Chapter 6

Conclusions and Future Work

This chapter of the thesis consists of two segments: a summary and a discussion on future research ideas.

6.1 Conclusions

The main contribution of this thesis is the development of approximation techniques for FSLPs and GFSLPs. Here, we have studied various FSLPs and GFSLPs and formulations involve B-operator, generalized fractional derivatives and TFDs.
Chapter 1 contains the introduction of the thesis including definitions of fractional integrals/derivatives operators and their properties. This chapter includes literature survey on SLPs and FSLPs describing the past and recent work in these areas. Additionally, it consists of the motivation of the proposed research work.
Chapter 2 presented a numerical approximation of the GFSLP with the mixed boundary conditions. Firstly, the well-posedness of the considered GFSLP is discussed. Then, a numerical method to solve the proposed GFSLP is described using

the finite difference method to discretized the B-operator involved in GFSLP. Further, the system of algebraic equations is generated and the set of approximated eigenvalues and corresponding eigenfunctions are shown and the convergence order of the proposed scheme is also presented which is close to one. The proposed scheme can also be extended in finding the solution of other FSLPs by varying the kernel in the B-operator. To establish an application, we have obtained the solution of the fractional diffusion equation defined in terms of the B-operator with the Prabhakar kernel.

In Chapter 3, we have presented a numerical technique to approximate the eigenvalues and eigenfunctions of GFSLP. For this, the GFSLP is transformed into an integral equation. Further, the generalized fractional integral is discretized using the Lagrange interpolation method, and a system of an algebraic equation is obtained. Solving the obtained system, we get the approximate eigenvalues and eigenfunctions. The rate of convergence of the proposed scheme is calculated numerically. It is observed that the solutions of GFSLP converge to the classical case as $\alpha \to 2$.

Chapter 4 investigated a numerical algorithm to solve the proposed FSLP. First, we demonstrated that the considered FSLP is well-posed. Following that, a numerical approach for solving the suggested FSLP is provided. We used the finite difference method and non-uniform node points to discretize CGFDs. A system of linear equations is then formed, and we determine the approximated eigenvalues and eigenfunctions. The numerical results obtained for the test cases show the convergence order is close to one. By changing the weight and scale functions, the suggested approach can easily be expanded to find solutions to different FSLPs. We derived the solution to the FDE provided in order to build an application.

Chapter 5 presented a numerical algorithm for the proposed TFSLP. Firstly, this TFSLP was transformed into an integral form using the composition rule of integral and derivatives. After this, we have converted this integral equation into a system of linear equation by discretization of tempered integral using linear approximation. After this, the equation was solved for a different order of derivatives $\alpha \in (0, 1]$ and $\alpha \in (1, 2]$ and for different values of tempered factor τ. Further, presented results and graph of numerical solutions are shown.

6.2 Future Work

There is an opportunity to demonstrate the theoretical results and expand the proposed research in a higher dimension. Other numerical approaches such as finite element method, projection methods, etc. can be applied to get the approximate eigenvalues and eigenfunctions of the studied FSLPs. Further, there is scope to study these FSLPs with mixed fractional boundary conditions. Developing analytical methods for the solutions of these FSLPs is also an interesting part to explore.

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