CHAPTER 4

TAGUCHI'S DESIGN OF EXPERIMENT, EXPERIMENTAL METHODOLOGY, AND GREY RELATIONAL OPTIMIZATION

The Taguchi-Grey analysis technique was applied in the present study to establish the relationship between the control parameters and the response characteristics. This chapter includes three parts: the experiment's design, the experiment methodology, and the Taguchi-Grey optimization method.

4.1 Taguchi Design of Experiment

The Minitab16 software is used for creating Taguchi experiments design by taking the number of control factors and levels of each factor. The Taguchi Design method is classified as the same level of design and mixed level of design. In the same level of experimental design, the level of each control parameter is equal to the design of the experiment. In mixed levels of design, the level of parameters is different. In this chapter, both methods are used for experimental design. The first experimental design is performed to optimize the multiple response characteristic of a four-cylinder-four-stroke diesel engine. Three control parameters, i.e., engine torque, engine speed, and fuel type, were elected with four levels each, as shown in Table 4.1. From the previous chapter investigation, it was found that the effect of engine power with varying torque at constant speed is found more compared to varying engine speed at constant torque, so for each parameter's contribution factor individually, both parameters have been taken as an input variable. The engine torque and speed levels are 14 N-m, 21 N-m, 28 N-m, and 35 N-m, and 1400 rpm, 1600 rpm, 1800 rpm, and 2000 rpm, respectively.

The first level of the fuel type is diesel fuel. The second, third, and fourth levels are decided according to OPB blends, SOB blends, water-emulsified diesel fuel, and nano additive incorporated water-emulsified biodiesel blend fuel. In Table 4.1, four levels of fuel type are diesel, OPB10, OPB20, and OPB30. The input parameters and their levels for various fuels are kept in Appendix A. The second experimental design was performed taking three control parameters, i.e., Engine torque, Engine speed, and fuel type were elected with four levels of engine torque and engine speed and two levels of fuel type as shown in Table 4.2. An experimental strategy was planned by applying the Taguchi method with an L16 orthogonal array, as shown in Table 4.3, Table 4.4, and Appendix B, for a limiting number of experiments. The L16 orthogonal array was selected based on the number of factors, level, and degree of freedom.

Degree of freedom (DOF) = (number of levels−1) ∗ number of factors + 1

$$\text{DOF} = (4 - 1) * 3 + 1 = 10$$

The orthogonal array, which has DOF more than or equal to 10, can be selected. L16 orthogonal array has DOF 16, and it is the only array that may be applied for factors 2 to 5 with level 4.

Table 4.1 Input parameters for the same level of design

Input Parameter	Designation	Level 1	Level 2	Level 3	Level 4
Engine torque (N-m)	A	14	21	28	35
Engine speed (rpm)	B	1400	1600	1800	2000
Fuel type	C	Diesel	OPB10	OPB20	OPB30

Table 4.2 Input parameters for the same level of design

Input Parameter	Designation	Level 1	Level 2	Level 3	Level 4
Engine torque (N-m)	A	14	21	28	35
Engine speed (rpm)	B	1400	1600	1800	2000
		Level 1		Level 2	
Fuel type	C	OPB20+WiDE5+CNT		SOB20+WiDE5+CNT	

Table 4.3 L16 orthogonal array for the same level of design

S.No.	Engine Torque (N-m)	Engine Speed (rpm)	Fuel type
1.	14	1400	Diesel
2.	14	1600	OPB10
3.	14	1800	OPB20
4.	14	2000	OPB30
5.	21	1400	OPB10
6.	21	1600	Diesel
7.	21	1800	OPB30
8.	21	2000	OPB20
9.	28	1400	OPB20
10.	28	1600	OPB30
11.	28	1800	Diesel
12.	28	2000	OPB10
13.	35	1400	OPB30
14.	35	1600	OPB20

15.	35	1800	OPB10
16.	35	2000	Diesel

Table 4.4 L16 orthogonal array for the mixed level of design

S.No.	Engine Torque (N-m)	Engine Speed (rpm)	Fuel type
1.	14	1400	OPB20+WiDE5+CNT
2.	14	1600	OPB20+WiDE5+CNT
3.	14	1800	SOB20+WiDE5+CNT
4.	14	2000	SOB20+WiDE5+CNT
5.	21	1400	OPB20+WiDE5+CNT
6.	21	1600	OPB20+WiDE5+CNT
7.	21	1800	SOB20+WiDE5+CNT
8.	21	2000	SOB20+WiDE5+CNT
9.	28	1400	SOB20+WiDE5+CNT
10.	28	1600	SOB20+WiDE5+CNT
11.	28	1800	OPB20+WiDE5+CNT
12.	28	2000	OPB20+WiDE5+CNT
13.	35	1400	SOB20+WiDE5+CNT
14.	35	1600	SOB20+WiDE5+CNT
15.	35	1800	OPB20+WiDE5+CNT
16.	35	2000	OPB20+WiDE5+CNT

4.2 Experimental Setup and Methodology

4.2.1Experimental Setup

The experiment was conducted in a four-cylinder, variable speed, four strokes, and water-cooled diesel engine setup designed and manufactured by DATACONE ENGINEERS PVT. Limited. The detailed specification of the engine is shown in Table 4.5. The experimental setup consists of the engine and dynamometer, fuel consumption measuring unit and temperature measuring unit, exhaust gas calorimeter, orifice meter, and AVL exhaust gas analyzer. The hydraulic dynamometer was used for loading the engine. The impeller of the hydraulic dynamometer was coupled with the output shaft of the engine, and the load on the engine was provided with controlling the water flow rate of the dynamometer casing. The load value after the balance reading was shown in the digital weighing device in kg. The engine's speed was measured using a digital tachometer by Lutron Electronics Company, Taiwan (MODEL: DT-2234C). The fuel volume flow rate is calculated by observing the time duration of specified fuel volume consumption through a digital stopwatch. The volume of fuel consumption was noted through the marked level of a burette in mL. The digital temperature indicator was used to measure temperature in ^{o}C. The temperatures at the different positions were measured using the k-type thermocouple. The engine exhaust and smoke emissions were marked using an AVL DiTEST gas 1000 BL exhaust gas analyzer and AVL DiTEST 480 smoke opacity meter, respectively. The schematic diagram of the experimental setup is shown in Fig.4.1. An image of the experimental setup is shown in Fig.4.2. The engine fuel flow rate, airflow rate, temperature of the coolant, the temperature of exhaust gases, the temperature of water in the calorimeter, and emission level were measured at different sets of control parameters.

Table 4.5 Engine descriptions

Engine attributes	Specification
Engine brand	TATA INDIGO
Cylinders number	Four
Stroke	4-stroke
Coolant	Water
Combustion type	Compression ignition
Compression ratio	21:1
Aspiration type	Turbocharged
Fuel inject system	Common Rail Direct Injection (CRDi)
Injection timing	23^{o}bTDC
Bore (mm)	75
Stroke length (mm)	79
Volume (CC)	1396

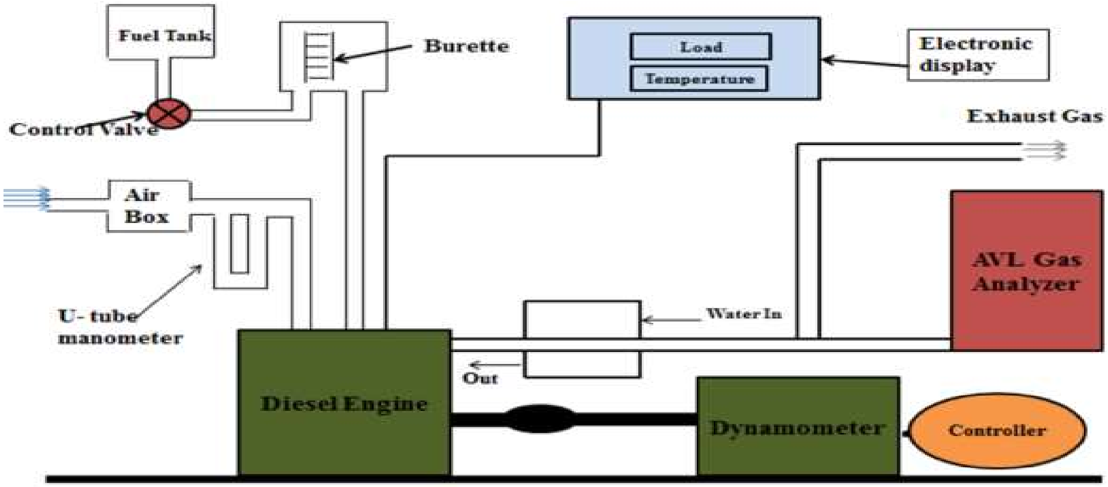

Figure 4.1 Schematic diagram of the experimental setup

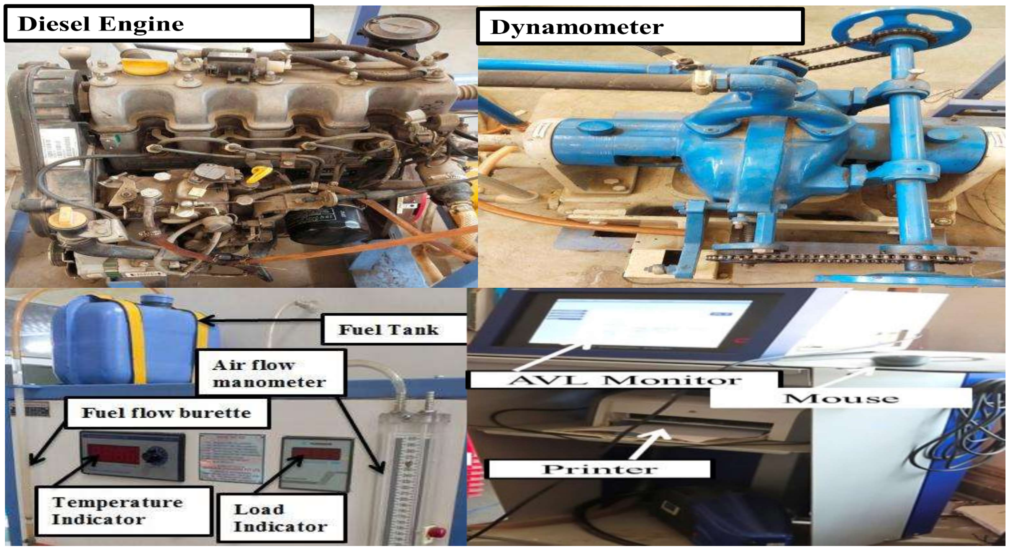

Figure 4.2 Actual diagram of the experimental setup

4.2.2 Experimental Methodology

The research engine was assigned to race with the diesel fuel for 12–15min to prerequisite the test setup. The coolant, i.e., water, was permitted at a flow rate of 240 lph. The engine raced for 8–10min to attain a steady state before changing fuel blends. The standard data was reported for the variable engine load 0-10kg with a difference of 2kg at constant engine speed 1500 rpm and variable engine speed1000-2000 rpm with a difference of 200 rpm at constant engine load10kg. The standard data was also noted according to the L16 orthogonal array obtained by Taguchi's experimental design. The fuel volume flow rate, manometer reading, and temperatures at all marked locations were recorded in each experiment. The fuel consumption rate, engine BTE, BSFC, exergy destruction rate, exergy efficiency, exergy sustainability, and entropy generation rate have been calculated using different thermodynamic equations at different control parameters. The engine energy and exergy performance parameters were measured for each performing circumstance. The engines's emission characteristics, i.e., HC, CO, CO_2, NO, and smoke opacity, were also obtained according to the input variables.

4.3 Mathematical Modelling

Input and output energy and exergy rate of the diesel engine is calculated, assuming it's a control volume **(Moran et al., 2014)**. The following assumption is made for evaluating the energy and exergy rate of diesel engines:

(i) The steady-state engine operation of the engine is assumed.
(ii) Exhaust gas follows the behavior of ideal gas.
(iii)A homogeneous combustion product is found inside the control volume.
(iv)The effect of kinetic and potential exergy rates is negligible.

The engine's effective power is calculated using the following expression,

$$BP = \frac{W.N}{2000} \qquad 4.1$$

Engine BTE is calculated using the following expression,

$$BTE = \frac{BP}{\dot{m}_f CV} \qquad 4.2$$

The following equation obtains the BSFC of the engine,

$$BSFC = \frac{\dot{m}_f}{BP} \qquad 4.3$$

An expression for the engine exergy balance is expressed (Aghbashlo et al., 2017),

$$\dot{Ex}_a + \dot{Ex}_f = \dot{Ex}_w + \dot{Ex}_l + \dot{Ex}_{des} + \dot{Ex}_c + \dot{Ex}_g^{Ph} + \dot{Ex}_g^{Ch} \qquad 4.4$$

Where $\dot{Ex}_a, \dot{Ex}_f, \dot{Ex}_w, \dot{Ex}_l, \dot{Ex}_{des},$ and $\dot{Ex}_c$ are the exergy flow rate of intake air, burning fuel, exergy rate of shaft work, rate of exergy loss to the surrounding, rate of exergy destruction, and rate of exergy loss to coolant, respectively. $\dot{Ex}_g^{Ph}$ and $\dot{Ex}_g^{Ch}$ is the physical and chemical exergy flow rate of exhaust gas.

The expression of the exergy flow rate of intake air is as follows (Aghbashlo et al., 2017),

$$\dot{Ex}_a = \dot{m}_a . [C_{p,a}\left(T_a - T_{ref} - T_{ref}\ \ln\frac{T_a}{T_{ref}}\right) + R\ T_{ref}\ \ln\frac{P_a}{P_{ref}} \quad 4.5$$

Where $C_{p,a}$ is the isobaric-specific heat of intake air, P_a andP_{ref} are the pressure of intake air and dead state pressure, respectively.

The expression of the fuel exergy rate is as follows (Aghbashlo et al., 2017),

$$\dot{Ex}_f = \dot{m}_f \emptyset\ LHV \quad 4.6$$

$$\emptyset = 1.0401 + 0.1728\left(\frac{H}{C}\right) + 0.0432\left(\frac{O}{C}\right) + 0.2169\left(\frac{S}{C}\right)\ [1 - 2.0268\ \left(\frac{H}{C}\right)] \quad 4.7$$

Where H, O, S, and C are a mass fraction of hydrogen, oxygen, sulfur, and carbon, respectively (López et al.,2014)

The shaft exergy rate is equivalent to the effective power of the engine and is expressed as follows:

$$\dot{Ex}_w = BP \quad 4.8$$

The rate of exergy loss to the environment surrounding is calculated using the following expression (Aghbashlo et al., 2017) :

$$\dot{Ex}_l = \dot{Q}_l\left(1 - \frac{T_{ref}}{T_b}\right) \quad 4.9$$

Where $\dot{Q}_l$ is the rate of loss of heat transfer, and T_b is ambient temperature.

The rate of loss of heat transfer is obtained by the energy balance method (Aghbashlo et al., 2017).

$$\dot{Q}_l = \dot{m}_a\ C_{pa}\left(T_a - T_{ref}\right) + \dot{m}_f\ LHV - \left(\dot{m}_a + \dot{m}_f\right)C_{pg}\left(T_g - T_{ref}\right) - \dot{m}_c C_{pc}\left(T_{c.out} - T_{c,in}\right) - EP \quad 4.10$$

Where, $T_{c.out}$ and $T_{c,}$ it is the exit and inlet coolant temperature.

The rate of exergy loss to the coolant is evaluated using the following expression (Aghbashlo et al., 2017):

$$\dot{Ex}_c = \dot{m}_c C_{pc}\left(T_{c.out} - T_{c,in} - T_{ref}\ln\frac{T_{c,out}}{T_{c,in}}\right) \quad 4.11$$

The expression for exhaust gas physical exergy rate is as follows (Song et al.,2013):

$$\dot{Ex}_g^{\ Ph} = (\dot{m}_a + \dot{m}_f)[C_{pg}\left(T_g - T_{ref} - T_{ref}\ln\frac{T_g}{T_{ref}}\right) + RT_{ref}\ln\frac{P_g}{P_{ref}} \quad 4.12$$

The expression for exhaust gas chemical exergy rate is as follows (Khanali et al.,2013):

$$\dot{Ex}_g^{\ Ch} = \dot{n}_g[\sum Y_i\,\varepsilon_i + \bar{R}T_{ref}\sum Y_i\ln(Y_i)] \quad 4.13$$

Where, $\dot{n}_g$ mole number of exhaust gas,Y_i is the mole fraction of ingredients, $\varepsilon_i$chemical exergies of each ingredient.

The expression of engine exergy efficiency of the diesel engine is calculated as follows:

$$EE = \frac{\dot{Ex}_w}{(\dot{Ex}_a + \dot{Ex}_f)} \quad 4.14$$

The engine sustainability index is calculated using the following expression (Khanali et al., 2013):

$$\mathrm{SI} = \frac{1}{(1 - \mathrm{EE})} \quad 4.15$$

The entropy generation rate is obtained using the following expression (Caliskan et al., 2009):

$$\dot{S}_{gen} = \frac{\dot{Ex}_{des}}{T_{ref}} \quad 4.16$$

4.4 Taguchi-Grey Relational Analysis

4.4.1 Signal-to-Noise Ratio (S/N Ratio) Analysis

The Taguchi design collects the output responses according to the orthogonal array, and the signal-to-noise (S/N) ratio for each output response has been recorded using Minitab16 software. For obtaining the S/N ratio, there are three conditions, i.e., larger is better, smaller is better, and Nominal is better. If the output response is such that its larger value is better, then the following correlation is used to calculate the S/N ratio (Kumar et al.,2016):

$$\frac{S}{N}HB = -10\ LOG({}^{1}/{}_{n}\sum_{i=1}^{n}\frac{1}{Y_i^2}) \qquad 4.17$$

For lower is a better response following correlation is used:

$$\frac{S}{N}\mathrm{LB} = -10\ LOG\ ({}^{1}/{}_{n}\sum_{i=1}^{n}Y_i^2) \qquad 4.18$$

For Nominal is a better response following correlation is used:

$$\frac{S}{N}\mathrm{NB} = -10\ LOG\ ({}^{1}/{}_{n}\sum_{i=1}^{n}Y_i - Y_m) \qquad 4.19$$

Where Y_i is the response value of i_{th} orthogonal array, Y_m is the mean value of all n response values, and n is a number of the orthogonal array.

The condition is selected according to the output response. The larger is better for the parameters, BTE, and exergy efficiency. For the BSFC, exergy destruction rate, NO, HC, CO, CO_2, and smoke opacity emission smaller is a better condition for the application. According to the S/N plot of each response parameter, the optimum input sets of variable conditions were obtained for each response parameter. The flow diagram of Taguchi analysis is shown in Fig.4.3. The calculated S/N ratio table according to L16 orthogonal is shown in Appendix B.

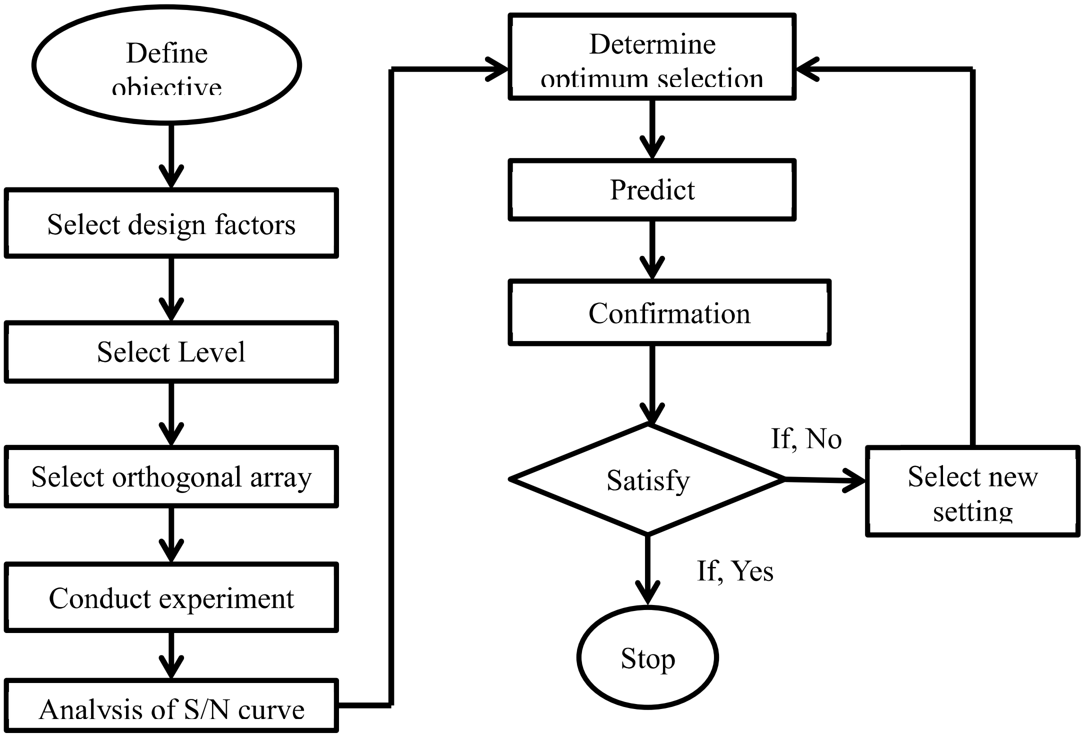

Fig.4.3 Flow chart of Taguchi analysis

4.4.2 Grey Relational Analysis

Taguchi's approach cannot determine the overall optimum condition, so the grey relational analysis was used. Fig.4.4 shows the flow diagram of the Grey analysis technique.

The first step of grey analysis is transforming data into a signal-to-noise ratio. Transformation is performed using Minitab 16 software. The second step of grey analysis is normalizing data between 0 and 1. Normalized data is prepared according to the higher is better or lower is better condition.

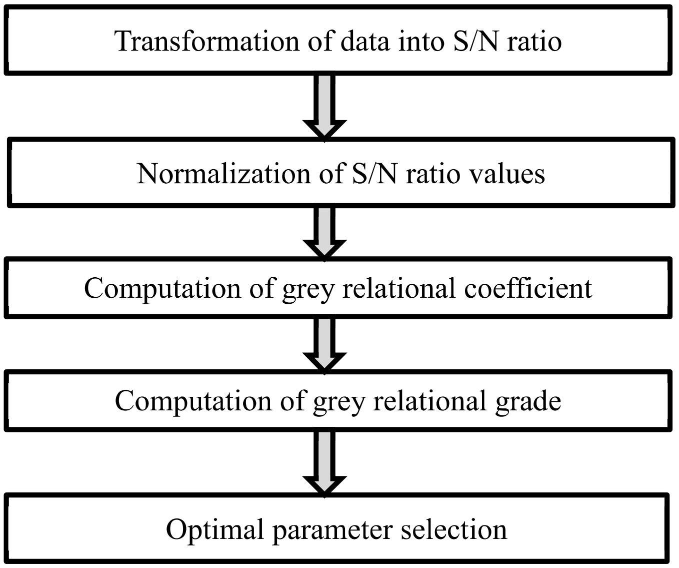

Fig. 4.4 Flow diagram of Grey analysis technique

If the output response is such that its higher value is better, then the following expression is used (Roy et al., 2014).

$$N_i(k) = \frac{\max x_i(k) - x_i(k)}{max\, x_i(k) - \min x_i\,(k)} \qquad 4.20$$

And, for lower is better;

$$N_i(k) = \frac{x_i(k) - \max x_i\,(k)}{max\, x_i(k) - \min x_i\,(k)} \qquad 4.21$$

Where$x_i(k)$ is the original sequence and $N_i(k)$is the sequence for Comparison, i=1,2,3…….n and k=1,2,3……,m. Where n is the total number of experiments and m is the total number of responses.

Deviation sequences (D_{oi}) for responses are calculated.

$$GRC = \frac{D_{min} + \Psi . D_{max}}{D_{oi}(k) + \Psi . D_{max}} \qquad 4.22$$

$D_{min}$is the minimum value of the absolute difference of all comparing sequences, and D_{max} is the maximum value of the absolute difference of all comparing arrangements. Ψ is the distinguished coefficient lies between 0 to 1. The present study's distinguish coefficient for each output response is 0.5.

Grey relation grade is calculated.

$$GRG = \frac{\sum_{K=1}^{K=m} GRC}{m} \tag{4.23}$$

The rank of each input parameter is obtained. The GRC, GRG, and Rank of various experiments are shown in Appendix C.

4.5 Uncertainty Analysis

The measuring range, accuracy, and percentage uncertainties of the used instruments have been presented in Table.4.6. Each of the experiments has been correlated with a few uncertainties. If R is a function of independent variables X_1, X_2, X_3……X_n and uncertainties of the individual variables are as E_1, E_2, E_3……… E_n, for variables such as temperature sensor error, load sensor error, exhaust gas sensors, etc., and then the uncertainty, E_R of the measured parameter has been calculated by the application of equation 4.24 (Krishnamoorthi et al., 2018, Ağbulut et al.,2020, Singh et al., 2020).

$$E_R = \pm\sqrt{\left[(\frac{\partial R}{\partial X_1}E_1)^2 + (\frac{\partial R}{\partial X_2}E_2)^2 + \left(\frac{\partial R}{\partial X_3}E_3)^2\right) + \ldots\left(\frac{\partial R}{\partial X_n}E_n)^2\right]} \tag{4.24}$$

The obtained uncertainties of different engine performances and the emission attributes have been laid out in Table 4.7. The evaluated engine performances and emissions were accepted to be reliable within the range of ± 3.32%.

Table 4.6 Details of measuring devices, their range, and measuring accuracy

Measured parameter	Range	Measuring Accuracy
Load	0-25kg	±0.10 kg
Time	-	0.01s
Engine speed	0-10,000rpm	±10rpm
Burette system	0-100cc	±0.1cc
Temperature	0-1000°C	±1°C
NO	0-5000ppm	±15ppm
HC	0-30,000ppm	±2ppm
CO	0-15 % vol	±0.2 %
CO_2	0-20% vol	±0.2%

Table 4.7 Details of evaluated engine parameters and percentage uncertainties

Engine parameter	Percentage uncertainties
Fuel mass rate	± 1.0%
Air mass rate	± 0.5%
Engine brake power	± 1.0%
BTE	± 1.5%
BSFC	± 1.2%
Exergy efficiency	±1.5%
Exergy destruction rate	± 1.7%

4.6 Highlights

This chapter details the experiment method, orthogonal Design, Taguchi-grey optimization techniques, and uncertainty analysis. The L16 orthogonal has been selected based on several factors and their levels. The orthogonal array was designed using the same level of design and mixed level of the design method. The engine performance and emission parameters have been observed to varying engine load and speed. The data was also collected according to the L16 orthogonal array. The following observations have been taken from the detailed analysis.

- The L16 orthogonal array has been designed considering three control parameters, i.e., engine load, engine speed, and fuel type, with four levels each. The four-level of engine loads and engine speeds are 14 N-m, 21 N-m, 28 N-m, and 35 N-m, and 1400 rpm, 1600 rpm, 1800 rpm, and 2000 rpm, respectively.
- The first level of the fuel type is diesel fuel, and the second, third, and fourth levels are decided according to OPB biodiesel blends, SOB biodiesel blends, water-emulsified diesel fuel, and nano additive incorporated water emulsified biodiesel blend fuel.
- The experiment was conducted in a four-cylinder, variable speed, four strokes, and water-cooled diesel engine setup designed and manufactured by DATACONE ENGINEERS PVT. Limited. However, The engine's speed was measured using a digital tachometer by Lutron Electronics Company, Taiwan (MODEL: DT-2234C).
- The digital temperature indicator was used to measure temperature in ^{o}C. The temperatures at the different positions were measured using the k-type thermocouple.

- The engine exhaust and smoke emissions were marked using an AVL DiTEST gas 1000 BL exhaust gas analyzer and AVL DiTEST 480 smoke opacity meter. The engines's emission characteristics, i.e., HC, CO, CO_2, NO, and smoke emissiona, were obtained according to the input variables.
- The signal-to-noise (S/N) ratio for each output response has been recorded using Minitab16 software according to larger is better, smaller is better, and nominal is better.
- The engine BTE, BSFC, exergy destruction rate, exergy efficiency, exergy sustainability, and entropy generation rate was calculated using different thermodynamic equations at different control parameters.
- The uncertainties of different engine performances and the emission attributes have been obtained, and the evaluated engine performances and emissions were accepted to be reliable within the range of ± 3.2%.

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