Chapter 2

Peer-to-Peer Energy Trading Frameworks and Methodologies: An Overview

2.1 Introduction

P2P energy trading allows prosumers (producers and consumers) to directly exchange surplus renewable energy without relying solely on the utility grid. This approach fosters local energy markets, reduces transmission losses, and enhances renewable energy utilization. This chapter discusses about different types of P2P energy trading models. Along with this, this chapter presents the preliminary methodologies that will be followed in the simulation studies carried out in the subsequent chapters. "Hong's 2m point estimate method" is used for stochastic modelling of renewable generation uncertainties. With the help of Game Theory, the problem is formulated and minimised in a decentralised way using alternating direction methods of multipliers (ADMM). Suitable incentives are decided to motivate the players to engage in P2P energy sharing. The receding horizon method incorporates the BESS in real-time market operations.

2.2 Types of P2P Energy trading models

The Peer-to-peer energy trading model can be classified in many ways, as shown in Figure 2.1.

1. Based on control and decision-making:

   (a) Centralized: In these models, control and decision-making functions are done by a

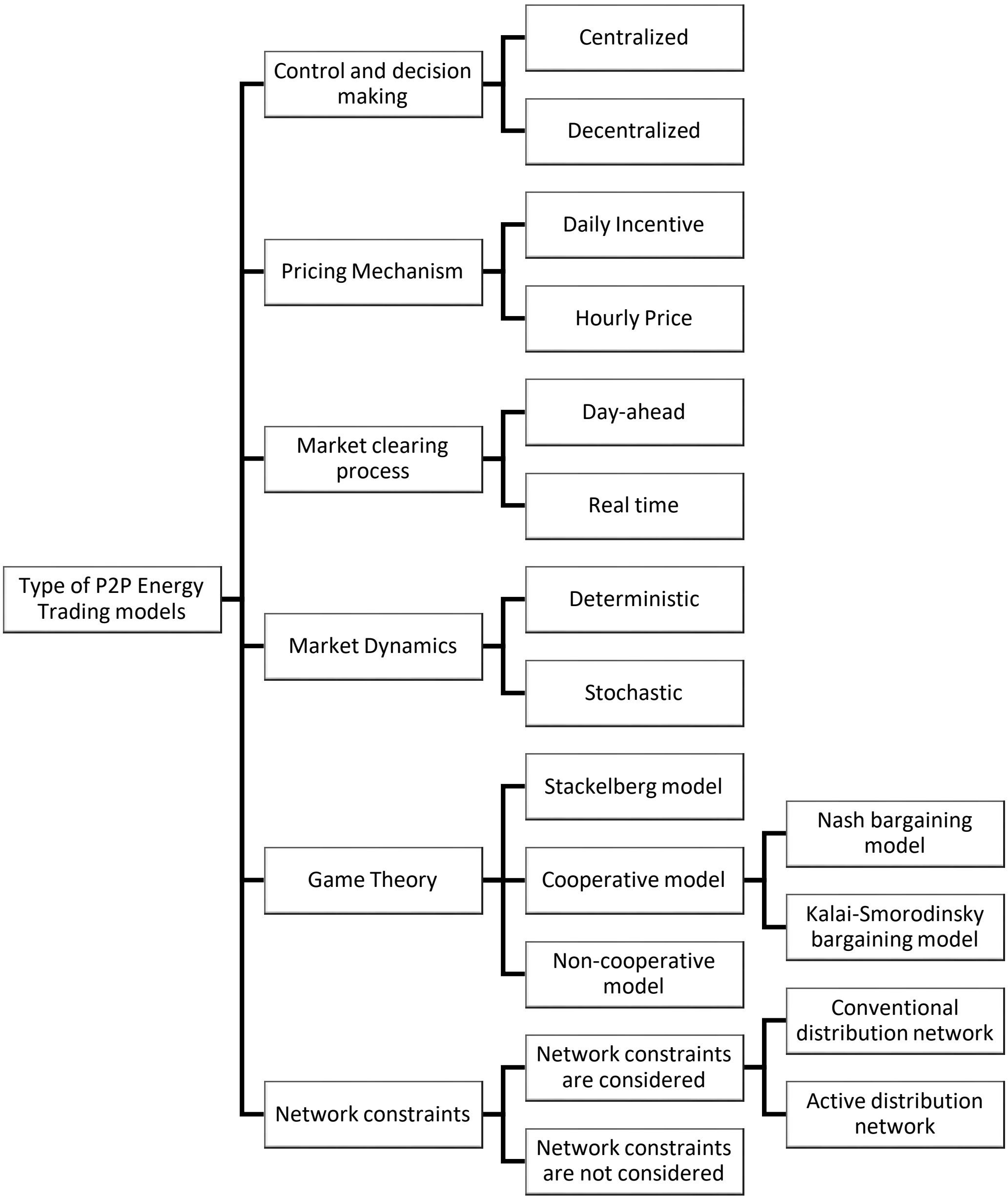

Figure 2.1: TYPES OF P2P ENERGY TRADING MODELS

single location or entity. A centralized entity makes decisions, and there is often a hierarchical structure.

   (b) Decentralized: In these models, decision-making functions are distributed among multiple entities or locations. Decision-making can be more localized, and a flatter organizational structure may exist.

2. Based on pricing mechanism:

   (a) Daily Incentive: Such models aim to encourage consumers to reduce their overall daily energy consumption by providing financial incentives for daily energy savings.

   (b) Hourly Price: Here, the goal is to efficiently manage the load demand during different periods of the day by encouraging them to shift their consumption to lower-cost periods.

3. Based on market clearing process:

   (a) Day-Ahead: The market operator clears the day-ahead market by matching bids and offers using forecasted data, thereby determining the market-clearing price for each hour of the next day.

   (b) Real-Time: The market operator clears the real-time market continuously, adjusting prices based on the immediate balance between supply and demand.

4. Based on market dynamics:

   (a) Deterministic: These models are suitable for stable and predictable markets.

   (b) Stochastic: These models are more appropriate for dynamic and uncertain markets.

5. Based on game theory: In an energy trading model, the game theory is an excellent tool for analyzing strategic interaction among consumers/ participants. Some of the common game models are as follows.

   (a) Stackelberg Model: A leader-follower structure is followed, making it a hierarchical decision-making model where one player becomes the leader and takes the decision first and then other players act as followers and respond to their leader's decision.

   (b) Cooperative Model: All the players work together in this model to achieve joint objectives that involve sharing information and coordinating the strategies. The cooperative model can be further classified into many categories. The two common categories are as follows:

    i. Nash Bargaining Model: In this model, the players are involved in iterative bargaining to find a mutually beneficial solution that maximizes their joint utility.
    ii. KS (Kalai-Smorodinsky) Bargain Model: A fair and Pareto optimal solution is aimed in these models, benefiting all the players by balancing fairness in the benefit distribution and maintaining overall efficiency.

  (c) Non-Cooperative Model: In this game theory model, the player decides independently in self-interested behaviour aiming to maximize their profit or utility irrespective of other strategies of players.

6. Based on consideration of network constraints: The framework for P2P energy trading among various consumers having excess renewable energy generations may or may not involve the consideration of distribution network constraints as explained below.

  (a) Network constraint not considered: Network constraints are not used in the optimization model.

  (b) Network constraint considered: Considering these constraints, the energy trading model can be further classified as follows.

    i. Conventional distribution networks: All elements are under DSO's direct or indirect control in these networks.
    ii. Active distribution networks: Some/all elements act independently. These elements include renewable energy sources, battery energy storage systems, combined heat and power (CHP) units, electric vehicles, fuel cells and many more. Some of them may be directly controlled by DSO, and some of them may be profit-oriented and thus act independently.

2.3 Challenges associated with P2P energy trading model

Apart from being innovative, these P2P energy trading models face several challenges. Some of them are as follows.

1. Balancing Local Generation and Demand: Matching the locally generated renewable energy with the local demand in real-time is quite difficult, especially when the load patterns of other consumers and renewable energy generation are uncertain.

2. Incentivizing Prosumers to Participate: Encouraging consumers to participate in local energy trade necessitates the implementation of a suitable pricing mechanism that fairly allocates benefits without infringing on their privacy.

3. Developing Suitable Pricing Mechanism: A proper incentivising mechanism is needed to make consumers participate in such energy trading. The main issue here is properly dividing the utility/ benefit among the participants.

4. Dealing with Renewable Energy Uncertainty: The intermittency of renewable energy is another big challenge for such an energy trading framework.

5. Maintaining Privacy of Participants: A player's strategies need to be secured to engage them seamlessly in performing P2P energy trading. The information flow and exchange between various entities can have privacy issues and implications.

6. Maintaining Network Constraints: The consumers are on the low voltage distribution side, and their active participation may violate the voltage and power limit.

7. Scalability: These models may work properly for a limited number of consumers. However, with the increase in the number of consumers, the degree of complexity of the system and the burden on communication infrastructure increases.

2.4 Recent Advancement in P2P energy trading

recent advancements in peer-to-peer (P2P) energy trading have been driven by the integration of innovative technologies and the growing interest in decentralized energy markets. Some notable examples are as follows.

- Power Ledger (India) [67] – The Uttar Pradesh government partnered with Powerledger and ISGF to pilot blockchain-based peer-to-peer (P2P) energy trading for rooftop solar power. This initiative, the first of its kind in India, demonstrated the feasibility of trading solar energy among households, lowering the market buy price by 43%. The successful project led to a regulatory framework that supports broader P2P trading, enhancing affordable energy distribution, especially in rural areas. In 2022, the project won a World Summit Award for its contribution to sustainability.

- Power Ledger (Australia) [68] – Power Ledger is one of the pioneering platforms in the P2P energy trading space. Using blockchain technology, it enables individuals to buy and sell excess solar energy directly with each other. In 2020, Power Ledger expanded its use to multiple projects worldwide, including a partnership with Singapore Power to test P2P trading among local solar producers. This platform ensures secure, transparent, and efficient transactions, allowing for decentralized energy trading while avoiding the need for central utilities.

- Brooklyn Microgrid (USA) [5] – Located in New York, the Brooklyn Microgrid is a community-driven P2P energy trading project where residents can buy and sell solar power using blockchain technology. The project uses smart meters to track energy consumption and production, allowing participants to trade energy directly, without relying on the centralized grid. The Brooklyn Microgrid has become a prominent example of how P2P trading can enhance energy independence and reduce reliance on large utility companies.

- SonnenCommunity (Germany) [69] – Sonnen, a German company, operates the SonnenCommunity, a platform that enables participants to trade surplus solar energy with one another. The company uses its home battery storage systems, combined with a smart grid, to facilitate local energy exchanges. This decentralized trading model has expanded to several countries, including the USA and Australia, and continues to demonstrate the potential for community-driven energy markets that leverage renewable energy and battery storage solutions.

These advancements exemplify how P2P energy trading is rapidly evolving with the help of blockchain, smart grids, and renewable energy integration. These projects not only contribute to more sustainable and efficient energy systems but also promote energy independence and local community empowerment.

2.5 Game Theory Concepts

In this thesis, various Game theory concepts have been incorporated to distribute the economic benefit fairly among participants engaged in P2P energy trading. The game theory is suitable for P2P energy trading due to the following reasons.

- Decentralized Decision-Making: In P2P energy trading, prosumers act independently, making decisions about selling, buying, or sharing energy. Game theory provides a framework to model their strategic interactions.

- Optimization of Benefits: Each participant aims to maximize its own benefit (e.g., minimizing costs or maximizing revenue), and game theory helps determine equilibrium strategies where no participant has an incentive to deviate.

- Fair and Efficient Energy Allocation: Cooperative game theory ensures that participants can form coalitions to share energy in a mutually beneficial way, while non-cooperative game theory helps model competitive scenarios where prosumers act individually.

- Market Stability and Convergence: Game theory ensures that energy transactions reach a stable state (equilibrium), where supply and demand are balanced efficiently

In general, under the following circumstances, Game Theory is found to be suitable.

- Multiple Independent Decision Makers Exist: In P2P/B2B/B2C trading, buildings and communities act as independent decision-makers, making it necessary to model their interactions strategically.

- Strategic Behavior Influences Outcomes: Each participant's decision depends on the decisions of others, making traditional optimization approaches less effective.

- Market-Based and Cooperative Energy Sharing Exists: Both competitive (non-cooperative) and collaborative (cooperative) scenarios can be modeled using game theory.

- Dynamic and Decentralized Systems Are Involved: When energy supply, demand, and pricing are dynamic, game theory provides adaptive and scalable solutions.

In this thesis, game theory is applied to model both cooperative and non-cooperative interactions in various P2P energy-sharing frameworks, ensuring efficient energy distribution and market stability.

2.5.1 Cooperative Game

In cooperative games, the participants form a coalition and work together for a common goal. The focus is on binding agreement, including collaboration and collective strategies. The optimal solution of these types of games can be calculated in the following manner.

2.5.1.1 Nash bargaining solution

The Nash bargaining solution is a concept in game theory that provides a way to allocate resources or divide the gains from cooperation between two or more players fairly and efficiently. It is named after the mathematician John Nash, who made significant contributions to the field of game theory. The Nash bargaining solution is based on the idea of a negotiation process between the players. It aims to find an outcome that both maximizes the joint welfare of the players and satisfies certain fairness criteria. The solution is considered a fair allocation because it represents a compromise that no player can improve upon unilaterally. Mathematically, the Nash bargaining solution can be defined as follows.

Consider a cooperative game with n players. Let u_i denote the utility or payoff function of player i, such that u_i is a function of the actions or strategies chosen by all players. The Nash bargaining solution seeks to find a feasible allocation of utilities $(u_1, u_2, ..., u_n)$ such that it maximizes the joint utility of the players while satisfying certain fairness properties.

The fairness properties are typically represented by axioms or conditions that the solution must satisfy. One commonly used set of conditions is known as the axioms of individual rationality, Pareto optimality, independence of irrelevant alternatives, and symmetry.

- Individual Rationality: Each player i should receive at least their reservation utility, denoted as R_i. This represents the minimum utility that player i is willing to accept.

$$u_i \geq R_i \ \ \forall i = 1, 2, ..., n.$$

- Pareto Optimality: The allocation should be Pareto efficient, meaning that there is no other feasible allocation that would make at least one player better off without making any other player worse off. Mathematically, there is no other feasible allocation $(u_1^{'}, u_2^{'}, ..., u_n^{'})$ such that $\prod(u_1^{'}, u_2^{'}, ..., u_n^{'}) \geq \prod(u_1^*, u_2^*, ..., u_n^*)$ where u^* is the pareto optimal.

- Independence of Irrelevant Alternatives: The solution should not depend on outcomes or utilities that are not relevant to the players' preferences.

- Symmetry: If players have similar utility functions, the solution should be symmetric, meaning that the allocation of utilities should be the same for players with identical preferences.

2.5.1.2 Kalai–Smorodinsky bargaining solution

The KS bargaining solution aims to achieve a compromise that maximizes the joint gains of all the players. An equitable and efficient solution is arrived at by striking a balance between the conflicting interests of the negotiating parties. The relative bargaining power of players is used for determining the solution [70], which reflects their ability to influence the bargaining power. The solution is a point in the bargaining space that is equidistant from the ideal outcomes of each player. This point is known as the Kalai-Smorodinsky point, and it represents a compromise solution that each party can agree upon. In game theory, the bargaining space can be conceptualized as a geometric space, where each point represents a possible outcome that can be reached through negotiation. The boundaries of this space are determined by the parties' reservation points, which represent their lowest acceptable outcome if they are unable to reach an agreement.

The modified Nash bargaining solution or Kalai-Smorodinsky (KS) bargaining solution is an approach to resolving bargaining problems that offers several advantages over traditional methods like the Nash bargaining solution. It aims to find a fair compromise between players by maintaining the ratio of their maximal gains. It uses the ”utopia point” or ”aspiration point” (best possible outcome for each player) and ”disagreement point” (worst acceptable outcome) as reference points.

In the KS bargaining solution, the ratio of distances from the disagreement point, d_i (a point where all players receive their reservation payoffs, i.e., the payoff they would receive if they did not reach an agreement) to the optimal point, c_i^* and the aspiration point, a_i (the point where all players receive the highest payoff they can imagine getting in the negotiation) is the same for all players. Now, the KS bargaining solution is a compromise between the two extreme points of disagreement and aspiration. Specifically, it is the point on the line connecting the two extreme points that is equidistant from them. In other words, it is the point that minimizes the sum of the distances to the disagreement point and the aspiration point.

If we consider two players in this negotiation, let's assume that they have different disagreement and aspiration points. Then, the KS bargaining solution is the point that satisfies the condition: $d_1^x + d_2^x = a_1^x + a_2^x$. Here d_1^x and d_2^x are the distances from the KS point to the disagreement points of players 1 and 2, respectively, and a_1^x and a_2^x are the distances from the KS point to the aspiration points of players 1 and 2, respectively.

Now, suppose that we change the disagreement and aspiration points of Player 1, but keep the ones of Player 2 the same. Then, the KS bargaining solution will change, but it will still satisfy the above condition. This is because the distances d_2^x and a_2^x remain the same, while the distances d_1^x and a_1^x change, but in such a way that their sum remains constant. Therefore, the ratio of d_1^x to a_1^x remains the same, and this ratio is the same for both players, meaning that the ratio of distances from the disagreement point of the optimal and aspiration point is the same for all the players. Therefore, for n player game, the KS bargaining solution, c_i^*, is given by

$$\underset{c}{max} \prod_{i}^{i \in n} (d_i - c_i) \tag{2.1}$$

subjected to

$$\frac{d_1 - c_1}{d_1 - a_1} = \frac{d_2 - c_2}{d_2 - a_2} = ... = \frac{d_n - c_n}{d_n - a_n} \tag{2.2}$$

The advantages of this method over traditional methods are given below.

- The KS solution ensures that players' relative gains are proportional to their maximum potential gains, leading to a more equitable outcome.
- Unlike the Nash solution, which satisfies independence of irrelevant alternatives, the KS solution considers the entire set of alternatives when determining the outcome.
- The KS solution often provides a more balanced compromise between competing objectives, especially in cases where one player has significantly more bargaining power.

By addressing these aspects, the KS bargaining method offers a more better approach for negotiations compared to traditional bargaining solutions.

2.5.2 Non-Cooperative Game

Unlike cooperative games, in a non-cooperative game, the participants act independently, aiming at maximising their benefits. The focus here is to achieve individual goals with their strategies irrespective of the strategy of others. The equilibrium point of such games is called the Nash Equilibrium, which is a point where no participants can improve their payoff by altering their strategy unilaterally, assuming the strategies of all the other participants remain unchanged.

Consider a non-cooperative game of n players Each player i have payoff u_i and strategy profile s_i. Then, the Nash equilibrium is represented by a strategy set $(s_1^*, s_2^*, s_3^*, ...s_n^*)$ such that,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \ \ \forall i = 1, 2, ..., n. \tag{2.3}$$

Here, s_{-i} denotes the players' strategies except for player i.

2.5.3 Difference between Cooperative and Non-Cooperative Game

Cooperative and non-cooperative games represent two distinct approaches to modeling interactions in energy trading scenarios. To illustrate the differences between these game types, a simple energy trading between various buildings equipped with flexible load, renewable generation resource. The access generated energy can be sold to the peer buildings or can to the grid back. The game can be formulated for this scenario with buildings as participants. The objective is to minimize the total energy consumption cost by changing is strategy which is the power exchanged with the grid and other buildings, and the load shifting. A cooperative and non-coopertive game can be formulated and the key difference between them will be the following.

- Objective: Cooperative games aim to maximize collective benefits, while non-cooperative games focus on individual profit maximization.
- Information Sharing: In cooperative games, participants may share information to optimize joint strategies. Non-cooperative games typically involve limited information exchange.
- Benefit Distribution: Cooperative games require mechanisms to fairly distribute collective gains, whereas in non-cooperative games, each participant's payoff is directly determined by market outcomes.
- Strategy Formation: Cooperative games involve joint strategy formulation, while non-cooperative games feature independent decision-making.
- Market Impact: Cooperative games can lead to more stable market outcomes and potentially higher overall efficiency, whereas non-cooperative games may result in more dynamic and competitive market conditions.

By understanding these differences, energy market participants and regulators can better design and participate in various trading mechanisms, balancing the benefits of cooperation with the dynamics of competition in the evolving energy landscape.

2.6 Assumptions for the Game-theoretic models used

For the game-theoretic model presented in the thesis, the following assumptions [71] were made.

- Agreement: In cooperative game theory, it is assumed that players engage in discussions or negotiations before the game begins, leading to a formal agreement among all participants. This assumption ensures that players can coordinate their strategies to achieve mutually beneficial outcomes. Conversely, in non-cooperative games, it is assumed that no binding agreements are made before the game starts, and each player acts independently to maximize their own payoff.

- Reliable Communication Infrastructure: The models in this study assume that the communication channels used by players are robust, free from disruptions, and do not introduce delays. This ensures that information exchange between players occurs seamlessly, which is crucial in strategic decision-making processes.

- Rational Player Behavior: It is assumed that all players behave rationally, meaning they make decisions that maximize their expected utility based on the available information. The possibility of irrational behavior, such as decision-making influenced by emotions or biases, is not considered in this study.

- Multiple Strategies for Each Player: Each player in the game is assumed to have access to multiple strategic choices. This ensures that the game captures a wide range of possible interactions and outcomes rather than being restricted to a limited set of actions.

- Common Knowledge: All players are aware of the game's rules, possible strategies, and payoffs, and they also know that all other players have this knowledge.

- Static Virtual Community Formation: The virtual community structure remains fixed during the optimization period, without considering dynamic adjustments based on changing conditions.

2.7 Choice of the Game-theoretic models

The selection of specific game-theoretic models in energy trading scenarios is crucial for accurately representing and analyzing complex interactions between market participants. The rationale for choosing particular models is based on several key factors:

- Market Structure and Participant Behavior: The choice between cooperative and non-cooperative game models depends on the market structure and the level of collaboration among participants.
  - Non-Cooperative Models: These are typically chosen when participants act independently and compete with each other.
  - Cooperative Models: These are selected when participants can form coalitions or have incentives to collaborate.
- Information Availability: The selection of game models also depends on the level of information available to participants.
  - Complete Information Games: When all participants have full knowledge of others' strategies and payoffs, models like the subgame perfect equilibrium can be used.
  - Incomplete Information Games: In scenarios where participants have limited information about others, Bayesian game models are more appropriate. This is common in energy markets where participants may not know others' cost structures or generation capacities.
- Complexity and Computational Feasibility: The choice of game model also depends on the computational resources available and the complexity of the problem.
  - Simplified Models: For large-scale systems or when quick decisions are needed, simplified game models like potential games or aggregative games might be preferred.
  - Complex Models: When detailed analysis is required and computational resources are available, more complex models like stochastic games or multi-level games can be employed.

By carefully considering these factors, researchers and practitioners can select the most appropriate game-theoretic models to accurately represent and analyze energy trading scenarios, leading to more insightful results and effective market designs.

The choice of a game-theoretic approach significantly impacts computational complexity, as different models require varying levels of computational resources to find equilibria or optimal strategies. When a large number of players are involved communication and computation complexity issues arises [52]. To reduce these complexities, in this thesis, the players (i.e. prosumers or buildings) have been grouped into virtual communities and Peer-to-peer (P2P) energy trading is divided into B2B (building-to-building), B2C (building-to-community) and C2C (community-to-community) energy trading. The computational complexity associated with some well-known game-theoretic approaches is outlined below.

- Cooperative Games [72]: In this type of game, players form alliances or coalitions, and the solution concepts generally involve solving optimization problems across different groups of players. This often requires evaluating multiple possible coalition structures, which can significantly increase computational effort.

- Non-Cooperative Games [73]: Here, each player independently optimizes their strategy without prior agreements with others. The best possible strategy is determined by iteratively solving and comparing different choices while taking into account the responses of other players.

- Stackelberg Games [74]: These hierarchical games involve a leader-follower dynamic, where one player makes a move first, and the others respond accordingly. Solving such games requires a combination of backward and forward motion, often leading to high computational complexity due to the need for extensive tree-based exploration.

- Mixed Strategy Games [75]: In these games, players do not rely on fixed choices but instead use probability distributions over multiple strategies. Finding equilibrium solutions involves solving nonlinear optimization problems, which can be computationally demanding, especially as the number of possible strategies increases.

2.8 Alternating direction method of multipliers

Alternating Direction Method of Multipliers (ADMM) [76] is an optimization algorithm used to solve optimization problems with certain structures or constraints. ADMM was developed to handle problems that involve a separable objective function and constraints that can be decomposed into smaller, more manageable subproblems. It allows for the decomposition of the global optimization problem into smaller subproblems that can be solved locally by each participant. The flowchart of ADMM algorithm is shown in Figure2.2. The basic idea behind

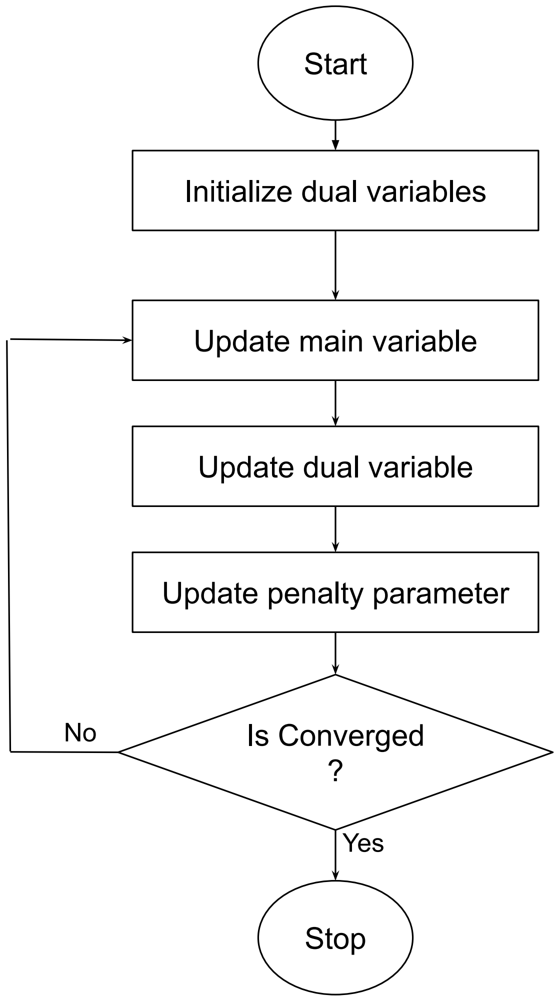

Figure 2.2: FLOWCHART OF ADMM ALGORITHM

ADMM is to divide the original problem into a series of smaller subproblems that can be solved separately. These subproblems are then solved iteratively, with each iteration consisting of three main steps:

1. **Splitting**: The original problem is split into smaller subproblems that can be solved

independently. This splitting is usually based on the structure of the objective function and the constraints.

2. **Solving**: Each subproblem is solved individually, typically using well-established optimization techniques. These subproblems can often be solved more efficiently and accurately than the original problem.

3. **Updating**: After solving the subproblems, the solutions are combined or updated to obtain an improved solution for the original problem. This step involves updating certain Lagrange multipliers or dual variables to enforce the consistency between the subproblem solutions.

Consider the following optimization problem:

$$minimize\ f(x) + g(z) \tag{2.4}$$

subject to

$$Ax + Bz = c \tag{2.5}$$

Where,

- x is the optimization variable of size n,
- z is the optimization variable of size m,
- A is a constraint matrix of size $p \times n$,
- B is a constraint matrix of size $p \times m$,
- c is a vector of size p,
- $f(x)$ is a convex function that depends only on x,
- $g(z)$ is a convex function that depends only on z.

The augmented Lagrangian is formed as,

$$L(x, z, y) = f(x) + g(z) + (y^T)(Ax + Bz - c) + (\rho/2)||Ax + Bz - c||^2 \tag{2.6}$$

where y is the dual variable, and ρ is the penalty factor. Combining the linear and quadratic terms and scaling the dual variable makes the ADMM more convenient. This scaled form of ADMM is given by

$$L(x, z, a) = f(x) + g(z) + (\rho/2)||Ax + Bz - c + a||^2 - (\rho/2)||a||^2 \tag{2.7}$$

where $a = (1/\rho)y$ and is called as the scaled dual variable.

The ADMM algorithm for solving this optimization problem can be described as follows.

Step 1: Initialization

Initialize the optimization variables: x^0, z^0, and a^0 (Lagrange multiplier or dual variable) to some initial values.

Step 2: Iteration

- For k = 0, 1, 2, ...
- Update x^{k+1}:

$$x^{k+1} = argmin\,\{f(x) + (\rho/2)\|Ax + Bz^k - c + a^k\|^2\} \tag{2.8}$$

- Update z^{k+1}:

$$z^{k+1} = argmin\,\{g(z) + (\rho/2)\|Ax^{k+1} + Bz - c + a^k\|^2\} \tag{2.9}$$

- Update a^{k+1}:

$$a^{k+1} = a^k + (Ax^{k+1} + Bz^{k+1} - c) \tag{2.10}$$

Step 3: Convergence check

Repeat Step 2 until convergence is achieved, typically by monitoring the change in the objective function or the optimization variables.

The primal and dual feasibility provide sufficient and necessary optimal conditions. At iteration $k + 1$ the primal residual r^{k+1} and dual residual s^{k+1} is given by

$$r^{k+1} = Ax^{k+1} + Bz^{k+1} - c \tag{2.11}$$

$$s^{k+1} = \rho A^T B(z^{k+1} - z^k) \tag{2.12}$$

As the ADMM proceeds, both residuals r^{k+1} and s^{k+1} converge to zero.

In ADMM, ρ is a parameter that controls the trade-off between the objective function and the constraint violation. Choosing an appropriate value for ρ is important for achieving good convergence properties. In the literature, many variations of this algorithm have been explored. One such variation includes the use of different penalty parameters for each iteration, as given below:

$$\rho^{k+1} := \begin{cases} \rho^k/\nu, & \text{if } \alpha r^k < s^k \\ \mu\rho^k & \text{if } r^k > s^k\alpha, \\ \rho^k & \text{otherwise .} \end{cases} \tag{2.13}$$

where ν, μ, and α are parameters.

The main idea behind ADMM is the alternating updates of the optimization variables x and z, while keeping the Lagrange multipliers fixed. The Lagrange multiplier acts as a coordination variable that enforces the consistency between the solutions of the subproblems. By iteratively updating x, z, and a, ADMM aims to find a solution that satisfies both the objective function and the constraints of the original problem.

In this thesis, the problem is formulated through game theory and various other concepts. However, the problem so formulated consists of various coupling constraints. The coupling constraints are those constraints in an optimization problem that link multiple variables of different participants or subsystems. These constraints are present in this work mainly due to P2P energy trading. For decentralized optimization, ADMM is used in this thesis to decouple these coupling constraints.

The Alternating Direction Method of Multipliers (ADMM) algorithm, while powerful for distributed optimization in peer-to-peer (P2P) energy trading, presents several computational challenges. This thesis addresses these challenges and implements mitigation strategies to enhance the algorithm's efficiency and applicability. The key computational challenges and their mitigations are as follows:

- Convergence Speed: ADMM may require many iterations to converge, especially for complex problems. To address this, this thesis employs careful tuning of the penalty parameter ρ, which plays a crucial role in the algorithm's convergence rate. The tuning is done based on primal and dual residuals.

- Communication Overhead: ADMM requires frequent information exchange between participants, which can lead to significant communication overhead. The thesis mitigates this by reducing the number of transactions by aggregating the buildings into virtual communities. Minimizing the amount of information shared between participants, focusing only on essential variables like energy exchange values and prices1.

- Computational Complexity: ADMM can be computationally intensive, especially for large-scale systems with many participants. This thesis mitigates this challenge by reducing the number of transactions by aggregating the buildings into virtual communities.
- Risk of communication failure: ADMM requires frequent information exchange regarding update of main and dual variables. Any failure can halt the system process. In this thesis, to mitigate this issue, it is assumed that the communication infrastructure is robust.

By addressing these computational challenges, the thesis presents an ADMM-based P2P energy trading system that is more efficient, scalable, and practical for real-world applications.

2.9 Hong's 2m Point Estimation Method (PEM)

The point estimation methods focus on the statistical information (first few central moments) of random input parameters to provide the moments of random output variables. Hong's 2m PEM uses the $2 \times m$ point concentrations for m random input parameters. Let $h(X)$ relate m random input parameters and Z output random variable as follows.

$$Z = h(x_1, x_2, ...x_k, ...x_m) \tag{2.14}$$

The i^{th} order central moment of x_k can be calculated as

$$M_i(x_k) = \int_{-\infty}^{\infty} (x - \mu_{x_k})^i f_{x_k}(x) dx. \tag{2.15}$$

Here, $f_{x_k}(x)$ is the probability density function of x_k. The standard central moment of x_k can be written as

$$\lambda_{x_k,i} = \frac{M_i(x_k)}{\sigma_{x_k}^i}, \quad i = 1, 2, 3... \tag{2.16}$$

The i^{th} concentration of x_k can be defined as a pair of a location $X_{k,i}$ and weight coefficient $\omega_{k,i}$. The location $X_{k,i}$ can be calculated as

$$X_{k,i} = \mu_{x_k} + \zeta_{x_k,i}\sigma_{x_k} \tag{2.17}$$

Here, standard location ζ_{x_k} can be defined as

$$\zeta_{x_k,i} = \frac{\lambda_{x_k,3}}{2} + (-1)^{3-i}\sqrt{m + \frac{\lambda_{x_k,3}^2}{4}}, \quad i = 1, 2 \tag{2.18}$$

The weight coefficient $\omega_{k,i}$ can be calculated as

$$\omega_{k,i} = \frac{\frac{1}{m}(-1)^i \zeta_{x_k,3-i}}{\zeta_{x_k}}, \tag{2.19}$$

where,

$$\zeta_{x_k} = 2\sqrt{m + \frac{\lambda_{x_k,3}^2}{4}}. \tag{2.20}$$

To implement Hong's 2m PEM, the function, $h(X)$, needs to be evaluated 2 times for each input random parameter x_k at $i = 1$ and 2 location points while keeping other input random parameters at mean, i.e., $h(\mu_{x_1}, \mu_{x_2}, ..., \mu_{x_{k-1}}, X_{i,x_k}, \mu_{x_{k+1}}, ...\mu_{x_m})$. The j^{th} moment of random output variables can be calculated as

$$\mathbb{E}(Z^j) = \sum_{k=1}^{m}\sum_{i=1}^{2} \omega_{k,i} Z_{k,i}^j \tag{2.21}$$

The detailed description of Hong's PEM can be found in [77]. This work considers the uncertainties related to solar and wind power generation. This makes the number of random parameters two. The expected strategies of all players, i.e., the first moment of random variables, will be used in the energy trading algorithm.

In this framework, discrete data [78] has been used for solar and wind generation. With the help of these data, statistical parameters like mean, standard deviations, central moments, and coefficient of skewness are calculated, which give the respective locations and weights for both solar and wind generations.

2.10 Receding Horizon

Receding Horizon Control (RHC) is also known as Model Predictive control [79]. It uses a mathematical model of the system to solve a moving horizon, finite, and closed loop problem. The optimization problem based on the receding horizon is solved at each time step, and only some part is taken as output. The MPC helps optimize a finite time horizon problem by only implementing the current time slot and then optimizing it repeatedly. It has the ability to anticipate future events and can take control actions accordingly.

For example, consider the problem of this thesis on battery management systems. The following steps will be followed:

- For the current time t, the model will solve and give the state-of-charge and other related values based on the T-hour optimization. For hours greater than t, forecasted PV generation and load values are used.

- For $t + 1$ hour, the previous hour results are considered as a parameter, and then again, T-hour optimization will occur in the same way as mentioned in the above step.

- The process continues until the end of the time period is reached.

This method is quite helpful in energy management systems designed for the real-time energy market, as discussed in further chapters, due to its adaptability and efficiency. Using the forecasted input data, this method optimizes the BESS to handle renewable energy uncertainties effectively and incorporates operational constraints like state of charge limits and degradation effects. This thesis uses this method to maximize the prosumer's benefit in real-time market operation by considering future market conditions over a prediction horizon and taking the just processed data for the past horizon.

2.11 Thesis Layout based on Energy Trading Models

Each chapter presents distinct concepts and methodologies that contribute to the overall analysis of Peer to Peer (P2P), Building to Building (B2B), Building to Community (B2C), and Community to Community (C2C) models. Each chapter addresses a unique aspect of the problem, with careful attention to the different contexts and applications as given in Table 2.1.

Table 2.1: Different Concepts Involved in Various Chapters of this Thesis

Concepts	Chapter 3	Chapter 4	Chapter 5	Chapter 6	Chapter 7
Peer-to-Peer Energy Trading	✓	✓	✓	✓	✓
Multi-community	×	✓	✓	✓	✓
Renewable Uncertainty	✓	×	✓	✓	✓
Game involved	Cooperative Game (Nash Bargaining)	Non-Cooperative Game	Cooperative Game (Nash Bargaining)	Cooperative Game (KS Bargaining)	Cooperative Game (KS Bargaining)
Network Contraints	×	×	✓	✓	✓
Active participation of DGs	×	×	×	✓	✓
Analysis of the effect of DGs' participation actively and marginal cost basis on LMPs	×	×	×	✓	×
Cost analysis of effect of network constraints in Day-ahead scheduling	×	×	×	×	✓
Real-time load scheduling	×	×	×	×	✓
Real-time BESS scheduling	×	×	×	×	✓

2.12 Summary

This chapter has provided a detailed discussion on P2P energy trading frameworks. The insights presented here form the foundation for the subsequent chapters, where these concepts are applied to develop innovative solutions for decentralized energy markets. The basic concepts of four methods viz. Hong's two-point estimation method, the Alternating direction method of multipliers, the Game theory, and the Receding Horizon have been described in this chapter. These methods will be used in the subsequent chapters to develop problem-specific solution approaches for energy trading frameworks.