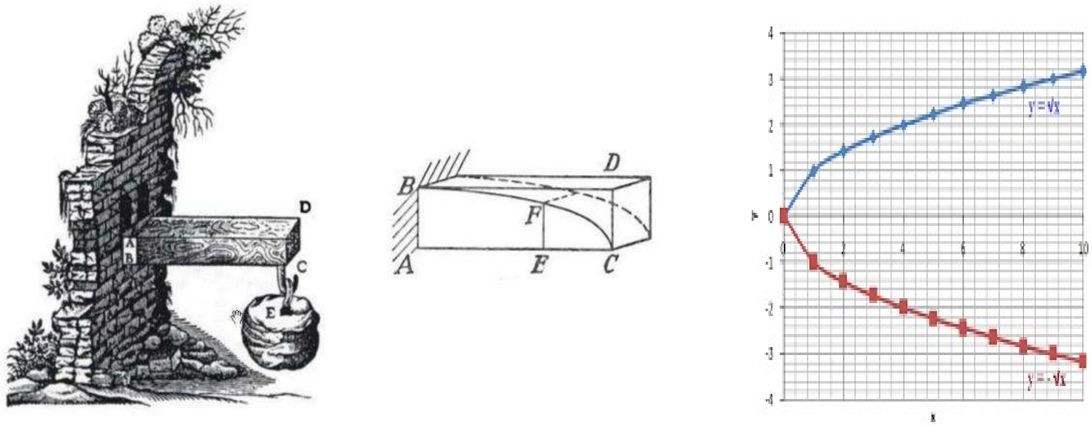


Figure 1.1: Illustration of Size, Shape and Topology optimization.

All of the different optimization methods have demonstrated their utility in real-world applications, among which shape optimization techniques have been gaining popularity over time (Upadhyay et al., 2021). This is because shape optimization can improve the overall shape of a structure without creating new holes or cut-outs while still meeting the constraints imposed on it. Shape optimization is seen to be more significant than size optimization since it enhances structural characteristics like structural weight and stress distribution. The design domain continuously changes throughout the whole shape design optimization process as a result of design parameter modifications and subsequent internal and/or exterior boundary modifications. Technically speaking, shape optimization is an iterative procedure for mapping out the best geometric design by combining design, analysis, and optimization tasks while adhering to predetermined design constraints. Shape optimization has been drawing attention from researchers across the globe for decades (Munk et al., 2015). The early practice of shape optimization goes back to the time of Galileo in 1638. Back then, he formulated a mathematical formulation to find the minimum weight of a cantilever beam as a parabolic beam/ shape of a square root function, as shown in Figure 1.2.



(a) (b)

Figure 1.2 a) Galileo's study on the shape of a cantilever beam in his 1638 publication (Ref.: Discorsi e Dimonstrazioni Matematich, Galileo Galile,1638),
b) Shape of a square root function (Ref.: Math Planet, Mattecentrum)

Since then, many researchers have worked in this field to get a proper shape that can be used in structures with ease. Categorising most of shape optimization algorithms can broadly be classified into two genres (Pathak et al., 2009): (1) Gradient-based method, (2) Zero-order/ Non-gradient based method.

(1) Gradient-based method: Mathematical programming approaches are generally used to handle shape optimization problems through gradient-based techniques. In mathematics, the problem may be expressed by Equation 1.2

$$\text{Minimize } f(x) \text{ subjected to } \begin{cases} g_i(x) \leq 0 & i = 1 \ldots \ldots \ldots N_1 \\ h_i(x) = 0 & i = N_1 + 1 \ldots \ldots \ldots N_2 \\ k_i(x) \geq 0 & i = N_2 + 1 \ldots \ldots \ldots N_{BC} \end{cases} \tag{1.2}$$

Where $x_{lk} \leq x_k \leq x_{uk} \quad k = 1 \ldots \ldots \ldots N_{DV}$

The f(x) is the objective function, and g, h, and k are less than, equality and greater than type constraints. 'x' is the design variable vector with lower x_{lk} and upper x_{uk} bounds for k^{th} design variable. N_{BC} and N_{DV} are the number of boundary conditions and design variables, respectively. The following are the main steps involved in gradient-based methods:

a. Selecting objective function

There are several options for choosing objective functions. Most commonly, the objective function is chosen to be the structure's weight or volume. The minimizing of maximum stresses on the boundaries may also serve as the foundation for the objective functions. The stress-based objective functions are:

i. Minimization of maximum Von Mises stress along the boundary.
ii. Difference of maximum and minimum tangential stress.
iii. Stress levelling.
iv. Weighted objective function.

b. Selection of constraints

The constraints enforced over an optimization method can be classified as geometric or behavioural constraints. The geometric constraints limit the range of acceptable structural types, shapes, and sizes, whereas the behavioural constraint limits the characteristics that describe the structure's state.

c. Sensitivity calculation

Calculating the first-order derivatives (gradients) of the structural response with respect to the design variables is one of the key steps in gradient-based shape optimization methods. The computation of these derivatives, known as sensitivity analysis, is a costly and time-consuming operation in structural optimization because of the implicit relationships between the behaviour and design variables. In optimum structural design, the following four sensitivity analysis techniques have become popular for computing the aforementioned derivatives:

i. Finite difference technique.
ii. Direct differentiation technique.
iii. Semi-analytical method.
iv. Continuum derivative approach

d. Numerical optimization methods

Numerical optimization techniques are used to tackle the majority of gradient-based shape optimization problems. The following are a few of the most significant numerical approaches that are typically employed for this purpose:

i. Sequential linear programming method (SLP).
ii. Penalty function methods.
iii. Feasible direction methods.
iv. Sequential quadratic programming methods (SQP).
v. Sequence of approximate methods.

e. Convergence aids

In order to accomplish the controlled convergence, the following are a few of the strategies that have been established.

i. The cutting plane method.
ii. Neighbourhood constraints.
iii. Method of inscribed hyper structures.
iv. Move limits methods.

(2) Zero-order/ Non-gradient method: Zero-order methods are often called non-gradient or gradient-less methods. These methods don't involve stress derivatives making their implementation in conventional finite element algorithms far more straightforward when compared to gradient-based methods. Schnack (1977, 1979) and Schnack et al. (1988) laid the foundation for the zero-order method, suggesting that stress minimization can be achieved by locally modifying the design boundary to homogenize the stress level. These approaches' fundamental strategy is to repeatedly change the shape of the boundary in order to maintain a constant, or nearly constant, stress distribution around the boundary, which has been demonstrated to be consistent with the goal of reducing peak stress. The non-gradient techniques may be categorised into the following groups.

a) Direct methods

These techniques are founded on the idea that the amount of material present under a boundary point affects the stress there. Intuitively, adding material while the stress is large and removing material when the stress is small will result in a wholly stressed boundary.

b) Curvature methods

The normal stress and the stress concentration effect are the two parameters that define the stress value at a boundary in a two-dimensional stress analysis problem. Since stress concentration effects and boundary curve curvature are closely associated, they can be employed in this application.

c) Pattern transformation methods

A methodology for altering the boundary's shape depending on the stress ratio in the boundary finite elements is called the pattern transformation method. For the first step, the calculation of the stress ratio for the boundary finite elements is done, followed by the second step, where the increase and decrease of the boundary element's size with respect to their corresponding stress ratio occurs.

1.2 CRITERIA FOR SHAPE OPTIMIZATION

Any type of optimization problem will require a set of criteria over which it will be optimized to attain a desired output. The criteria refer to the combination of global objectives and constraints for the analysis. Regarding the optimization criteria, designs can be categorized as either single-objective (possibly with multiple criteria) or multi-objective. The goal of a single-objective optimization problem is to find the best solution for either a specific criterion or a set of criteria for, e.g. Minimization of either stress or weight or cost etc., under the criteria of having a certain thickness or maximum deflection or stiffness etc. At the same time, A multi-objective optimization problem refers to finding a set of optimal solutions that satisfy different and often competing (or conflicting) objectives. There are several criteria that can be used in shape optimization:

1) Stress criteria: The optimization process can be constrained by the maximum allowable stress in the structure, ensuring that the final design is safe and able to withstand the loads it will be subjected to.
2) Displacement criteria: The final design can also be constrained by the maximum allowable displacement, which ensures that the structure will not experience excessive movement during loading.
3) Stiffness criteria: The optimization process can be constrained by the required stiffness of the structure, which ensures that the final design will have the necessary rigidity to support the loads it will be subjected to.

4) Natural frequency criteria: The optimization process can be constrained by the natural frequencies of the structure, which ensures that the final design will have the desired dynamic behaviour.
5) Buckling criteria: The optimization process can also be constrained by the buckling behaviour of the structure, which ensures that the final design will be stable under compression.
6) Mass criteria: The optimization process can be constrained by the total mass of the structure, which ensures that the final design will be lightweight and efficient.
7) Aerodynamic criteria: In the case of wind turbine blades or aircraft structures, the optimization process can be constrained by the aerodynamic performance of the structure, which ensures that the final design will have the desired lift and drag characteristics.
8) Cost criteria: The optimization process can be constrained by the cost of the materials used in the final design, ensuring that the final design will be cost-effective.
9) Environmental criteria: The optimization process can also be constrained by the environmental impact of the final design, ensuring that the final design will be sustainable and have minimal impact on the environment.

Work towards attaining an optimized shape has been based on several criteria, viz., fully stressed design criteria, minimum weight criteria, deflection criteria, minimum cost criteria, thickness criteria, stiffness criteria, frequency criteria etc. These criteria have sometimes been used individually or in combination with one another as per the output desired by the user. However, most of the shape optimization work, which is based on zero-order methods, has mostly been single-criteria.

The present work presents a single-objective (achieving desired stress by changing geometry) and a dual-objective (achieving desired stress and weight reduction) multiple criteria (having minimized weight, controlled deflection, minimal cover etc., simultaneously as per the requirement) based approach towards shape optimization of structural elements.

1.3 OBJECTIVE AND SCOPE

Since Zero-order/ non-gradient methods do not involve stress derivatives and are simpler to implement, the current thesis intends to suggest a new multi-criteria driven integrated zero-order procedure for shape optimization of structural elements used in the engineering domain. The suggested technique optimizes the shape of structural elements using a fuzzy-controlled integrated zero-order methodology, which includes design elements and automated mesh construction with mesh refinement at each iteration. The fuzzy membership function is used to handle the nodal movement and convergence monitoring. The shape modifications are based on achieving the selected target maximum shear stress (σ_t) at all points, as close as possible to the target value. The methodology has been implemented as software called GSO (Gradientless shape optimization) in FORTRAN language. The suggested methodology is then compared to an existing software OptiStruct (a part of the software suite HyperWorks from Altair engineering), which works on a gradient-based method and is widely used and accepted for shape optimization problems. A few examples under different types of loading, support condition and different other constraints have also been discussed.

Further, the suggested methodology is used to optimize the steel-concrete-steel (SCS) sandwich beams. The method presented in this work is combined with a novel strategy of modifying the shape of the faceplates and core at the interface of the SCS sandwich beam while keeping the overall shape unchanged. This approach is highly effective in determining the optimized shape of the faceplates and core. The efficacy of the method is demonstrated through several examples, where the boundary conditions and shape of the sandwich beam are varied to show the versatility of the approach.

The capability of the suggested methodology is then increased to incorporate the optimization of cable layout in pre-stressed beams synergistically with the change in the shape of the concrete. The pre-stressing cable is modelled as a curvilinear three-noded bar element tracing a B-spline profile. The concrete is modified from its initial shape using the previously suggested integrated zero-order technique, whilst the cable is modified employing a coherent algorithm designed towards the modification of ordinates of knot vector. The entire process is synergistically iterative in nature. For the incorporation of these new capabilities, another in-house software labelled "Simultaneous Shape and Cable Optimization" (SSCO) which is an extension of the GSO, is developed. The doubly optimized pre-stressed concrete beams are then also studied for the effect of span length using SSCO to find out the optimum number of spans with maximum material saving for a given optimized pre-stressed beam under a given predefined load.

1.4 LAYOUT OF THE THESIS

The present thesis constitutes of eight chapters.

Chapter 1 introduces structural optimization and different methods that have been developed to achieve the same. The chapter then emphasizes the efficiency of shape optimization and different methods that has been developed with time. Discussion on different criteria used during optimization has also been presented. The chapter then discusses, in brief, the effectiveness of shape optimization, followed by the objective and scope of the present study. Finally, the chapter concludes with the layout of the thesis.

Chapter 2 presents a survey of the literature available in the field of shape optimization. It first presents a general overview of shape optimization and problems related to its finite element modelling, followed by different algorithms that have been used in gradientless/non-gradient/zero-order shape optimization across the decades. The chapter then presents a review of the literatures on shape optimization of beams, sandwich beams, and pre-stressed beams concluding with the list of research gaps identified to be worked on.

Chapter 3 presents the suggested multi-criteria driven fuzzy controlled zero order approach for shape optimization of structural elements. The chapter discusses the process of model generation and finite element methodology (FEM) involved in achieving the optimized shape under the given constraints. A software based on the suggested methodology is also created, which is discussed in this chapter. The chapter then concludes with a few numerical illustrations with the results and discussions on it to show the effectiveness of the proposed approach.

Chapter 4 presents a result-based comparative study of the current suggested method, which is based on the non-gradient/zero-order/gradientless approach with a heavily used gradient-based methodology for structural shape optimization. The chapter compares the two methodologies on the basis of mesh convergence, iterations, runtime, weight reduction achieved, maximum deflection present, and the overall shape of the optimized structure using a few numerical illustrations. The chapter then concludes with a discussion of the results obtained.

Chapter 5 presents the application of the proposed methodology of shape optimization in Steel-concrete-steel (SCS) beams in a novel way to achieve its optimized form. The chapter starts with the introduction of SCS beams in the field of construction. It then discusses the model generation and the problem formulation for the optimization of SCS beams. The chapter then presents the validation study and a few numerical illustrations to show the effectiveness of the proposed methodology on SCS beams and concludes with the discussions on acquired results.

Chapter 6 presents another multi-criteria zero-order methodology to obtain synergistically optimized pre-stressed concrete beams, which are optimized for their concrete shape and cable layout simultaneously. The chapter begins by introducing pre-stressed beams and then proceeds to describe the methodology used to achieve the optimized shape. Software based on this methodology is also developed and discussed in this chapter. The chapter includes numerical examples to demonstrate the effectiveness of the proposed methodology and concludes with a discussion of the results obtained.

Chapter 7 uses the methodology presented in chapter 6 to ascertain the effects of span length on the synergistically optimized pre-stressed beams and to find out the optimum number of spans while keeping the other factors, such as the length, imposed loading, and pre-stressing force constant. The chapter concludes with a discussion of the obtained results.

Chapter 8, the final chapter, is dedicated to presenting the conclusions and recommendations of the thesis. At the beginning of the chapter, a summary of the main findings and conclusions of each objective of the thesis are presented. Additionally, at the end of the chapter, further research recommendations are provided based on the experiences gained throughout the study, highlighting areas where additional research is needed or could be beneficial.