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6.2 Comparison with existing MDS EAQECCs 150

List of Symbols

Symbol	Description
p	Prime number
q	Prime power
\mathbb{F}_q	Finite field of order q
\mathbb{F}_q^*	Set of nonzero elements of \mathbb{F}_q
J	$\{1, 2, 3, 4, 5, 6\}$ (used in Chapter 2)
\mathcal{S}	$\mathbb{F}_q[u, v, w]/\langle u^3 - u, v^2 - v, w^2 - w, uv, vu, uw, wu, vw - wv \rangle$
\mathcal{R}	$\mathbb{F}_q[u_1, u_2, \dots, u_r]/\langle u_i^3 - u_i, u_i u_j - u_j u_i \rangle$
\mathcal{T}	$\mathbb{F}_q[u_1, u_2, \dots, u_r]/\langle f_j(u_j), u_i u_j - u_j u_i \rangle$
$\mathcal{U}(R)$	Set of units (whose multiplicative inverse exist) in the ring R
\mathcal{C}	Codes over \mathbb{F}_q (except for the codes obtained by decomposition of codes over rings)
\mathcal{C}	Codes over rings
$\langle f(y) \rangle$	Ideal generated by $f(y)$
$f^*(y)$	Reciprocal polynomial of $f(y)$
$f^\dagger(y)$	Left monic skew reciprocal polynomial of $f(y)$
$\bar{f}^\dagger(y)$	Hermitian left monic skew reciprocal polynomial of $f(y)$
\mathcal{C}	Code over \mathbb{F}_q
\mathcal{C}	Code over a ring
$a \mid b$	a divides b
\equiv	Congruent to
θ	An automorphism of \mathbb{F}_q
Θ	An automorphism of ring R , where $R = \mathcal{S}, \mathcal{R}, \mathcal{T}$
$[n, k, d]$	A classical linear code with length n , dimension k , and minimum distance d

Symbol	Description
$[[n, k, d]]$	A quantum code with length n , dimension k , and minimum distance d
$[[n, k, d; c]]$	An EAQECCs with length n , dimension k , and minimum distance d using c ebits

Abbreviations

Abbreviation	Description
AMDS	Almost Maximum Distance Separable
BKLC	Best Known Linear Code
CSS	Calderbank-Shor-Steane
EAQECC	Entanglement-Assisted Quantum Error-Correcting Code
GCRD	Greatest Common Right Divisor
LCD	Linear Complementary Dual
MDS	Maximum Distance Separable
QECC	Quantum Error-Correcting Code

