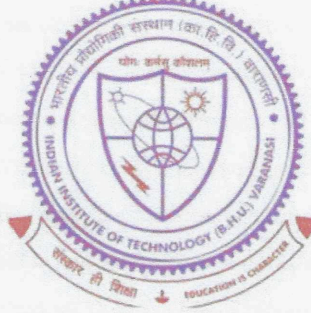


EXTENDED ABSTRACT

MATHEMATICAL MODELING ON THERMOELASTICITY AND PORO-THERMOELASTICITY



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EXTENDED ABSTRACT

'Thermoelasticity' is a branch of engineering and physics concerned with the combined behavior of materials subject to both thermal and mechanical loads. It investigates how temperature affects the ability of a solid to deform and vice versa. In many engineering applications, thermoelasticity is crucial, such as the design of components for high-temperature environments and comprehension of the behavior of materials in nuclear power plants, aerospace engineering, and other industries with high thermal fluctuations. Thermoelastic effects are taken into account in seismology and earthquake engineering to comprehend how temperature fluctuations might affect the stress distribution and mechanical characteristics of geological materials. Another application can be found in medical treatment, specifically in the monitoring of treatment responses, cancer treatment, and other medical procedures that include the controlled heating of tissues. A comprehensive understanding of temperature and stress distributions inside biological tissue is required to deliver therapeutic care of the highest caliber in thermal treatments. The thermoelasticity theory is also the basis for numerous other disciplines of mechanics and physics, including porothermoelasticity, viscothermoelasticity, pizothermoelasticity, magneto-thermoelasticity, and many other sub-branches that have attracted the interest of researchers. The basic laws on which the thermoelasticity theory has been established are the equation of motion, the constitutive laws, and the compatibility equations. Biot (1956) gave the first complete description of thermoelasticity theory in order to look at various thermoelastic problems. In this theory that is based on Fourier's law of heat conduction, the energy conservation equation is parabolic, while the equation of motion is a hyperbolic type partial differential equation. A physically improbable behavior is produced by this heat conduction equation, that of thermal wave propagating at infinite speed. Additionally, it is reported in the literature (see Chandrasekharaiah (1998) and references therein) that Biot's theory results in unfavorable outcomes of the problems involving temperature close to absolute zero, high heat flux, extreme thermal gradients, and short-time behavior, as in laser-material interactions. Therefore, during the past few decades, significant efforts have been made to better grasp the limitations of classical thermoelasticity theory.

The generalized thermoelasticity theories that were presented by Lord and Shulman (1967) and Green and Lindsay (1972) are two examples of well-established ideas that overcome the drawback of Biot's theory and admit finite speed of thermal wave. Lord-Shulman (LS) theory proposes a modified thermoelasticity theory based on Cattaneo-Vernotte law (1958; 1958; 1961) of heat conduction that introduces a single thermal relaxation time resulting in changes to Fourier's law, whereas the Green-Lindsay (GL) theory proposes an alternative theory of thermoelasticity by introducing a pair of relaxation times. In this framework, the constitutive relations are modified with the inclusion of temperature-rate terms and this GL theory is also known as the temperature-rate dependent thermoelasticity theory. In the 1990s, Green and Naghdi (1991-1993) suggested three different

thermoelastic theories based on continuum mechanics axioms, with the key variation being the type of heat conduction they proposed. The linearized version of the first model (GN-I) of Green-Naghdi theory resembles classical theory and the second one (GN-II theory), also called as thermoelasticity theory without energy dissipation, predicts finite speed propagation of an undamped thermal wave. However, the third version of Green-Naghdi theory (GN-III theory) is the most general one from which the GN-I and GN-II models can be extracted as special cases. The exponential decay of solutions can be obtained in this case, however, this theory also predicts the instantaneous propagation of thermal waves (see Puri and Jordan (2004)) and has the same drawback as the conventional theory. Recently, Quintanilla (2019) used the Moore-Gibson-Thompson (MGT) heat conduction equation to come up with a new theory of thermoelasticity. This theoretical framework is a generalization of GN-III theory and has garnered significant interest among researchers. Furthermore, researchers have extended these thermoelastic models in various contexts like the two-temperature thermoelastic model, porothermoelastic model, bio-heat transfer models, etc.

The present thesis is devoted to the study of some generalized thermoelastic and porothermoelastic models, focusing on analysis of a variety of their characteristics. The focus of the thesis lies in the analysis of three distinct theories within the field of thermoelasticity. These theories comprise the Moore-Gibson-Thompson (MGT) thermoelasticity theory, the modified temperature-rate dependent two-temperature (MTRDTT) thermoelasticity theory, and the generalized porothermoelasticity theory. Thus, it is separated into three segments on this premise. The first part examines the MGT model, encompassing Chapter 2 and Chapter 3, while the second part consists of Chapter 4, which provides a detailed elaboration of the MTRDTT thermoelasticity theory. Further, the porothermoelastic model is examined in a variety of contexts within Chapter 6, which is the last part of the thesis. The objective of this thesis is to conduct a comprehensive theoretical and numerical analysis of these thermoelastic models and to explore their applications in several disciplines.

The thesis is composed of Six chapters and is structured as follows:

Chapter 1 introduces the subject of the thesis. It provides an overview of the advancement of several generalized thermoelasticity theories and subsequently proceeds to perform an extensive review of literature related to the present thesis. The objective of the thesis is presented at the end of this chapter.

Chapter 2 presents a theoretical discussion of the Moore-Gibson-Thompson thermoelastic model and derives the domain of influence theorems in different contexts. This theorem implies that a solution of a given system under MGT theory vanishes outside the bounded domain for a prescribed bounded support of thermo-mechanical loading and for a finite time. This chapter is divided into two subchapters. In **Subchapter 2.1**, domain of influence results are obtained for potential-temperature disturbance under the MGT thermoelasticity theory. An identity regarding the potential of displacement and temperature is established in the context of a mixed initial-boundary value problem. Then, the domain of influence theorem has been derived based on this

identity by considering certain conditions to be satisfied by the material parameters. In continuation, **Subchapter 2.2** describes a thermoelastic process corresponding to the natural stress-heat-flux problem in the context of MGT model and aims to prove the domain of influence theorem for the problem. The fundamental equations for an isotropic and homogeneous medium are represented in terms of stress and heat flux to obtain the results. Similar to the subchapter 2.1, an energy identity is first presented and then the domain of influence theorem regarding this identity is established. This chapter establishes the finite propagation of thermoelastic disturbance under the MGT theory and analyzes that the propagation speed depends on the parameters of thermoelastic material. It is further shown that the domain of influence results for the present contexts reduce to the domain of influence result derived in LS thermoelasticity theory under some conditions.

Chapter 3 comprises two subchapters that focus on further investigations of the MGT thermoelastic model. **Subchapter 3.1** examines harmonic plane wave propagation inside an isotropic, unbounded, and homogeneous thermoelastic medium under the MGT thermoelasticity theory. The asymptotic expressions of various important characteristics of the wave fields such as phase velocity, specific loss, and penetration depth for elastic wave as well as thermal mode wave are obtained for limiting cases of very high and low frequency values. Some important observations about the prediction of the MGT model in comparison to the other existing models, namely LS, GN-III, and Biot models are highlighted. Furthermore, the effects of material parameters on the propagation of waves are analyzed in a detailed manner. It is observed that the effect of rate of the thermal conductivity on wave fields is significant for a small range of frequency values and the effect of thermal relaxation parameter on wave components is prominent when frequency increases. The results predicted here are in complete agreement with the domain of influence results as obtained in chapter 2. **Subchapter 3.2** completes the discussion on MGT thermoelastic model by applying it to address a problem of biological tissue in the presence of superficial cancer. In order to investigate thermo-mechanical interactions in biological tissue caused by laser irradiation, a numerical approach based on the Legendre wavelet method is proposed. A thorough parametric analysis is carried out to examine the effects of some material and geometrical parameters on the behavior of various fields such as the displacement, temperature, stress, and the tumor-normal tissue interface. The outcomes are further compared to the predictions of other existing models. By observing the non-zero values of all field variables within a finite region and their disappearance beyond the tumor-tissue interface point, the MGT thermoelastic model is further shown to predict the finite speed of thermoelastic wave propagation.

Chapter 4 is devoted to the second part of the thesis consisting of two subchapters that deal with the modified temperature-rate dependent two-temperature (MTRDTT) thermoelasticity theory (2019). This thermoelastic model is a generalization of the Green-Lindsay model. **Subchapter 4.1** aims to establish some theoretical results of the MTRDTT thermoelasticity theory. The basic governing equations of this model are considered for the homogeneous and anisotropic material. Further, an alternative formulation of a mixed initial-boundary

value problem is presented incorporating initial data into the field equations. Then, the variational principle of the convolution type is established by using this formulation. Lastly, a reciprocal principle is derived in the present context. **Subchapter 4.2** discusses the thermoelastic interactions in a linear, isotropic, and homogeneous unbounded solid resulting from a continuous line heat source. The problem is studied with the unified version of the two-temperature relation to compare the results for displacement, temperatures, and stresses in the MTRDTT model with the corresponding results of the two-temperature Green-Lindsay (TTGL) model. By using Laplace and Hankel transforms, explicit expressions for these field variables are obtained for the short-time approximated solution of the problem. Further, some important observations about these two models are highlighted. Also, the two-temperature parameter is observed to have a prominent effect on all the field variables. It is further observed that unlike the GL model that predicts finite speed for elastic as well as for thermal waves, the temperature-rate dependent two-temperature models do not admit finite wave speed of the thermal wave.

Chapter 5 deals with the last part of the thesis, which analyzes the porothermoelasticity theory (LSPTE) (2007) that is derived in the frame of Lord-Shulman theory. Two subchapters are presented in this chapter. **Subchapter 5.1** provides some theoretical findings for the LS porothermoelasticity theory to better comprehend the theory and to lay the groundwork for the creation of various numerical approaches. Similar to subchapter 4.1, a convolution-type variational principle is derived by taking into account the alternative characterization of the problem. This type of variational principle is helpful because it finds all the governing equations as well as the initial conditions and boundary conditions. Further, the continuous dependence result of the solution on initial data and external supply terms (heat source and body force) is established in this chapter. **Subchapter 5.2** proposes a numerical method based on the Legendre wavelet for the thermo-mechanical coupling analysis of poroelastic material considering temperature-dependent thermal conductivity. In the proposed method, after linearization, the time domain is discretized by using a finite-difference approach, and subsequently space domain is approximated with the use of Legendre wavelets. The method is finally implemented to obtain the numerical solution of an initial boundary value problem of porothermoelasticity for a suitable material. The behavior of displacement, temperature, and stress of solid and fluid phases are demonstrated. Lastly, the impacts of porosity, relaxation parameters, and variable thermal conductivity are examined in detail from the numerical results of the problem. The outcomes demonstrate that the aforementioned parameters significantly influence the field variables. Computer simulations reveal that the proposed method is computationally efficient, quick, dependable, and relatively good even with the small number of collocation points.

Chapter 6 summarizes the work carried out in the present thesis and provides the scope for further research in the related areas. The references cited in the thesis are appended at the end of the thesis.

The above mentioned results forming the subject matter of the thesis led to the following publications:

1. **Komal Jangid** and Santwana Mukhopadhyay. "A domain of influence theorem under MGT thermoelasticity theory." *Mathematics and Mechanics of Solids* 26(2), 285-295 (2021)
2. **Komal Jangid** and Santwana Mukhopadhyay. "A domain of influence theorem for a natural stress–heat-flux problem in the Moore–Gibson–Thompson thermoelasticity theory." *Acta Mechanica* 232, 177-187 (2021)
3. **Komal Jangid** and Santwana Mukhopadhyay. "On propagation of harmonic plane waves under the Moore- Gibson-Thompson thermoelasticity theory." *Waves in Random and Complex Media*, 1-24 (2021)
4. **Komal Jangid** and Santwana Mukhopadhyay. "Legendre wavelet collocation method for investigating thermo-mechanical responses on biological tissue during laser irradiation." (Under review in an International Journal)
5. **Komal Jangid** and Santwana Mukhopadhyay. "Variational and reciprocal principles on the temperature-rate dependent two-temperature thermoelasticity theory." *Journal of Thermal Stresses* 43(7), 816-828 (2020)
6. **Komal Jangid** and Santwana Mukhopadhyay. "Thermoelastic interactions on temperature-rate-dependent two-temperature thermoelasticity in an infinite medium subjected to a line heat source." *Zeitschrift für angewandte Mathematik und Physik* 73, 196 (2022)
7. **Komal Jangid** and Santwana Mukhopadhyay. "Variational principle and continuous dependence results on the generalized poro-thermoelasticity theory with one relaxation parameter." *Continuum Mechanics and Thermodynamics* 34, 867-881 (2022)
8. **Komal Jangid** and Santwana Mukhopadhyay. "Application of Legendre wavelet collocation method to the analysis of poro-thermoelastic coupling with variable thermal conductivity." *Computers and Mathematics with Applications* 146, 1-11 (2023)

Publications Apart from Thesis:

1. Manushi Gupta, **Komal Jangid**, and Santwana Mukhopadhyay. "Domain of influence results of dual-phase-lag thermoelasticity theory for natural stress–heat-flux problem." *Zeitschrift für angewandte Mathematik und Physik* 73(4), 169 (2022)
2. Bhagwan Singh, **Komal Jangid**, and Santwana Mukhopadhyay. "Legendre wavelet method on size dependent bending analysis of nanobeam under nonlocal strain gradient theory." (Under review in an International Journal)

Conferences and Workshops:

1. Attended an Online workshop on “**Recent Trends in Modelling & its applications**” held at PG college Gazipur during September 04-08, 2022.
2. Participated in **International Conference (Online) on dynamical systems, control and their applications** held at IIT Roorkee during July 01-03, 2022 and presented a work with the title “A domain of influence theorem under MGT thermoelasticity theory.”
3. Participated in **International Conference of CMSE (Online)** held at BITS-Pilani, Hyderabad during April 22-24, 2022 and presented a work with the title “A domain of influence theorem for a natural stress-heat-flux problem in the Moore-Gibson-Thompson thermoelasticity theory.”
4. Attended an Online workshop on “**Recent Development in Mathematical Modelling in Engineering Sciences**” held at NIT Uttarakhand during December 27-31, 2021.

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