PREFACE

The study of reaction-diffusion problems has long been a cornerstone of mathematical modeling in fields ranging from biology and chemistry to physics and engineering. These models describe how substances or populations interact and spread over time and space, capturing the essence of processes like chemical reactions, biological pattern formation, and ecological dynamics. However, classical reaction-diffusion models often assume integer-order derivatives, which may not fully capture the complexities of real-world systems where memory effects, anomalous diffusion, or spatial heterogeneities play significant roles.

In recent years, fractional calculus has emerged as a powerful tool for extending classical models to include non-integer or fractional derivatives. This generalization provides a more flexible framework for modeling processes that exhibit non-locality and history dependence, which are frequently observed in natural systems. Despite its growing importance, the study of fractional-order reaction-diffusion problems remains a challenging and rapidly evolving area of research. The primary objective of this thesis is to find the numerical solution of some fractional-order reaction-diffusion problems by developing efficient and accurate methods. Special attention is given to the spectral and operational matrix methods that can handle the unique demands of fractional order models.

This thesis has six chapters; Chapter 1 serves as an introduction to the thesis, covering topics such as the diffusion process and the reaction-advection-diffusion equation. It also includes a historical background of fractional calculus theory and its related definitions. Basic concepts of fractional order reaction-advection-diffusion equations and a literature review on the topic are discussed in this chapter. Additionally, the methods used in the thesis rely heavily on Lucas and Vieta-Fibonacci polynomials, so a brief introduction to these polynomials, along with some of their important properties, is provided in this chapter.

In chapter 2, the author has developed a numerical method to solve the two-dimensional nonlinear and multi-term time-fractional diffusion equations with the help of Lucas's operational matrix. In the proposed method, solutions to the problems are expressed in terms of Lucas polynomial as basis function. The concerned method provides a highly accurate numerical solution. The accuracy of the approximate solution of the problem can be increased by expanding the polynomial terms. The accuracy and efficiency of the concerned method have been authenticated through error analyses with some existing problems whose solutions are already known.

In chapter 3, a new scheme has been developed for the numerical solution of the fractional order reaction advection-diffusion equation. To approximate the problem, the authors used Vieta-Fibonacci polynomials as basis functions and derived the operational matrices with the said polynomials for integer and fractional order Caputo differential operators for the first time. Lastly, the method is used to solve an unsolved reaction-advection diffusion model and observe the effect on solute concentration due to changes in various parameters of the model.

In Chapter 4, an innovative approach for the numerical solution of a three-component time-fractional order Brusselator reaction-diffusion system using the Vieta-Fibonacci wavelet and collocation method has been developed. The proposed method involves the derivation of operational matrices for both integer and fractional order derivatives. The existence, uniqueness of the solution, and Ulam-Hyers stability of the model are rigorously discussed. The numerical experiments showcase the method's superior performance in terms of accuracy and computational efficiency. The application of the Vieta-Fibonacci wavelet method to the three-component fractional order Brusselator reaction-diffusion system marks a significant advancement in the field of computational mathematics.

In chapter 5, we consider a nonlinear reaction-diffusion equation with a Caputo-Fabrizio derivative, and its solution is obtained by the finite difference collocation method. First, we approximate the Caputo-Fabrizio derivative with the aid of shifted Legendre polynomials. To deal with the time derivative, a finite difference scheme is applied, and to deal with the spatial Caputo-Fabrizio derivative, the shifted Legendre spectral collocation method is used. To signify the efficiency and validity of the developed scheme, a few numerical examples are solved, and the absolute error between exact and numerical results is presented in tabular form.

In Chapter 6, overall work has been concluded, and also the future works have been presented.