CHAPTER III

MATHEMATICAL FORMULATION OF CENTRALIZED AND DECENTRALIZED OPF BASED CONGESTION MANAGEMENT WITH SOCIAL WELFARE

3.1. INTRODUCTION

In the previous chapter, a brief introduction about the competitive electricity market and congestion management has been discussed. As the demand for deregulation of electric utilities is on the rise, the selection of objective function for optimization of economic system operation is becoming more and more critical. After the restructuring of the electric power industry, profit generating companies have been developed to deliver electric energy in the power market. They try to formulate the multi-transaction structures in the system for establishing a competitive market. The independent regulated transmission system operators (TSOs) manage the operations of the transmission system. The congestion management becomes one of the central issues of optimal power flow in transmission networks [89, 104 and 105]. The conventional optimizations and congestion management methods are mainly based on economic dispatch that enables to achieve minimum generation cost in a single transaction environment [88, 116]. However, this problem becomes more complicated if multi-transaction operation is involved in the system [96]. The consumer benefits are also included in the problem objective of the congestion management to achieve maximum social welfare using economical optimal dispatch. An interchange of information among the involved market players is organized to obtain a global solution by setting up a common control centre [99]. The congestion pricing and cost allocation based methodologies are used in this centre for optimizing the benefits of participants [95, 103, 108 and 111].

The present investigation deals with the congestion management in multi-transaction power market structure with bilateral/multilateral contracts between generators and costumers using centralized and decentralized decision supports [37-42]. In a multi-transaction environment, centralized and decentralized decision support based power system models are defined as follows:

i. *Centralized decision based power system operation* - This system consists of several electric transactions which are controlled by a centralized dispatching control like a tight pool.
ii. *Decentralized decision based power system operation* - In this system, each transaction acts independently and has its own dispatching control.

The congestion management solution with maximization of social welfare is obtained by optimal power flow (OPF) based rescheduling of active powers of generators and demands of loads. Additionally, the loss and its cost due to individual transactions have also been included in the problem formulation.

3.2. CENTRALIZED DECISION SUPPORT BASED SYSTEM OPERATION

In deregulated power system, because of the present trends of bilateral contracts in the electricity market, the role of independent system operator (ISO) is increasing. Under this scenario, the role of ISO is to create a set of rules that ensure sufficient control over producers and consumers to maintain an acceptable level of power system security and reliability [51-52]. The generators located with the larger magnitude of active and reactive powers can support to manage the congestion by their rescheduling of active and reactive power with OPF solution in the system. A centralized OPF (COPF) based decision support has been used to manage the operations of various systems and manage the congestion. In COPF, all the participants of the market jointly operate in the system as a group and find the solutions as shown in Fig. 3.1. COPF can provide an efficient solution for global optimization and congestion management, but it has some apparent disadvantages.

i. All the private information of the generators and consumers are required by the system operator to run the OPF program.
ii. The system operator collects the entire cost of surplus caused by congestion, but the allocation of these surpluses is a debatable issue.

These factors affect the performance of the market operations. Simultaneously, the independencies of transactions are also completely restricted by the COPF. Therefore, decentralized OPF (DOPF) based decision support has been proposed to improve the performance of the market with multi-transactions.

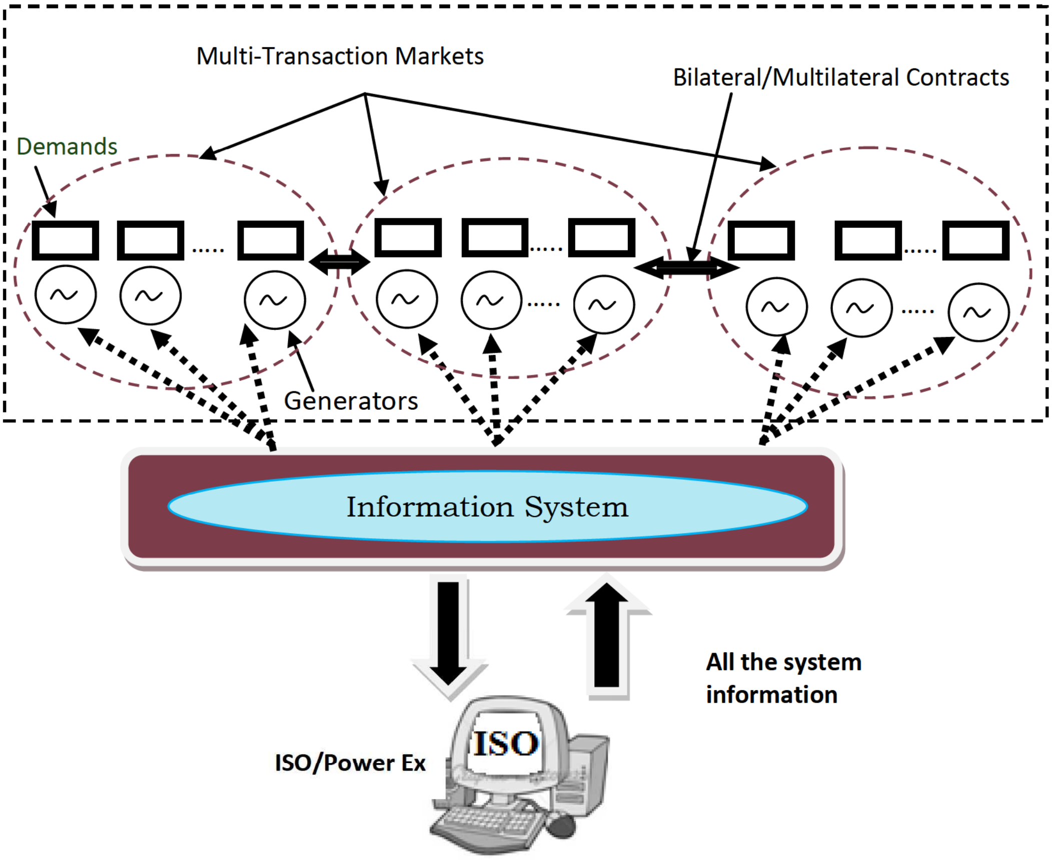

Fig. 3.1 Structure of centralized decision support based system operation in a multi transaction market model.

3.3. DECENTRALIZED DECISION SUPPORT BASED SYSTEM OPERATION

In a multi-transaction system, all the transactions are bound by some rules of coordination which are established by the centralized authority. Thus, this authority becomes a supreme power in the market. To establish an improved competition in the market, it is essential that all the transactions are free to optimize their operations and improve their profits. A decentralized decision support can play a vital role in market competition [40]. As the electric power industry is undergoing restructuring, it results into higher degree of decentralized decision making in the power system. This change has been affecting long term expansion planning of independent investors with less centralized coordination. After the restructuring of the electric power industry, all the power generating and distribution companies are trying to obtain the improved profits by delivering the electric energy in a competitive market. In such cases, independent regulated TSOs manage the operation of the transmission system. Therefore by setting a multi-transaction operation in the market, the congestion management becomes one of the central issues in the market [94-96].

The optimal rescheduling is one of the frequently used methods among various reported methods for the congestion management [92]. The OPF based rescheduling has been used in the present work for the congestion management including the social welfare in multi-transaction power market. In this case, ISOs maintain a schedule for load and generation to maximize the social welfare of the system without violating the system constraints. In this way, the consumers also participate in the formation of problem objective for maximizing the social welfare. The overall decentralized system operation can be represented by a schematic arrangement as shown in Fig. 3.2. The power transfer distribution factor (PTDF) calculation has been used for identifying the congested lines in the system [47, 109, 100 and 116]. In the preset work, these PTDF based sensitivities are directly used for observing the lines flows during optimization procedure. The DOPF solution provides an efficient solution without affecting the liberty of transactions in the market.

The recent trends in electricity market are towards large multinational electricity markets, such as, the internal electricity market (IEM) in Europe [10 and 11]. In this model, the whole system is partitioned into individual markets and coordination based trading is proposed among the partitions. The line information (line index) is exchanged at the end of each iteration in optimization, until the final convergence is achieved. Different decomposition methods for dividing the interconnected electricity markets into individual markets have also been reported [5]. After decomposition of the single joint market, a DOPF approach of congestion management is usually applied [97, 99-102]. An efficient economic dispatch in competitive electricity market has been analyzed to solve seam problems by establishing computational algorithm. The DOPF algorithm is an iterative algorithm in which the market agent of each transaction iteratively solves a modified OPF sub-problem for its own transaction and shares the line information with other transactions. In this way multi-agent based market structure is formed where these agents exchange their necessary information of generations and demands on the basis of sensitivity information declared by the ISO. The ISO then sends a control signal to each agent of transaction for rescheduling of their generations and demands to alleviate the congestion of the system with maximum social welfare.

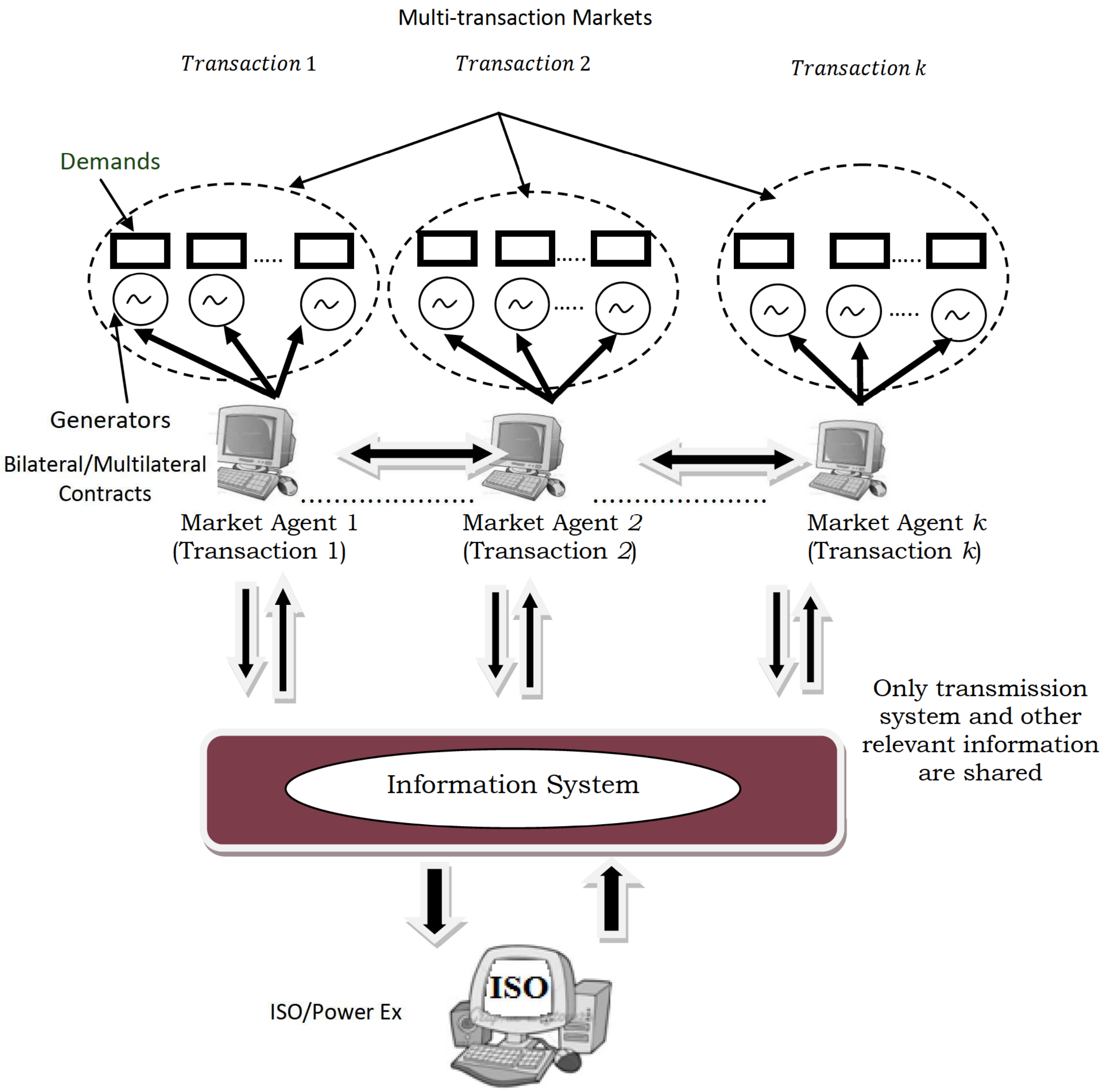

Fig. 3.2 Structure of decentralized decision support based system operation in a multi-transaction market model.

3.4. PROBLEM FORMULATION FOR CONGESTION MANAGEMENT WITH SOCIAL WELFARE

The optimal rescheduling in a power market has been used as a congestion management solution with economic operation. The application of a suitable optimum power flow method can provide a minimum rescheduling cost of generation and maximum benefit obtained by consumer in a power market. In this work, the participation of appropriate generators and demands are selected and the congestion mitigation process can be taken care by OPF formulation. In the present work, the line limits are considered as inequality constraints with other system components constraints. In the problem objective, the power rescheduling methodology is applied for analyzing the congestion management in this work [95].

The objective function of social welfare maximization using conventional OPF formulation can be mathematically defined as

$$Max\ f(x) = \sum_{j=1}^{N_d} B_i\left(P_{D_j}, Q_{D_j}\right) - \sum_{i=1}^{N_g} C_i\left(P_{G_i}, Q_{G_i}\right) \tag{3.1}$$

Subjected to the following constraints

Nonlinear equality constraints of the power mismatch equations in rectangular coordinates at a bus are given by

$$P_i(x) = P_{G_i} - P_{D_j} = 0 \tag{3.2}$$

$$Q_i(x) = Q_{G_i} - Q_{D_j} = 0 \tag{3.3}$$

where is loss

Nonlinear inequality constraints

$$h_i^{min} \leq h_i(x) \leq h_i^{max} \tag{3.4}$$

where P_{G_i} and Q_{G_i} real and reactive powers of generator at bus i, respectively; P_{D_j} and Q_{D_j} the real and reactive power loads respectively; P_i and Q_i the power injections at the node and are given by

$$P_i = |V_i| \sum_{j=1}^{n} \{(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij})\}|V_j| \tag{3.5}$$

$$Q_i = \sum_{j=1}^{n} V_i V_j [G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij})] \tag{3.6}$$

where $\theta_{ij} = (\theta_i - \theta_j)$ and V_i and θ_i are the magnitude and angle of the voltage at bus i, respectively; n is the total number of system bus. The transmission MVA limit is represented by

$$(P_{ij})^2 + (Q_{ij})^2 \leq (S^{max}{}_{ij})^2 \tag{3.7}$$

where $S^{max}{}_{ij}$ is the maximum MVA limit of the transmission line connected between bus i and j. In the present work, maximum limit of transmission line ij is considered as $S^{max}{}_{ij} = P^{max}{}_{ij}$. The active and reactive power flows P_{ij} and Q_{ij} in lines of the system are given by

$$P_{ij} = -V_i^2 G_{ij} + V_i V_j (G_{ij} cos\theta_{ij} + B_{ij} sin\theta_{ij}) \tag{3.8}$$

$$Q_{ij} = -V_i V_j + [G_{ij} sin\theta_{ij} - B_{ij} cos\theta_{ij}] + V_i^2 (V_j B_{ij} - \frac{b_{ij}}{2}) \tag{3.9}$$

where

G_{ij}	:	is conductance of ij^{th} line;
B_{ij}	:	is susceptance of ij^{th} line;
b_{ij}	:	is line charging susceptance of ij^{th} line;
$x = [P_G, Q_G, P_D, Q_D, t, \theta, V]^T$	:	is the vector of variables;
N_g	:	the number of generators bus;
N_d	:	the number of demand bus;
$P_i(x)$	:	bus active power mismatch equations;
$Q_i(x)$	:	bus reactive power mismatch equations;
$h(x)$	:	functional inequality constraints including line voltage magnitude constraints, simple inequality constraints of variables such as generator power, generator reactive power, transformer tap ratio;
t	:	the vector of transformer tap ratios;
B_j	:	the benefit function of consumers;
C_i	:	the generation cost function of generator.

3.4.1. Generator Cost Function

The real power generation cost function of each generator is modeled by a quadratic function as

$$C_i(P_{gi}) = a_i + b_i P_{gi} + c_i P_{gi}^2 \ (\$/h) \tag{3.10}$$

where a_i, b_i and c_i are predetermined cost coefficients. Usually, the reactive power costs of generators are also included in problem objectives and are known as opportunity cost [2]. The reactive power output of a generator reduces its active power generation capability which can serve at least as spinning reserve, and the corresponding implicit financial loss to generators is modeled as an opportunity cost. For simplicity, the reactive power cost of generator from the approximated capability curve which can be modeled as

$$C_{Qi}(Q_{Gi}) = [C_i(P_{Gi}^{max}) - C_{Qi}(\sqrt{P_{Gi}^{max\,2} - Q_{Gi}^2})]k \qquad (3.11)$$

where k is the profit rate of active power generation. This usually lies between 5 to 10%. In this work, only the costs of active power generations have been considered for formulating the problem objective.

3.4.2. Consumers Benefit Function [37]

The benefit of consumer is a function of demand which is mathematically represented as $B_j(P_{D_j})$. This is analogous to the cost functions of the supplier. After defining the benefit of consumer, the *Social Welfare* in the system can be defined as the difference of total consumer's benefits and the total supplier's cost and mathematically this can be represented as $\{B_j\left(P_{D_j}\right) -$ *CiPGi*. The consumer demand function is a function of the benefit of the consumer at the j^{th} bus. This can be represented as $D_j = f\left\{B_j\left(P_{D_j}\right)\right\}$, where D_j is the demand at the *j*th bus. In fact, the demand function D_j is the inverse of first derivative of benefit function of consumer with respect to load at the j^{th} bus and is represented as $D_j = \left(\frac{\partial B_j\left(P_{D_j}\right)}{\partial P_{D_j}}\right)^{-1}$. In the present case, the consumer's benefit function B_j is assumed as a concave function. This is always in an increasing order and helps in ensuring the maximum social welfare in the system with existing function (ignoring the convexity of the constraints). Mostly, the consumer's gains more benefit by more consuming more power. The sample plot of consumer benefit for real power and its derivative is shown in Fig. 3.3.

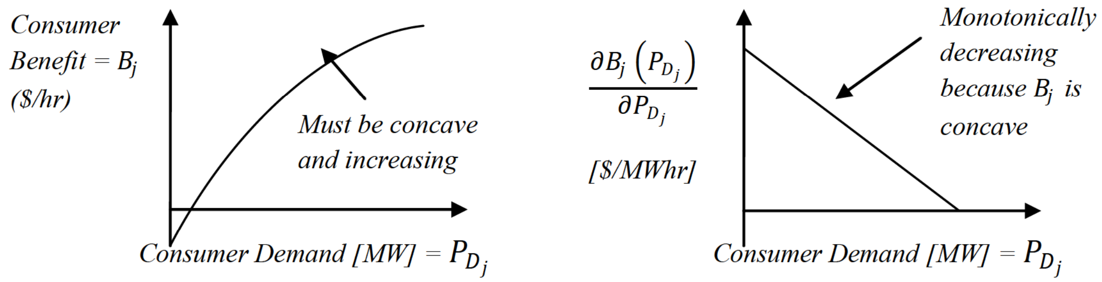

Fig. 3.3 Consumer benefit and derivative of benefit.

Let us consider that the consumer price is $P_j\left(P_{D_j}\right)$ and the demand function is the inverse of $\frac{\partial B_j\left(P_{D_j}\right)}{\partial P_{D_j}}$ at the j^{th} bus in addition to the base consumer price and base demand value is $P_{j,base}\left(P_{D_j}\right)$ and $P_{D_{j,base}}$ respectively. Then the slope m_{price} can be specified by the line as shown in Fig. 3.4.

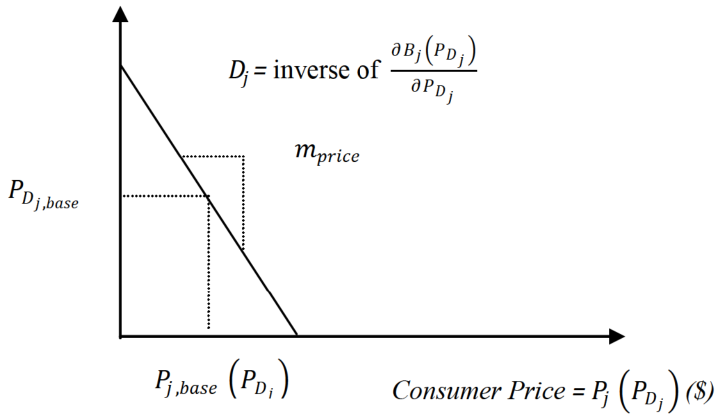

Fig. 3.4 Consumer demand for real power.

Therefore on the basis of relationship as shown in Fig. 3.4, the consumer demand function can be written as

$$D_j = \left\{P_{D_{j,base}} + M_{price} \times P_{j,base}\left(P_{D_j}\right)\right\} - M_{price} \times P_j\left(P_{D_j}\right) \tag{3.12}$$

where M_{price} is a diagonal matrix by summing the entire m_{price}. The consumer benefit function is a quadratic function and this is represented as

$$B_j\left(P_{D_j}\right) = P_{D_{j,base}}{}^{T} \times \left\{M_{price}{}^{-1} \times P_{D_{j,base}} + P_{j,base}\left(P_{D_j}\right)\right\}$$
$$- {}^{1}/_{2}\, P_{D_{j,base}}{}^{T} \times M_{price}{}^{-1} \times P_j\left(P_{D_j}\right) \tag{3.13}$$

3.4.3. Power Transfer Distribution Factor (PTDF) Calculation [109]

In present work, PTDF sensitivity indices are used to identify the congested lines during change of power transactions in between participants. In a bilateral/multilateral contract, some group of generators and consumers make coordination for transaction of power into a specific amount within the system. This coordination forms a linear relation between the participants. These relationships have a coefficient which relates the amount of power transaction and the flow in the line is known as the power transfer distribution factor *(PTDF)*. This is considered as a sensitivity factor because it relates the unit change in transaction with the change in line flows. The *PTDF* is a fractional part of transaction between participants which flows in a given line. In its simplest form, a transaction is a specific amount of power injected into one bus and withdrawn from another bus. In the present work, the nodal PTDF sensitivities are used for observing the power flows in the lines. In the nodal PTDF calculations, the changes in transactions are with respect to slack bus of the system. Thus, the PTDF of slack bus is considered to be zero. A transaction is denoted by $Trans\ (P_i, P_j)$, where bus P_i is the power injection bus and P_j is the sink bus. The specific amount of the power flow due to transaction is P_l where l is the specific line in between two buses. The *PTDF* is defined as the fraction of amount of transaction $Trans\ (P_i, P_j)$, flowing over line l. The *PTDF* is denoted as

$$PTDF = \frac{\Delta P_l}{\Delta Trans\ (P_i, P_j)} \tag{3.42}$$

In the present case, the change in line flows associated with a new traction is given as

$$(P_l^{new}) = (PTDF) \times Trans(P_i, P_j)^{new} \tag{3.43}$$

where P_l^{new} is the new value of power flow in the l^{th} line for the proposed new transaction $Trans(P_i, P_j)^{new}$ in MW.

3.4.4. Power Loss Calculations

3.4.4.1. Power loss calculation for COPF

In COPF formulation the total active and reactive power loss P_{Loss} and Q_{Loss} in the system is allocated on the slack bus and this is obtained by AC load flow

(ACLF) solution. The cost of power loss is also allocated on the slack bus. This can be given as

$$P_{Loss} = \sum_1^{n_l} P_{L,l} \tag{3.44}$$

$$Q_{Loss} = \sum_1^{n_l} Q_{L,l} \tag{3.45}$$

where $P_{L,l}$ and $Q_{L,l}$ are the real and reactive power loss in the line l connected between i and j is given by

$$P_{L,l} = G_{ij}\,(V_i^2 + V_j^2 - 2V_iV_j\cos(\theta_i - \theta_j)) \tag{3.46}$$

$$Q_{L,l} = V_i^2\left(B_{ij} - b_{\frac{ij}{2}}\right) + V_j^2\left(B_{ij} - b_{\frac{ij}{2}}\right) - 2V_iV_j\cos(\theta_i - \theta_j)) \tag{3.47}$$

3.4.4.2. Power loss calculation for DOPF [17]

In DOPF, the losses due to various transactions in the system are calculated by incremental transmission loss (ITL) coefficients based sensitivity indices. These indices directly allocate the losses of the system on individual generators of the various transactions. In this method, the ITL coefficients proportionally allocate the losses to the generators and demands of the system. These coefficients are easily obtained from a converged power flow. The ITL of a given bus provides the change in total losses produced by an incremental change in the power injected in that bus. Therefore

$$K_i = \frac{\partial P_{Loss}}{\partial (P_{G_i} - P_{D_j})} \tag{3.48}$$

where K_i is the ITL corresponding to bus i. In this calculation, the ITL coefficient of the slack bus is consider as zero. The computations of the losses allocated to generator and demand can be represented as

$$L_{G_i} = P_{G_i}\frac{\partial P_{Loss}}{\partial P_{G_i}} = P_{G_i}K_i$$

(3.49)

$$L_{D_j} = P_{D_j}\frac{\partial P_{Loss}}{\partial P_{D_j}} = -P_{D_j}K_j \tag{3.50}$$

Due to nonlinearities in the system, the sum of these allocated losses do not match total actual (measured) losses. If L' is the allocated losses and L is total actual (measured) losses, then

$$L \neq \sum_{i=1}^{N_G} L_{G_i} + \sum_{j=1}^{N_D} L_{D_j} = \sum_{i=1}^{N_G} P_{G_i}K_i - \sum_{j=1}^{N_D} P_{D_j}K_j = L' \tag{3.51}$$

Thus, the normalization procedure is used to allocate the exact amount of losses in the system. The exact amount of losses after normalization can be represented as

$$L = L'\frac{L}{L'} = \left(\sum_{i=1}^{N_G} P_{G_i}K_i - \sum_{j=1}^{N_D} P_{D_j}K_j\right)\frac{L}{L'} = \sum_{i=1}^{N_G} P_{G_i}K_i{}' - \sum_{j=1}^{N_D} P_{D_j}K_j{}' \qquad (3.52)$$

where $K_i^{'} = K_i(\frac{L}{L'})$ and $K_j^{'} = K_j(\frac{L}{L'})$ are the normalized ITL coefficients for the generator and load buses respectively. Therefore, the losses allocated to every generator and demand is

$$L_{G_i}^{'} = P_{G_i}K_i{}' \text{ and } L_{D_j}^{'} = -P_{D_j}K_j{}' \qquad (3.53)$$

In the present work, the losses are allocated on generator buses of various transactions and the cost of losses are allocated on their respective generators. The costs of losses due to various transactions are calculated using cost coefficients of slack bus generation cost. The cost coefficient of slack bus is used to evaluate the loss cost since the losses are supplied by the slack bus.

3.4.5. Mathematical Formulation for Centralized OPF (COPF)

The centralized decision based multi-transaction power market includes the demand biding in the objective of the problem. The power suppliers and the consumers mutually share the system information and optimize the generations and demands schedules for maximizing the total social welfare of the power market without violating the system constraints. In the proposed mathematical model, the congestion management problem has been solved using the rescheduling of generators and loads based methodology considering the line limits in the problem formulation. The line flows and total active power loss have been determined using AC load flow (ACLF) method. Thus a centralized decision based solution methodology to obtain the maximum social welfare with congestion management is given as

$$Max\, f\left(P_{G_i}, P_{D_j}\right) = \sum_{k=1}^{T}\left\{\sum_{\substack{j=1\\ j\epsilon D(k)}}^{N_d} B_j{}^k\left(P_{D_j}{}^k\right) - \sum_{\substack{i=1\\ i\epsilon G(k)}}^{N_g} C_i{}^k\left(P_{G_i}{}^k\right)\right\} \qquad (3.54)$$

$$Subject\ to \sum_{k=1}^{T}\left\{\sum_{i=1}^{Ng} P_{G_i}{}^k - \sum_{j=1}^{Nd} P_{D_j}{}^k\right\} = \sum_{k=1}^{T} P_{Loss}^k \qquad (3.55)$$

$$P_{G_i}^{k,min} \leq P_{G_i}^k \leq P_{G_i}^{k,max} \qquad (3.56)$$

$$Q_{G_i}^{k,min} \leq Q_{G_i}^k \leq Q_{G_i}^{k,max} \qquad (3.57)$$

$$P_{D_i}^{k,min} \leq P_{D_i}^k \leq P_{D_i}^{k,max} \qquad (3.58)$$

$$V_i^{min} \leq V_i \leq V_i^{max} \quad (3.59)$$

$$\sum_{k=1}^{T} P_{l,}^{k} \leq P_l{}^{max} \quad (3.60)$$

where

N_g	:	number of generator bus;
N_d	:	number of demand bus;
T	:	total number of transaction;
k	:	index for set of transaction in the system; ($k \epsilon T$)
$P_{G_i}^k$	:	active power output of i^{th} generator in transaction k;
$Q_{G_i}^k$	:	reactive power output of i^{th} generator in transaction k;
$P_{D_j}^k$	:	active power demand by consumer j in transaction k;
P_{Loss}^k	:	active power loss in transaction k;
$B_j{}^k$	:	benefit function of demand j of transaction k;
$C_i{}^k$	:	cost function of generator i of transaction k;
C_{slack}	:	cost function of slack bus generator;
V_i	:	voltage magnitude of i^{th} bus;
V_i^{min}, and V_i^{max}	:	minimum and maximum voltage limits at bus i in the system;
$P_{l,}^k$	:	power flow in l^{th} transmission line due to transaction k;
$P_{l,}^{max}$	:	maximum power flow limit of l^{th} transmission line;
$D(k)$	:	set of consumers in transaction k; ($k \epsilon T$)
$G(k)$	:	set of generators in transaction k; ($k \epsilon T$)

The objective of Eq. (3.54) is to maximize the social welfare in the system. The first term represents the demands benefits and the second term represents the generation costs of all generators. The total power loss due to load flow has also been included in this problem formulation. The equality constraint in Eq. (3.55) denotes the active power balance for each transaction considering the losses. The inequality constraints in Eqs. (3.56) and (3.57) denote the output active and reactive power limits of generators. The demand limits of consumers are represented in Eq. (3.58). Inequalities constraints of Eq. (3.59) denote the voltage limits at different buses and Eq. (3.60) denote the line capacity limits of lines which are participate in congestion management.

3.4.6. Mathematical Formulation for Decentralized OPF (DOPF) using Optimal Resource Allocation (ORA) Approach

In DOPF model, transactions independently optimize the schedules of their own generators and demands. In this case, the main optimization problem has been decomposed into sub-optimization problems. The COPF model is converted into DOPF model using optimal resource allocation (ORA) based approach [100] by rewriting Eqs. (3.54-3.60) to simpler form and by defining a decision variable of contract u^k for transaction k as

$$u^k \overset{def.}{\triangleq} f\left(P_{G_i}{}^k, P_{D_j}{}^k\right) \qquad (3.61)$$

where $(k \epsilon T)$.

Taking Eq. (3.61) into consideration, the welfare of transaction k can be defined as

$$w^k\left(u^k\right) \overset{def.}{\triangleq} \sum_{\substack{j=1 \\ j\epsilon D(k)}}^{Nd} B_j^k\left(P_{D_j}{}^k\right) - \sum_{\substack{i=1 \\ i\epsilon G(k)}}^{Ng} C_i^k\left(P_{G_i}{}^k\right) - C_{slack}^k\left(P_{Loss}^k\right) \qquad (3.62)$$

The last term of objective function expressed by Eq. (3.62) represents the cost of loss incurred due to transaction k. The power losses due to various transactions are allocated by ITL coefficients. In the present objective, all the generators and demands in transaction k can be re-scheduled to maximize their welfare without violating the local constraints as in Eqs. (3.55)-(3.59). These local constraints are the operational limits of participants in an individual transaction without interfering with the influence of other transactions. Only the line limits are influenced by all the transactions. Thus, the actual COPF model can be rewritten as

$$max \sum_{k=1}^{T} w^k\left(u^k\right) \qquad (3.63)$$

$$subject\ to \sum_{k=1}^{T} P_l^k \le P_l{}^{max} \qquad (3.64)$$

In DOPF, the line capacity of individual transaction is allocated by the ORA method, in which the generation levels of generators and demands of consumers are optimally re-scheduled according to an index α for the transmission line, l. This index is defined as resource allocation weighting matrix (RAWM). Therefore, if the index of transaction k on the line l is represented by $\alpha_l{}^k$, then the allocated maximum transmission limit for transaction k on line l can be defined as

$$P_l^{k,max} \le P_l{}^{max} \,.\, \alpha_l{}^k \qquad (3.65)$$

where $\sum_{k=1}^{T} {\alpha_l}^k = 1$.

Thus, applying the above method of conversion of COPF problem to DOPF problem using RAWM index ${\alpha_l}^k$ results into following DOPF model

$$max \quad \sum_{k=1}^{T} w^k(u^k) \tag{3.66}$$

$$subject\ to \sum_{k=1}^{T}\left\{\sum_{i=1}^{Ng} {P_{G_i}}^k - \sum_{j=1}^{Nd} {P_{D_j}}^k - P_{Loss}^k\right\} = 0 \tag{3.67}$$

$$P_{G_i}^{k,min} \leq P_{G_i}^k \leq P_{G_i}^{k,max} \tag{3.68}$$

$$Q_{G_i}^{k,min} \leq Q_{G_i}^k \leq Q_{G_i}^{k,max} \tag{3.69}$$

$$P_{D_i}^{k,min} \leq P_{D_i}^k \leq P_{D_i}^{k,max} \tag{3.70}$$

$$V_i^{min} \leq V_i \leq V_i^{max} \tag{3.71}$$

$$\sum_{k=1}^{T} P_l^k \leq \sum_{k=1}^{T} {\alpha_l}^k . {P_l}^{max} \tag{3.72}$$

$$\sum_{k=1}^{T} {\alpha_l}^k = 1 \tag{3.73}$$

The counter flow of power in lines may result into negative value of ${\alpha_l}^k$. This equality condition is only true for congested lines. In the present work, the OPF problem of Eq. (3.66) is decomposed into k independent sub-OPF problems. These sub-problems have been solved by transactions under a fixed value ${\alpha_l}^k$ which is assigned by ISO for all the transactions. Therefore the objective of Eq. (3.66) can be redefined as

$$S(\alpha) \overset{def.}{\triangleq} max_u \quad \left\{\sum_{k=1}^{T} w^k(u^k)\Big|_{\sum_{k=1}^{T} P_l^k \leq \sum_{k=1}^{T} {\alpha_l}^k . {P_l}^{max}}\right\} \tag{3.74}$$

which is equivalent to $S(\alpha) = \sum_{k=1}^{T} S^k(\alpha)$ and $S^k(\alpha)$ which can be defined as

$$S^k(\alpha) \overset{def.}{\triangleq} max_{u^k} \quad \left\{w_k(u_k)\big|_{P_l^k \leq {P_l}^{max} . {\alpha_l}^k}\right\} \tag{3.75}$$

The solution for ORA based problem as given in Eq. (3.74) should satisfy optimality conditions. These optimality conditions can be derived using Karush Kuhn Tucker (KKT) necessary conditions. Thus after applying the KKT condition

$$\frac{\partial S(\alpha)}{\partial\, {\alpha_l}^1} = \frac{\partial S(\alpha)}{\partial\, {\alpha_l}^2} = \cdots \frac{\partial S(\alpha)}{\partial\, {\alpha_l}^k} \tag{3.76}$$

The Eq. (3.76) can be used as sensitivity information by the ISO. After applying the steps described in ref. [100], the following relations are obtained

$$\frac{\partial S(\alpha)}{\partial\, {\alpha_l}^k} = \frac{\partial S^k(\alpha)}{\partial\, {\alpha_l}^k} \tag{3.77}$$

$$\frac{\partial S^k(\alpha)}{\partial P_l^{k,max}} = \lambda_l^k \tag{3.78}$$

$$\frac{\partial S^k(\alpha)}{\partial\, {\alpha_l}^k} = \lambda_l^k . P_l^{max} \tag{3.79}$$

In Eq. (3.78), λ_l^k is the Lagrange multiplier of line l in transaction k, which is solved using sub-OPF problem of Eq. (3.75). These Lagrange multipliers denote the change of welfare obtained with respect to change of allocated maximum transfer capacity $P_l^{k,max}$ of the l^{th} line in transaction k. Further, from Eq. (3.65), it can be explained that the value of $P_l^{k,max}$ also depends upon ${\alpha_l}^k$. Because of this, the welfare of k^{th} transaction varies due to change in ${\alpha_l}^k$ as given in Eq. (3.79). Therefore, λ_l^k of a line l also provides an optimality necessary condition for Eq. (3.74). Now, by taking the gradient of $S(\alpha)$ of Eq. (3.74), the following expression is obtained

$$\nabla S(\alpha) = \begin{bmatrix} \frac{\partial S(\alpha)}{\partial {\alpha_1}^1} & \cdots & \frac{\partial S(\alpha)}{\partial {\alpha_1}^k} \\ \vdots & \ddots & \vdots \\ \frac{\partial S(\alpha)}{\partial {\alpha_l}^1} & \cdots & \frac{\partial S(\alpha)}{\partial {\alpha_l}^k} \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial S(\alpha)}{\partial \alpha_1}\right)^k \\ \vdots \\ \left(\frac{\partial S(\alpha)}{\partial \alpha_l}\right)^k \end{bmatrix} = \begin{bmatrix} \lambda_1^1 . P_1^{max} & \dots & \lambda_1^k . P_1^{max} \\ \vdots & \ddots & \vdots \\ \lambda_l^1 . P_l^{max} & \dots & \lambda_l^k . P_l^{max} \end{bmatrix}$$

$$= [P_1^{max} \dots \dots \dots P_l^{max}]_{1\times l} [\lambda_l^k]_{l\times k} \tag{3.80}$$

Thus Eq. (3.76) can be represented as

$$\lambda_l^1 = \lambda_l^2 = \cdots = \lambda_l^k \tag{3.81}$$

The condition expressed in Eq. (3.81) shows that the social welfare of a system can be maximized if the Lagrange multiplier of l^{th} line due to all transactions are the same. This results into equal economic efficiency of a line for all transactions. Therefore, Eqs. (3.73) and (3.81) are the optimality and necessary conditions of Eq. (3.74). In ORA based optimization problems, the optimality and KKT necessary conditions for the transactions are equivalent to the optimality and KKT necessary conditions of COPF. A detailed mathematical proof of this is given in ref. [100]. The problem of congestion can be managed by selecting the correct value of ${\alpha_l}^k$ index for transactions which provides maximum social welfare in the system. However, these values should satisfy the aforesaid optimality conditions. If any of these are not satisfied, then the values of ${\alpha_l}^k$ are updated according to the line limits using gradient projection method [100] and congestion in the system can be managed. The congestion management is applied to the system by master and slave operations of optimization problems. The ISO optimizes ${\alpha_l}^k$ as master operation and the solution of the sub-optimization problem u^k as the slave operation to obtain the maximum social welfare of the system with congestion management.

3.4.7. Congestion Management Methodology using DOPF

In the proposed DOPF model, the congestion in the congested lines caused by transactions is determined using power transfer distribution factor (PTDF) calculations. Thereafter, the ISO optimally allocate the capacity of transmission lines for individual transaction k on the basis of RAWM index $\alpha_l{}^k$. Then, it is send by ISO to all transactions, out of which the first transaction optimizes its own generations and demands schedules using OPF with $\alpha_l{}^k$ signal. Subsequently, the next transaction starts optimizing its own generations and demands schedules after optimizing the first transaction. Similarly, every transaction optimizes its own schedules and this process continues till all the transactions complete their optimization procedure. Thus, the maximum social welfare of the market is achieved using serial computation process of optimization. Further, if the lines are congested due to new schedules, ISO updates the rescheduling signal $\alpha_l{}^k$ for all transactions.

3.5. DOPF BASED CONGESTION MANAGEMENT ALGORITHM

The steps of congestion management using IP-PSO based OPF method is as follows:

Step 1) Identify the congested lines in the system using PTDF.

Step 2) Set the initial RAWM indices for all the lines corresponding to transaction k such that condition mentioned in (3.73) is satisfied, $\sum_{k=1}^{T} \alpha_l{}^k = 1$.

Step 3) Solve the optimization problem expressed by (3.66) for all transactions.

Step 4) Find the Lagrange multipliers λ_l^k corresponding to various transactions.

Step 5) Check the condition for Lagrange multipliers for various transactions.
If it is close enough for all lines according to (3.81), i.e.
$\lambda_l^1 = \lambda_l^2 = \cdots = \lambda_l^k$, *then* go to step 6. *Else* update RAWM indices α_l^k for the various transactions and go to step 3.

Step 6) STOP

3.6. CONCLUSION

This chapter presents the problem formulation of centralized and decentralized decision based optimal power flow operations in power market. The congestion management in transmission system with social welfare maximization in multi-transaction is considered as a problem objective in COPF and DOPF approaches. In the DOPF based approach, the centralized system optimization problem can be decomposed into sub-optimization problems. The transactions in the power market can optimize their own generators and demands schedules on the basis of rescheduling signal $\alpha_l{}^k$ and participate into congestion management operations. The PTDF calculations are directly useful to identify the congested lines and according to that the system operators can reschedule the generation levels of generators and demands of consumers to relieve the congestion from the system. The performance of COPF and DOPF formulations using IP, PSO and proposed IP-PSO methods on various test systems are evaluated and discussed in the subsequent chapters.