

ABSTRACT

For the past few decades, finite element limit analysis (FELA) is used as an effective tool for providing rigorous solutions to different two-dimensional and three-dimensional geotechnical stability problems. The FELA technique is quite advantageous as it is based on strong theoretical background, possesses all the advantages of finite elements, does not require any strict implementation of the constitutive equations, and limits the true collapse load within lower and upper bounds. The present thesis focuses on employing the FELA technique to compute the collapse load for various axisymmetric/ plane-strain footings rested over saturated and unsaturated strata. Broadly speaking, the thesis consists of two sections. In the first section, the lower-bound (LB) FELA technique is presented and then extensively implemented to compute the ultimate bearing capacity (UBC) of ring footings resting on homogenous/ layered geomaterials and subjected to different loading arrangements. In the second section, the upper-bound (UB) FELA technique is modified to evaluate the ultimate bearing capacity of plane-strain footing resting on partially saturated sandy soil and subjected to various hydromechanical properties with due consideration of ground water table fluctuation and climatic changes. The problems are solved by writing suitable codes in 'MATLAB (R2019a)'.

The LB-FELA technique is demonstrated using two-dimensional axisymmetric elements, three-dimensional yield criterion, and an interior point method-based nonlinear optimization process. The yield surfaces that are chosen to represent the cohesive-frictional soils, undrained clays, and rocky stratum are based on the Mohr-Coulomb, Tresca, and Hoek-Brown yield criteria, respectively. In the three-dimensional principal stress space, the Mohr-Coulomb, and Hoek-Brown yield surface looks like

linear and curvilinear hexagonal pyramids, respectively. These yield surfaces exhibit sharp discontinuities, which act as singular points and create problems in determining the unique gradient and Hessian at every stress-points. Therefore, a smoothening exercise is carried out in order to remove the “edge discontinuities” and the “tip discontinuity” present in the yield surfaces. The apex of the hexagonal pyramids, as viewed in the meridian plane, and the sharp corners of the yield hexagon present in the π -plane are adequately smoothened with the aid of the hyperbolic approximations and the trigonometric rounding-off techniques, respectively. However, the Tresca yield criterion is hydrostatic, stress-independent, and resembles a hexagonal prism indicating that the smoothening is to be done solely on the π -plane. In this process, a C^2 -continuous yield surface is obtained. Notably, in the present axisymmetric formulation, there is no need to consider the inherent assumption associated with the hoop stress in the traditional analysis. The efficacy of this technique is verified by computing the bearing capacity factor, N_c , for axially loaded circular piles embedded into completely saturated and undrained clayey strata whose cohesion increases linearly with depth. The design charts are prepared by varying the pile geometry, the roughness of the pile base and shaft, and the non-dimensional parameter controlling the rate of cohesion increment. The roughness along the pile shaft seems more dominating than the roughness below the pile base. After the limiting collapse state is obtained, the failure pattern is drawn by verifying the closeness of the Mohr stress circle (of each and every nodal point) to the Mohr yield circle. For Tresca soil, the diameter of the Mohr yield circle is the cohesive strength at that specific node.

With reference to the application of the nonlinear LB-FELA technique, a series of numerical simulations are carried out on ring footings corresponding to various combinations of underlying geomaterials and varying loading configurations. To start

off, a rigorous investigation has been performed to understand the impact of the material strength, geometrical configuration, surcharge pressure, and soil-footing interface condition on the behaviour of ring footing resting over sands, which is underlain by fully cohesive strata. The results are presented in the form of efficiency factor (η); where η is defined as the ratio of the UBC of ring footing in the presence of a sand layer to the UBC of ring footing placed over the clayey layer alone. It is found that even the inclusion of a thin sand layer between the footing and clays improves the bearing capacity significantly. The improvement in the bearing capacity occurs continuously with the decreases in (i) radius ratio and (ii) undrained cohesive strength (of the bottom clays), and the increases in (i) surcharge pressure and (ii) drained friction angle (of the top sand layer). There exists a certain optimum thickness of the sand layer beyond which no further improvement in the bearing capacity occurs. A number of design charts are prepared by including the effect of footing roughness and the external surcharge pressure.

An in-depth analysis is performed to understand the load-carrying capacity of ring footing placed over two-layered soils and subjected to various vertical loadings applied over certain portions of the footing. Three different partial loadings are considered, namely, inner half, middle half, and outer half loading. The chosen layered soils are: (i) drained sand over another drained sand, (ii) undrained clays overlying another layer of undrained clays, and (iii) drained sand underlain by undrained clays. The numerical simulations are done for both smooth and rough ring footing. A parametric study has been performed by varying internal friction angle of sand (ϕ), undrained cohesion of clays ($c_u/(\gamma r_0)$), radius ratio of ring footing (r_i/r_0), thickness of upper layer soil (h/r_0), surcharge pressure ($q/\gamma r_0$) and different loading positions. The efficiency factor (η) and normalized bearing capacity charts have been prepared to

represent the results. The normalized bearing capacity is found to be highest for the inner half and least for the outer half loading. The magnitude of η and optimum h/r_0 are found to be independent of loading positions. The extent of the failure zone for different ring geometries, underlying layer properties, top layer thicknesses, surcharge pressures, soil-footing interfaces, and loading positions are thoroughly verified.

An attempt has also been made to evaluate the UBC of ring footing resting on rock medium by employing the *GSI*-classification-based isotropic Generalized Hoek-Brown yield criterion. The design charts have been prepared for different load configurations corresponding to varying rock strength, ring size, and surcharge pressure. It is found that the bearing capacity factor gets highly influenced by loading position. Complete loading always yields a lower UBC than its partial loading counterpart. The ring footings are preferable when the loading is placed over the mid-portion or outer portion of the annular section; nevertheless, inner half loadings yield the maximum bearing capacity irrespective of any other factors. A proper validation has been done with the results reported in the previous literature.

In the second part of the thesis, the conventional upper bound (UB) FELA technique is modified for performing the stability analysis of plane-strain geotechnical structures resting on an unsaturated medium. This modification has been carried out with the help of the suction stress-based effective stress tensor and by redefining the Mohr-Coulomb yield circle and yield envelope in the two-dimensional stress space. The spatial variation of matric suction in the vadose zone has been developed based on the van-Genuchten *soil water retention curve* (SWRC), Darcy's linear flow law, and Gardner's hydraulic conductivity function. The developed technique is flexible enough to consider the effects of seasonal changes and groundwater table (GWT) fluctuations.

The proposed modified UB-FELA technique is used to analyze the strip footing by considering the steady-state as well as transient water flow in the vadose zone.

A detailed investigation is carried out to estimate the UBC of strip footing by varying the mechanical strengths, unsaturated soil properties, GWT depths, and climatic conditions. The numerical solutions have been presented in terms of the bearing capacity factors with respect to the unit weight and surcharge pressure. An attempt has also been made to determine the UBC of strip footings placed over unsaturated sandy soil and subjected to different eccentric and inclined loads. By considering a wide range of soil strength and SWRC model parameters, a rigorous parametric study is carried out, and a series of UBC charts are prepared to represent the combined effect of GWT, hydromechanical soil properties, loading position, and load orientation on the strength characteristics of the strip foundation. It is well observed that if the load is placed anywhere but along the central axis of the footing, the bearing capacity reduces significantly. The analysis reveals the existence of a specific GWT position, especially for the sands having a larger capillary saturation zone, at which the load-withstand capacity of the soil becomes maximum.

In the last part of the thesis, a series of numerical analyses with the help of the developed UB-FELA formulation is performed to examine the effect of transient infiltration on the UBC of strip footing. The vertical flow within the vadose zone is realized by considering Richards one-dimensional transient flow equation. The temporal UBC profiles are plotted for different air entry values, infiltration rates, and GWT positions. The temporal profiles of suction stress are used to describe the time-dependent variation of UBC.

Failure patterns (for LB-FELA) and nodal velocity contours (for UB-FELA) are illustrated, adding valuable insights and further corroborating the numerical findings.

The LB-FELA and UB-FELA formulations provided in the thesis can be further used to solve other axisymmetric saturated and plane-strain unsaturated problems. The design charts provided in this thesis may be useful to the design engineers.