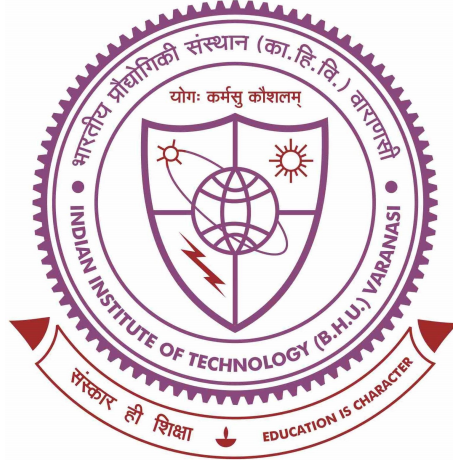


**A Study on Fractal Dimension of Different Types of
Functions and their Fractional Integrals and Approximation
Aspects**



The thesis submitted in partial fulfillment

for the Award of Degree

DOCTOR OF PHILOSOPHY

by

Megha Pandey

DEPARTMENT OF MATHEMATICAL SCIENCES

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November 2023

CERTIFICATE

It is certified that the work contained in this thesis titled "*A Study on Fractal Dimension of Different Types of Functions and their Fractional Integrals and Approximation Aspects*" by *Megha Pandey* has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of the Comprehensive Examination, Candidacy, and SOTA for the award of a Ph.D. degree.

T. Som.
14/11/23

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DECLARATION BY THE CANDIDATE

I, **Megha Pandey**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Prof. Tanmoy Som** from **July 2019 to November 2023** at the **Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi**. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports, dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

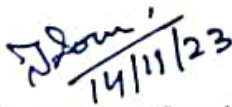
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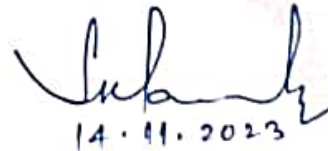

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It is certified that the above statement made by the student is correct to the best of my/our knowledge.


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To

My Mom Mrs. Sarita Pandey

My Daddy Mr. Dineshwar Pandey

&

My Brother Aakash Pandey

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