MATHEMATICAL MODELING ON THERMOELASTICITY AND PORO-THERMOELASTICITY

Thesis submitted in partial fulfillment for the award of degree

Doctor of Philosophy

by

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CONTENTS

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It is certified that the work contained in the thesis titled **"MATHEMATICAL MODELING ON THERMOELASTICITY AND PORO-THERMOELASTICITY"** by **KOMAL JANGID** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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I, **KOMAL JANGID**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Prof. SANTWANA MUKHOPADHYAY** from January, 2019 to September, 2023 at the **Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi**. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, theses, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

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Dedicated

to

My Parents

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Komal Jangid

LIST OF FIGURES

LIST OF SYMBOLS

σ_{ij}	Stress tensor for the solid phase, Pa or N m^{-2} or kg $m^{-1}s^{-2}$
σ	Stress tensor for the fluid phase, Pa or N m^{-2} or kg $m^{-1}s^{-2}$
u_i	Displacement vector for the solid phase, m
U_i	Displacement vector for the fluid phase, m
e_{ij}	Strain tensor for the solid phase, dimensionless
ϵ	Strain tensor for the fluid phase, dimensionless
q_i	Heat-flux, W m^{-2}
T_0	Uniform reference temperature, K
θ	Thermodynamic temperature/Temperature variation from the uniform reference temperature, K
ϕ	Conductive temperature, K
ν	Thermal displacement, K s
F_i	Body force per unit mass, N kg^{-1}
H	Heat source per unit mass, J s^{-1} kg^{-1} or N m s^{-1} kg^{-1}
S	Entropy per unit mass, J K^{-1} kg^{-1}

ρ^s	$\rho^s = (1-\beta)\rho^{s*}$, density of the solid phase per unit volume of bulk, kg m^{-3}
ρ^f	$\rho^f = \beta\rho^{f*}$, density of the fluid phase per unit volume of bulk, kg m^{-3}
ρ^{s*}, ρ^{f*}	Density of the solid and fluid phases, kg m^{-3}
ρ	$\rho = \rho^s + \rho^f$, density of the material, kg m^{-3}
K_{ij}/ K	Thermal conductivity tensor/ Thermal conductivity constant, W m^{-1} K^{-1}
K^*_{ij}/ K^*	Thermal conductivity rate tensor/ Thermal conductivity rate constant, W m^{-1} K^{-1} s^{-1}
C_{ijkl}	Elasticity tensor, kg m^{-1}s^{-2}
λ, μ	Lame's constants of material, kg m^{-1}s^{-2}
γ_{ij}	Thermoelasticity tensor, kg m^{-1} s^{-2} K^{-1}
$\gamma = (3\lambda + 2\mu)\alpha_t$	Thermoelasticity constant, kg m^{-1} s^{-2} K^{-1}
α_t	Linear thermal expansion of the material, K^{-1}
β	Porosity, dimensionless
C_E	Specific heat at constant strain, J kg^{-1} K^{-1}
τ_q	Phase-lag of heat-flux vector, s
τ_1, τ_2	Thermal relaxation parameters, s
$\boldsymbol{\nabla}$	Gradient operator
∇^2	Laplacian operator
δ_{ij}	Kronecker delta, dimensionless

$\boldsymbol{x} = (x_1, x_2, x_3)/(x, y, z)$	Cartesian coordinates, m
t	Physical time, s

Note: Throughout the thesis, the subscripted comma notations are used to denote the partial derivatives with respect to the space variables. The over-headed dots denote partial derivatives with respect to time variable, t. Subscripts i, j, k, l take the values 1, 2, 3 and summation is implied by index repetition. Moreover, the superscripted s is used to denote the solid phase, f is used to denote the fluid phase, sf and fs are used to represent the couplings between the solid and fluid phases of the physical quantities.