# CHAPTER 6

# STATISTICAL MODELLING OF TRIBOLOGICAL PARAMETERS USING RESPONSE SURFACE METHODOLOGY

## 6.1 INTRODUCTION

The tribological properties of a material are influenced by various factors, and investigating the impact of each factor in combination with others can result in numerous unnecessary experiments. Thus, statistical modelling serves as a valuable tool that not only eliminates redundancy during experimentation but also aids in optimizing the objective function.

With above view in mind, we employ a statistical technique as Response Surface Methodology (RSM) to predict and optimize the tribological properties (wear rate and coefficient of friction (COF)) in both dry and lubricating sliding conditions for the TiC reinforced composites. Response Surface Methodology (RSM) serves as a statistical approach to fine-tune the process parameters, ensuring an efficient and effective optimization of the responses [Vettivel et al., 2013]. In this study, Design Expert-13 software package was used for RSM and the input factors were applied load, sliding distance, and the weight percentage of TiC reinforcement. The output response and input variables are mathematically correlated using response surface methodology (RSM). The desirability function method of the developed model was used to optimize the input variables for wear rate and COF. Further a series of confirmation tests were run with the optimized input parameters in order to verify the estimated wear rate and COF.

## 6.2 STATISTICAL MODELLING OF WEAR RATE AND COF IN DRY SLIDING CONDITION

### 6.2.1 Central composite design (CCD)

Central Composite Design (CCD) of experiments on Design Expert-13 software was employed to study the influence of input factors on wear rate and COF. Three factors with three level of variables were used in present investigation as outlined in Table 6.1. The Table 6.2 present the design matrix of 20 iterations for input factors at three different levels using the central composite design (CCD) technique for determining wear rate and coefficient of friction (COF) of composites. The ANOVA test was also run for the constructed model in order to specify the significance of the model for both the responses wear rate and COF, respectively.

**Table 6.1** Input factors and levels for the composites under dry sliding condition

| **Factor** | **Unit** | **-α** | **+α** | **Low (-1)** | **High (+1)** |
|---|---|---|---|---|---|
| A: Load | N | 20 | 40 | 20 | 40 |
| B: Sliding distance | M | 2000 | 4000 | 2000 | 4000 |
| C: TiC content | Weight % | 1.5 | 4.5 | 1.5 | 4.5 |

**Table 6.2** Design of experiment with input factors and responses

| **Std** | **Run** | **Factor 1** | **Factor 2** | **Factor 3** | **Response 1** | **Response 2** |
|---|---|---|---|---|---|---|
| | | **A: Load**<br>**N** | **B: Sliding distance**<br>**m** | **C: TiC**<br>**wt. %** | **Wear rate**<br>**$\times 10^{-4}$ mm$^3$.m$^{-1}$** | **COF** |
| 13 | 1 | 30 | 3000 | 1.5 | 2.28 | 0.478 |
| 6 | 2 | 40 | 2000 | 4.5 | 1.93 | 0.567 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 5 | 3 | 20 | 2000 | 4.5 | 1.1 | 0.44 |
| 1 | 4 | 20 | 2000 | 1.5 | 1.67 | 0.374 |
| 9 | 5 | 20 | 3000 | 3 | 1.15 | 0.431 |
| 8 | 6 | 40 | 4000 | 4.5 | 1.67 | 0.615 |
| 10 | 7 | 40 | 3000 | 3 | 2.45 | 0.547 |
| 3 | 8 | 20 | 4000 | 1.5 | 1.32 | 0.411 |
| 19 | 9 | 30 | 3000 | 3 | 1.93 | 0.486 |
| 15 | 10 | 30 | 3000 | 3 | 1.94 | 0.494 |
| 7 | 11 | 20 | 4000 | 4.5 | 0.78 | 0.492 |
| 20 | 12 | 30 | 3000 | 3 | 1.92 | 0.487 |
| 2 | 13 | 40 | 2000 | 1.5 | 3.17 | 0.511 |
| 4 | 14 | 40 | 4000 | 1.5 | 2.91 | 0.551 |
| 11 | 15 | 30 | 2000 | 3 | 1.95 | 0.472 |
| 12 | 16 | 30 | 4000 | 3 | 1.75 | 0.513 |
| 17 | 17 | 30 | 3000 | 3 | 1.83 | 0.493 |
| 14 | 18 | 30 | 3000 | 4.5 | 1.2 | 0.532 |
| 18 | 19 | 30 | 3000 | 3 | 1.82 | 0.494 |
| 16 | 20 | 30 | 3000 | 3 | 1.83 | 0.487 |

### 6.2.2 Quadratic model and analysis of variance

Response Surface Methodology (RSM) was employed to assess experimental data with the aim of building a statistical model for optimizing the responses. The analysis involved examining linear, two-factor interaction (2FI), and quadratic models to determine the mathematical relationships between the input factors and the corresponding output responses. However, a quadratic model was readily suggested the developed model.

Analysis of variance (ANOVA) was utilized to assess the effectiveness of the developed model, while the determination coefficient ($R^2$) was found to gauge the accuracy and level of fit of the model [Radhika et al., 2015]. A high $R^2$ value, approaching 1, along with closely aligned adjusted $R^2$ and predicted $R^2$ values, gives a strong indication that the developed model appropriately fits the experimental results.

**Table 6.3** ANOVA for optimisation of wear rate

| Source | $d_f$ | Sum of Squares | Mean Square | F-value | p-value | |
|---|---|---|---|---|---|---|
| **Model** | 9 | 6.40 | 0.7112 | 134.54 | < 0.0001 | **significant** |
| A-Load | 1.0 | 3.82 | 3.82 | 722.54 | < 0.0001 | |
| B-Sliding distance | 1.0 | 0.1877 | 0.1877 | 35.51 | < 0.0001 | |
| C-TiC | 1.0 | 2.12 | 2.12 | 400.32 | < 0.0001 | |
| AB | 1.0 | 0.0008 | 0.0008 | 0.1513 | 0.7054 | |
| AC | 1.0 | 0.2592 | 0.2592 | 49.04 | < 0.0001 | |
| BC | 1.0 | 0.0002 | 0.0002 | 0.0378 | 0.8497 | |
| $A^2$ | 1.0 | 0.0001 | 0.0001 | 0.0155 | 0.9035 | |
| $B^2$ | 1.0 | 0.0011 | 0.0011 | 0.1988 | 0.6652 | |
| $C^2$ | 1.0 | 0.0118 | 0.0118 | 2.23 | 0.1663 | |
| Residual | 10.0 | 0.0529 | 0.0053 | | | |
| **Lack of Fit** | 5.0 | 0.0375 | 0.0075 | 2.43 | 0.1758 | **not significant** |
| Pure Error | 5.0 | 0.0154 | 0.0031 | | | |
| Cor Total | 19.0 | 6.45 | | | | |

Table 6.3 presents the ANOVA results for optimization of the wear rate concerning the input factors. The table demonstrates the significance of the model through the inclusion of the p-value. A p-value less than 0.05 shows significance of the developed statistical model as illustrated in the ANOVA table. It means the probability for prediction of wear rate by the developed model has 99% confidence level. The factors which are having p-value less than 0.05 are significant model factors that majorly contribute in the calculation of wear rate. The factors A, B and C i.e., load, sliding distance and wt.% of TiC and AC are significant factors that affect the wear rate since their p-value is lower than 0.0001 ($p\text{-value}<0.05$). Also, the model F-value is 134.54 which means that there is 0.01% chance that such a large F value could occur due to noise. However, the higher F-value suggests the higher contribution of a factor in wear rate calculation. Load (A) is

having maximum F- value as 722.54 implying that it is the dominant factor for wear rate followed by TiC (C): 400.32, Load* TiC (AC): 49.04 and Sliding distance (B): 35.51, respectively. The ‘Lack of Fit’ was found to be not significant which is good as we want the model to fit. Thus, it implies that model can be successfully applied for prediction and optimization of the wear rate response. From Table 6.4, it was suggested that model is quadratic and value of $R^2$= 99.18% and the adjusted $R^2$= 98.44% implied that developed model is consistent with less than 2% of the total variance. Their high values meant a greater relation between the wear rate’s actual and predicted values.

**Table 6.4** Statistics model summary of wear rate

| **Source** | **Std. Dev.** | **$R^2$** | **Pred. $R^2$** | **Adj. $R^2$** | **PRESS** | |
|---|---|---|---|---|---|---|
| Linear | 0.1437 | 0.9488 | 0.8901 | 0.9392 | 0.7092 | |
| 2FI | 0.0735 | 0.9891 | 0.9760 | 0.9841 | 0.2479 | |
| **Quadratic** | **0.0727** | **0.9918** | **0.9616** | **0.9844** | **0.1548** | **Suggested** |
| Cubic | 0.0750 | 0.9948 | -2.4947 | 0.9834 | 22.55 | Aliased |

The Table 6.5 illustrates ANOVA result for COF corresponding to the input factors. The significance of model is acceptable as the p-value <0.0001 which is less than 0.05. It implies the probability for prediction of COF by the developed model has 99% confidence level. The factors having p-value less than 0.05 are significant model factors that majorly contribute in the calculation of COF. The factors A, B and C i.e., load, sliding distance and wt.% of TiC are significant factors that affect COF since their p-value is lower than 0.0001 (p-value<0.05). Also, the model F- value is 208.90 which means that there is 0.01% chance that such a large F value could occur because of noise. However, the higher F- value suggests higher contribution of a factor in COF calculation. Load (A) is having maximum F- value as 1336.17 implying it is dominant factor for COF followed by TiC (C): 340.48 and Sliding distance (B): 157.03, respectively. The ‘Lack of Fit’ was also found to be not significant which is good as we want the model to fit. Thus, it means the developed model can be effectively used for prediction and optimization of COF response.

**Table 6.5** ANOVA for optimisation of COF

| **Source** | **$d_f$** | **Sum of Squares** | **Mean Square** | **F-value** | **p-value** | |
|---|---|---|---|---|---|---|
| **Model** | 9.0 | 0.0569 | 0.0063 | 208.90 | < 0.0001 | **significant** |
| A-Load | 1.0 | 0.0413 | 0.0413 | 1366.17 | < 0.0001 | |
| B-Sliding distance | 1.0 | 0.0048 | 0.0048 | 157.03 | < 0.0001 | |
| C-TiC | 1.0 | 0.0103 | 0.0103 | 340.48 | < 0.0001 | |
| AB | 1.0 | 1.250E-07 | 1.250E-07 | 0.0041 | 0.9500 | |
| AC | 1.0 | 0.0001 | 0.0001 | 3.01 | 0.1134 | |
| BC | 1.0 | 0.0001 | 0.0001 | 2.18 | 0.1702 | |
| $A^2$ | 1.0 | 0.0001 | 0.0001 | 2.44 | 0.1493 | |
| $B^2$ | 1.0 | 7.778E-06 | 7.778E-06 | 0.2570 | 0.6232 | |
| $C^2$ | 1.0 | 0.0003 | 0.0003 | 10.63 | 0.0086 | |
| Residual | 10.0 | 0.0003 | 0.0000 | | | |
| **Lack of Fit** | 5.0 | 0.0002 | 0.0000 | 3.04 | 0.1236 | **not significant** |
| Pure Error | 5.0 | 0.0001 | 0.0000 | | | |
| Cor Total | 19.0 | 0.0572 | | | | |

Also from Table 6.6, a quadratic model is suggested and the $R^2$ value and the adjusted $R^2$ value are 0.9947 and 0.9899, respectively. They signify that developed model is good enough for optimization of COF.

**Table 6.6** Statistics model summary of COF

| Source | Std. Dev. | $R^2$ | Pred. $R^2$ | Adj. $R^2$ | PRESS | |
|---|---|---|---|---|---|---|
| Linear | 0.0071 | 0.9860 | 0.9777 | 0.9834 | 0.0013 | |
| 2FI | 0.0070 | 0.9888 | 0.9738 | 0.9836 | 0.0015 | |
| **Quadratic** | **0.0055** | **0.9947** | **0.9691** | **0.9899** | **0.0018** | **Suggested** |
| Cubic | 0.0046 | 0.9978 | -0.1444 | 0.9929 | 0.0655 | Aliased |

### 6.2.3 Regression equation for wear rate and COF

All the coefficients were examined for its significance using F- test on Design expert software. These coefficients were used to establish a mathematical expression to evaluate wear rate and COF. The regression equations Eqn. 6.1 and Eqn. 6.2 for the model are developed for calculating the wear rate and COF, respectively. The equations are in quadratic form and relate all input factors. These also includes the ‘square’ and ‘multiplication’ terms of individual factors. With the help of equations, wear rate and coefficient of friction values can be obtained for any value of input factors (A: Load, B: Sliding distance and C: TiC wt. %).

***Wear rate*** $= 0.172182 + 0.098073*A - 0.000274*B + 0.237879*C + 1e^{-06}*AB - 0.012*AC - 3.33333e^{-06}*BC - 0.000055A^2 + 1.95455e^{-08}*B^2 - 0.029091C^2$ *(Eqn. 6.1)*

***COF*** $= +0.146648 + 0.010252*A + 0.000027*B - 0.006448*C - 1.25e^{-08}*AB - 0.000225*AC + 1.91667e^{-06}*BC - 0.000052*A^2 - 1.68182e^{-09}*B^2 + 0.004808*C^2$ *(Eqn. 6.2)*

Figure 6.1 shows surface and contour plot (2D and 3D) for wear rate at various input variables. Figure 6.1 (a-c) illustrates surface and contour plots showing influence of load and sliding distance on wear rate of TiC reinforced composites with varying weight % of TiC content (1.5-4.5). The dots in graph represent design points. The red dots mean points lies above the predicted values while white dots mean point lies below the predicted values. The contour and 3D surface images employ distinct colors to signify the range of wear rates. Blue represents the minimum wear rate, green indicates an average wear rate, and yellowish-red signifies the maximum wear rate. The contours reveal that the behavior of wear rate is significantly influenced by the sliding distance, load, and the wt.% of TiC. It is inferred that as the load is increased a consistent rise is observed in wear rate, whereas wear rate value decreases slightly as sliding distance is more. Also, from Figs. 6.1(a-c) it is observed that as the weight percentage of TiC increases, there is a noticeable decrease in wear rate as a greater number of hard TiC particles carry the load and provide hindrance to surface deformation thereby retarding the gradual wear of the composite.

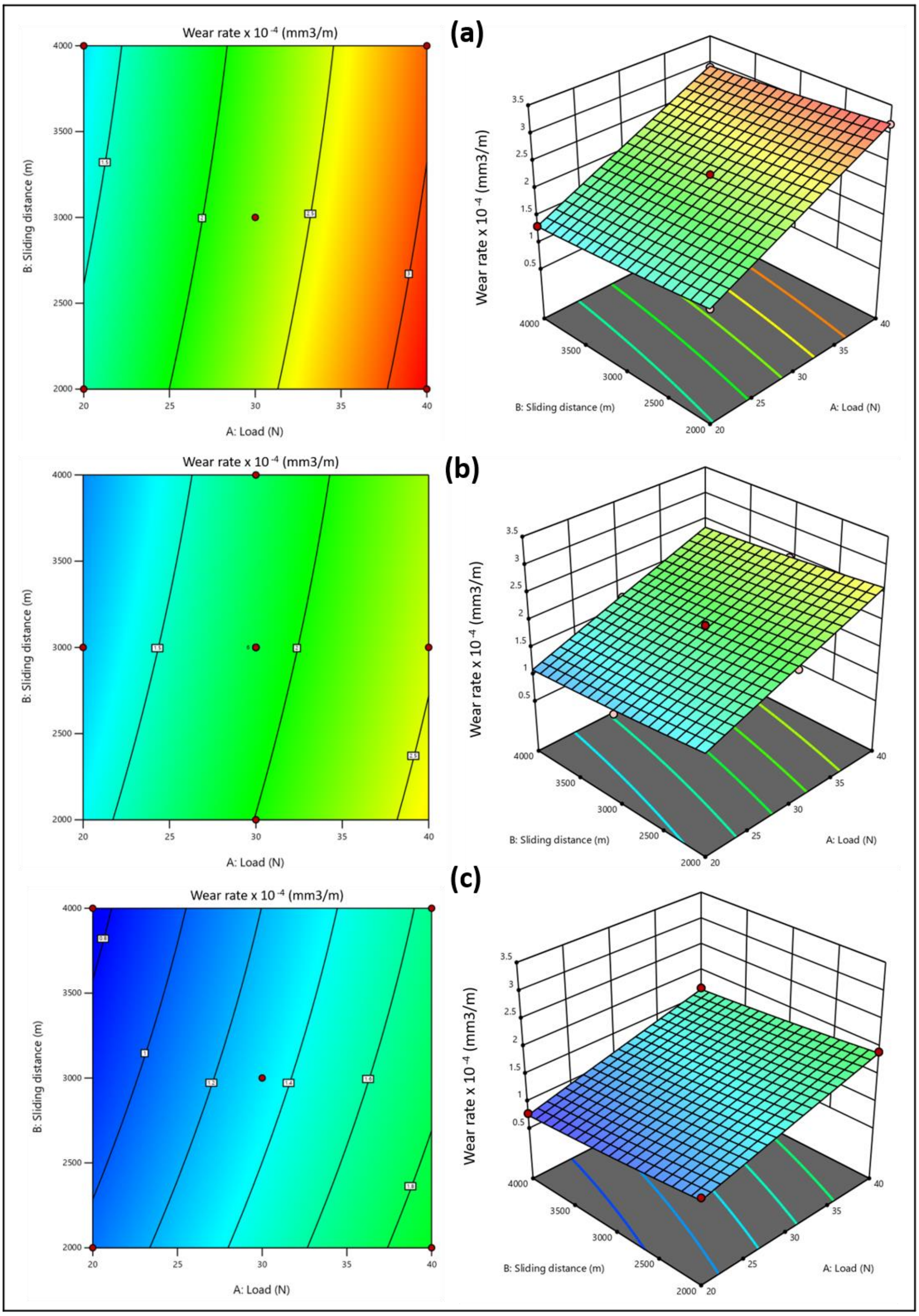

**Fig. 6.1** 2D and 3D- surface and contour plots showing wear rate variation with applied load and sliding distance for (a) T1.5 (b) T3.0 (c) T4.5

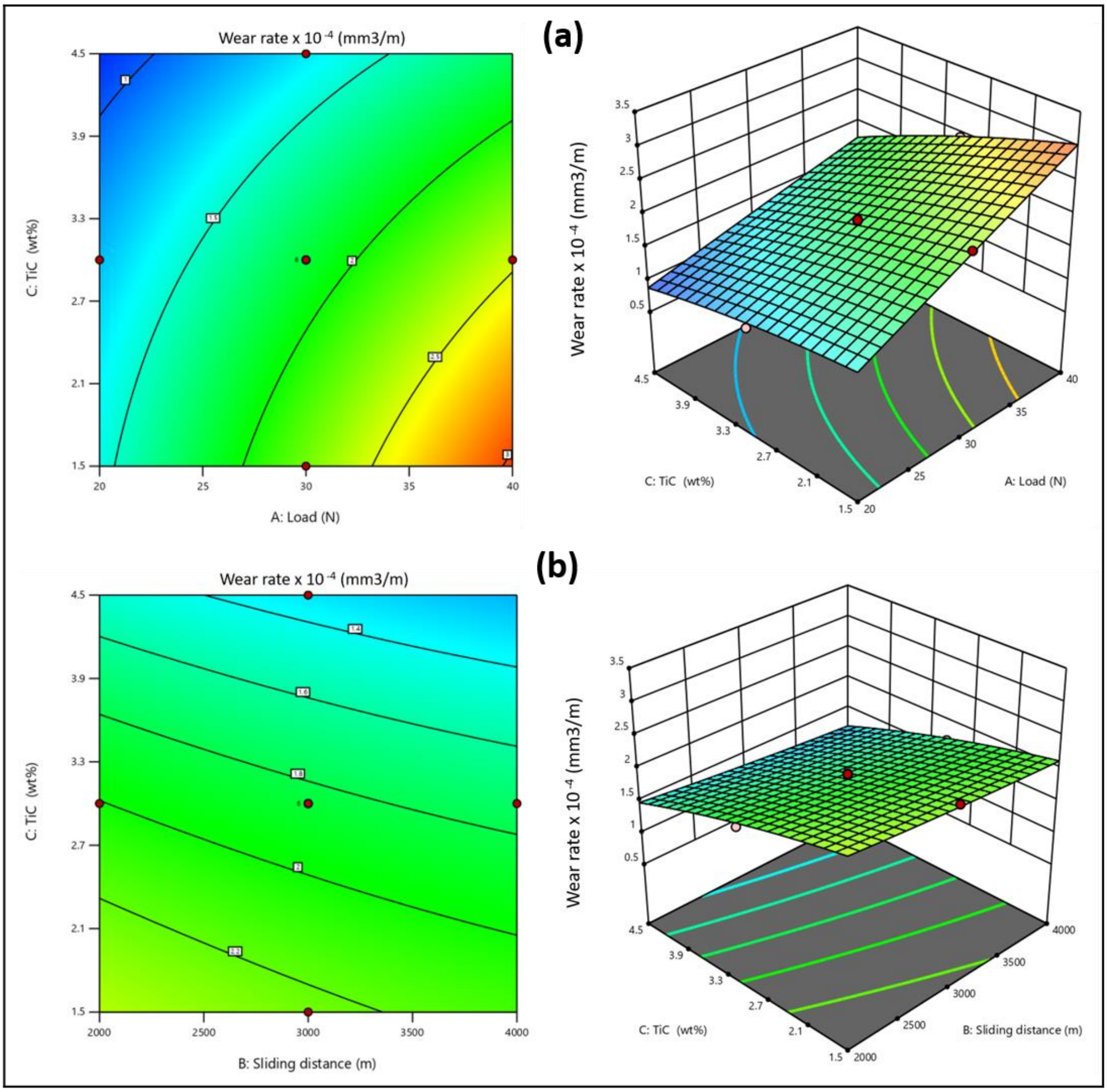

**Fig. 6.2** 2D and 3D- surface and contour plots for wear rate (a) applied load vs wt.% of TiC and (b) sliding distance vs wt.% of TiC

Figure 6.2 (a) shows the surface and contour plot demonstrating influence of load and TiC wt.% on wear rate at sliding distance of 3000 m. It was found that a considerable change in wear rate at extreme values of reinforcement and load. The surface plots depict that wear rate value falls the increase in reinforcement percentage which may be attributed to hard reinforcement particles, which act as a protective barrier for the matrix, preventing significant deformation and delamination under dry sliding conditions. It was also observed that wear rate rises with load due to more plastic deformation of the matrix.

For analysing combined influence of TiC content and sliding distance, the load was fixed at 30 N as shown in Figure 6.2 (b) A decrease in wear rate is seen with higher content of particle dispersion and with rise in sliding distance. Further minimum wear rate is observed at maximum reinforcement content and 4000 m sliding distance.

Figure 6.3 provides 2D and 3D surface and contour plot representations illustrating how the input variables influence the coefficient of friction (COF). Figure 6.3 (a-c) illustrates contour plots showing influence of load and sliding distance on COF with varying TiC wt.%. As the load and sliding distance increase, there is a corresponding elevation in the coefficient of friction (COF) value. However, as the reinforcement content is increased from 1.5 to 4.5 wt.% TiC (Fig 6.3 (a) to Fig. 6.3(c)) it is seen that there is a gradual increase in COF at the respective load and sliding distance.

Figure 6.4 (a) illustrates the surface and contour plot demonstrating the influence of load and TiC wt.% on COF at sliding distance of 3000 m. It was clearly suggested that as the TiC content rises, the coefficient of friction (COF) experiences an increase at lower applied loads. This can be attributed to the interaction of surface irregularities (asperities) and the presence of the hard reinforcement phase. However, as the applied load further increases along with the TiC particles, the COF also rises to maximum value at 4.5 wt.% TiC and 4000 m distance.

Figure 6.4 (b) shows the combined influence of TiC content and sliding distance on COF when load is constant at 30 N. The 3D surface plot illustrates a slight increase in the coefficient of friction (COF) with the increasing sliding distance and substantial rise when TiC content is more.

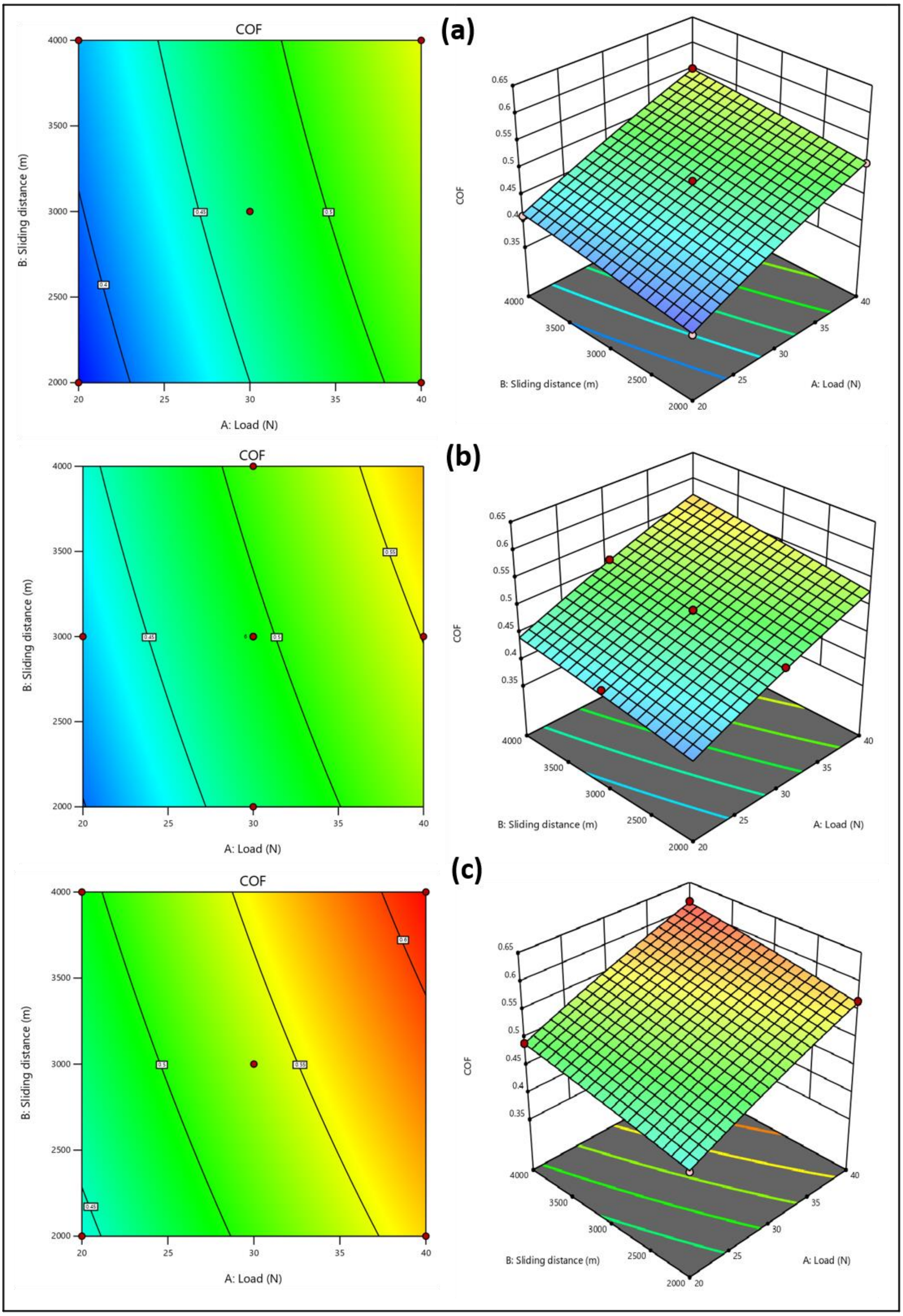

**Fig. 6.3** 2D and 3D- surface and contour plots showing COF variation with applied load and sliding distance for (a) T1.5 (b) T3.0 (c) T4.5

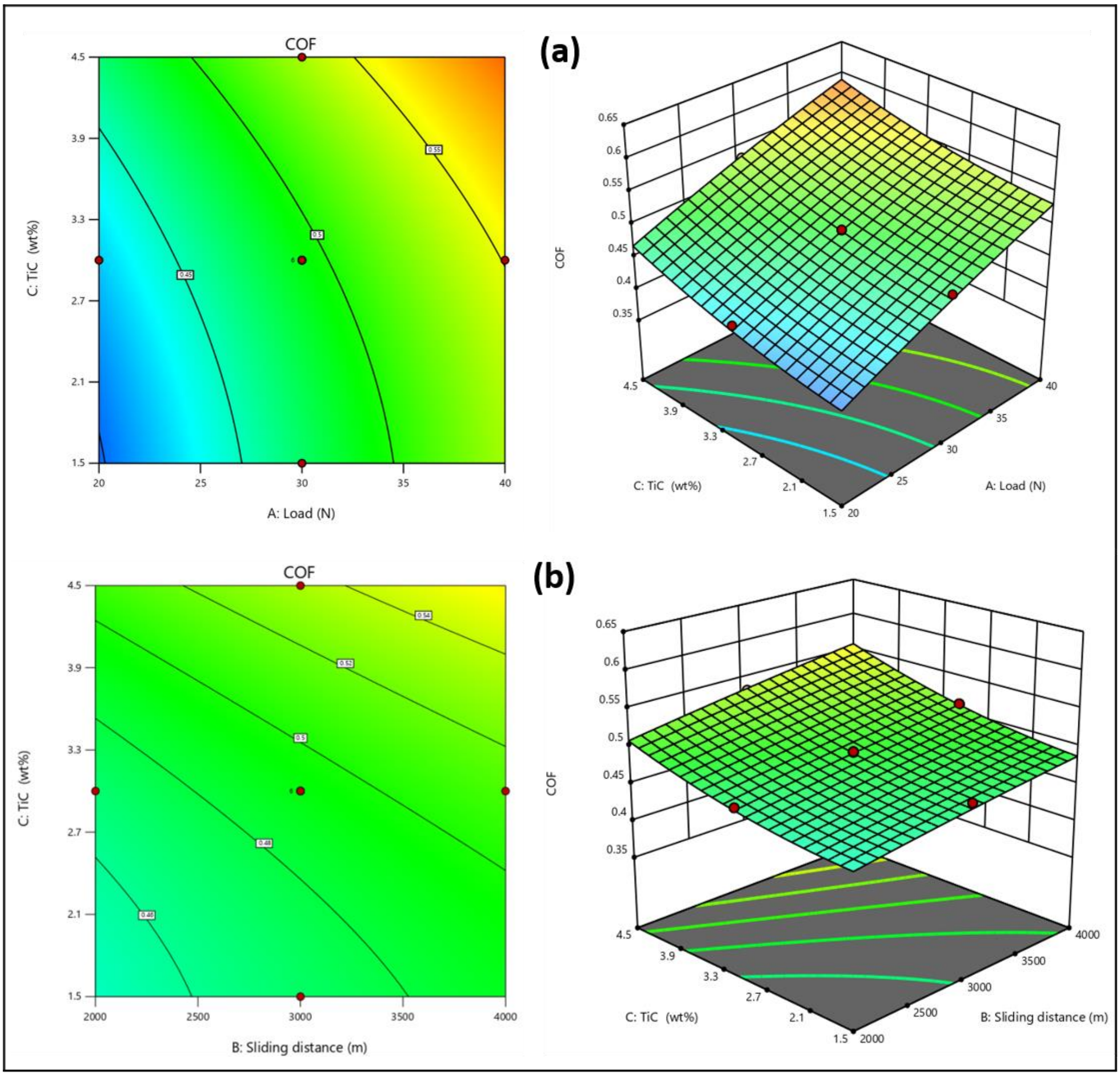

**Fig. 6.4** 2D and 3D- surface and contour plots for COF (a) applied load vs wt.% of TiC and (b) sliding distance vs wt.% of TiC

Figures 6.5 and 6.6 illustrates the relationship between the actual (experimental) values and the values predicted by the model, along with the normal probability distribution for the residuals of wear rate and coefficient of friction (COF), respectively.

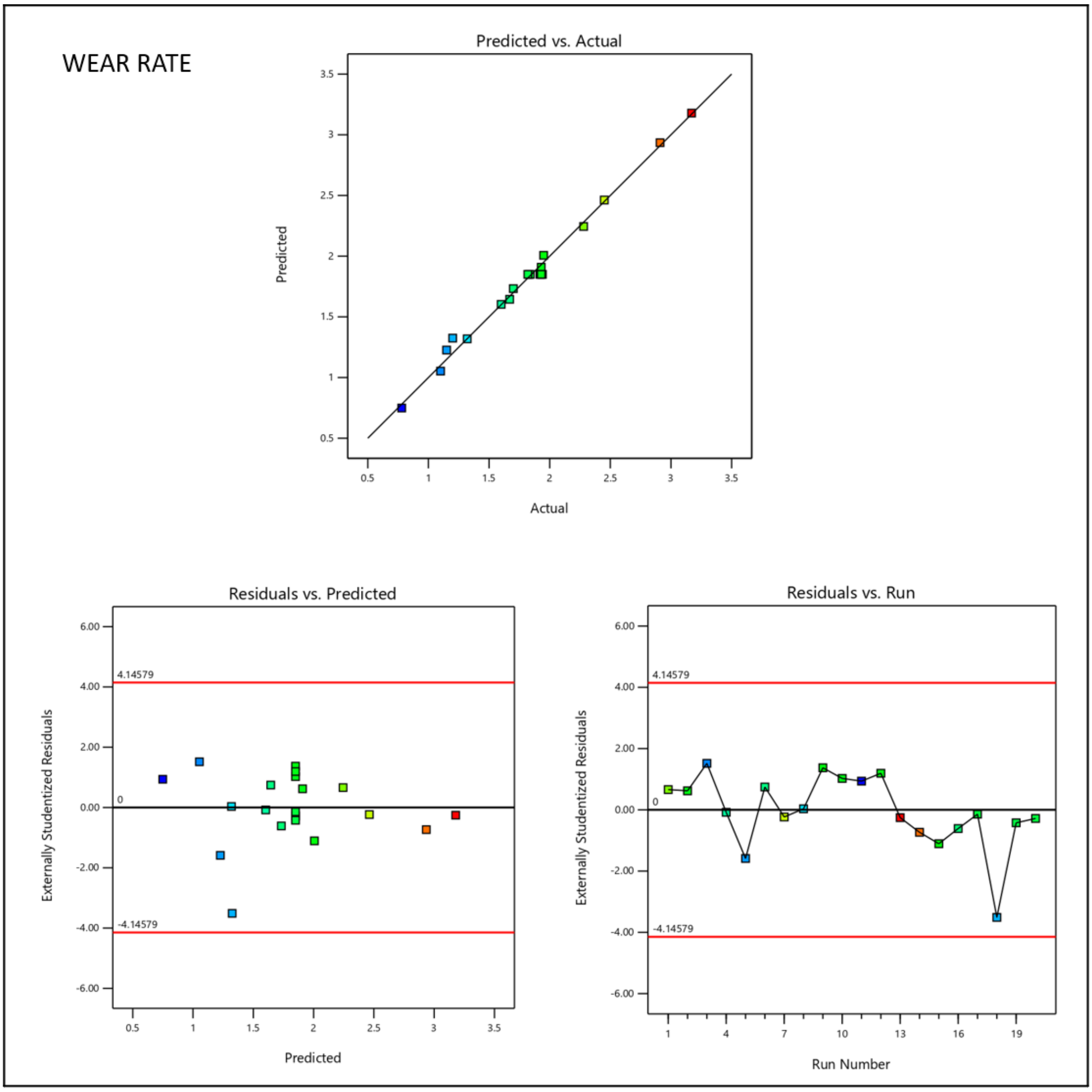

**Fig. 6.5** Variation of experimental and predicted values and normal probability for residuals of wear rate

The close proximity of the actual values with the inclined lines in the Figure 6.5 and Figure 6.6 suggests that the established model is well-suited for predicting the wear rate and coefficient of friction (COF) of the TiC reinforced composite. Additionally, the plots for residuals against run order for wear rate and COF of the composite depict that the residuals are evenly distributed within a range and exhibit a distinct correlation. This further implies that the developed model is satisfactory for describing the data.

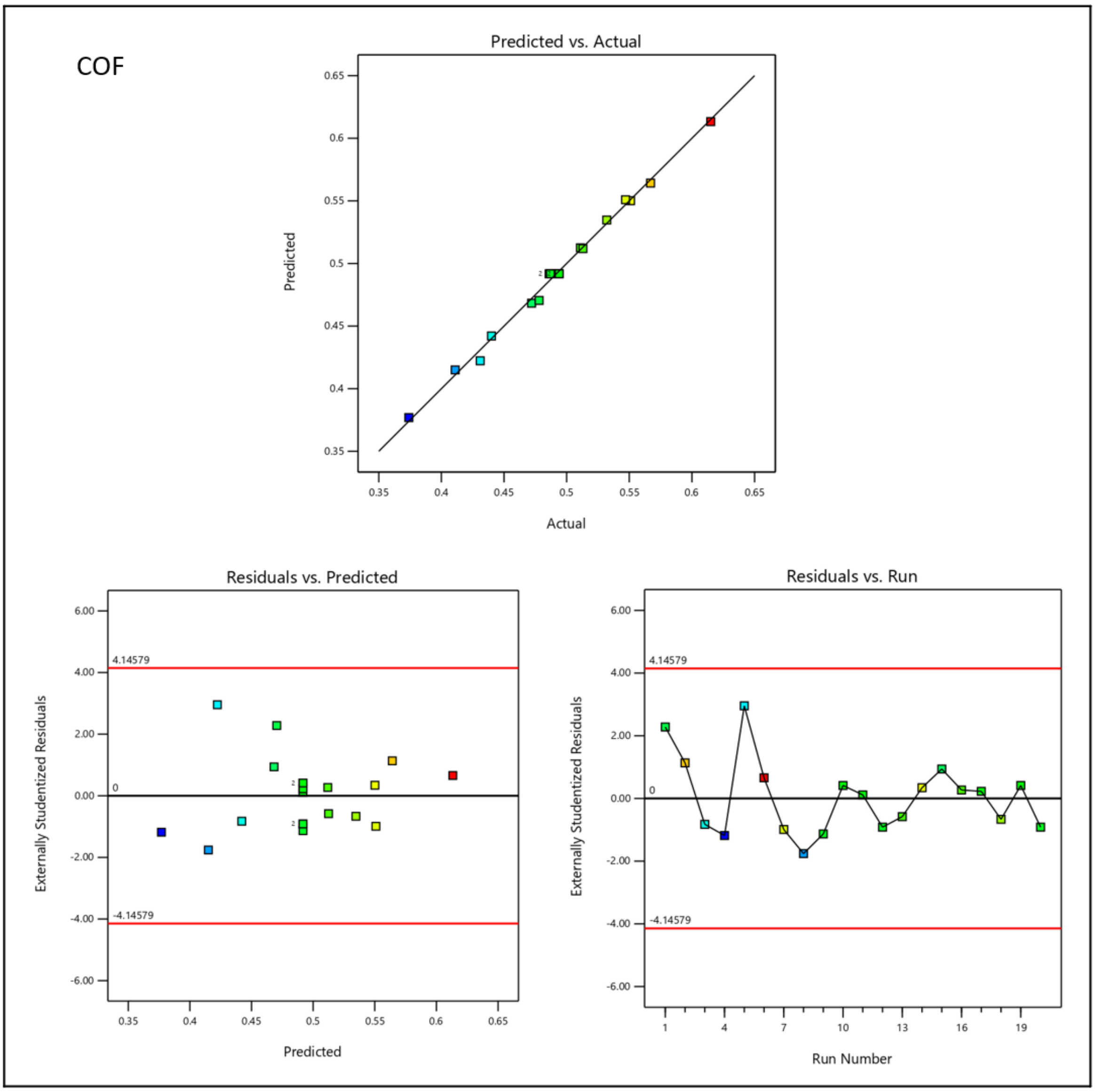

**Fig. 6.6** Variation of experimental and predicted values and normal probability for residuals of COF

Figure 6.7 (a) and (b) illustrates the perturbation plots which shows the effect of individual factors on wear rate and COF, respectively. The perturbation graph was plotted by taking a reference point. The reference point so selected was the central level of all individual factors (Load-30 N, sliding distance-3000 m and 3 wt.% TiC content). It was inferred that wear rate of the composites increases with rise in load while wear rate decreases with rise in reinforcement content and sliding distance. Also, COF of composites enhances with the rise in load, reinforcement content and with increase of sliding distance.

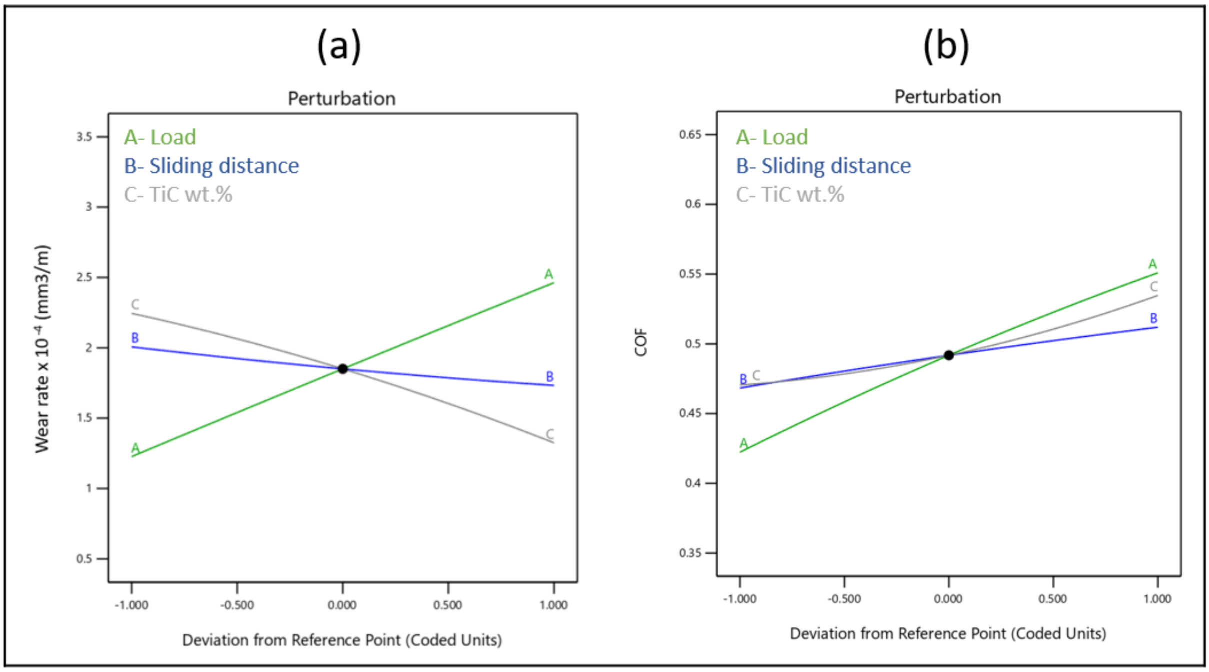

**Fig. 6.7** Influence of individual factors on wear rate and COF.

### 6.2.4 Validation of model and optimization of tribological parameters

Identifying optimal parameters is essential for the processes and minimizing the need for numerous experiments. The goal of optimization is to identify the input factors that lead to the best wear rate and coefficient of friction (COF). This becomes challenging when multiple input variables influence the response. The desirability-based approach is a widely utilized method for parameter optimization where desirability values range from 0 to 1, and a value close to 1 indicates proximity to the target. In the context of Response Surface Methodology (RSM), a range of variables was developed to optimize both wear rate and COF. Among the numerous sets of input variables, those having highest desirability (close to 1) were chosen as the optimal conditions for both the wear rate and COF as depicted in Figure 6.8.

Figure 6.8 illustrates the influence of individual factors on the wear rate and COF. These input parameters are chosen to achieve the lowest possible wear rate and coefficient of friction under these specific conditions. It implies that at specific values of input parameters (load, sliding distance, wt.% TiC), the wear rate and COF obtained are minimum. Optimized predicted values of wear rate and COF occur at input parameters when A-20 N load; B-2000 m sliding distance and C-3.08 wt.% TiC. Further, from Figure 6.8, the optimum values of wear rate and COF are $1.3781\times10^{-4}$ $mm^3/m$ and 0.4004, respectively. The relation among the input parameters at minimum load, minimum sliding distance, and central wt.% of TiC (3.08wt.%) are the most optimized parameters for minimum wear rate and COF.

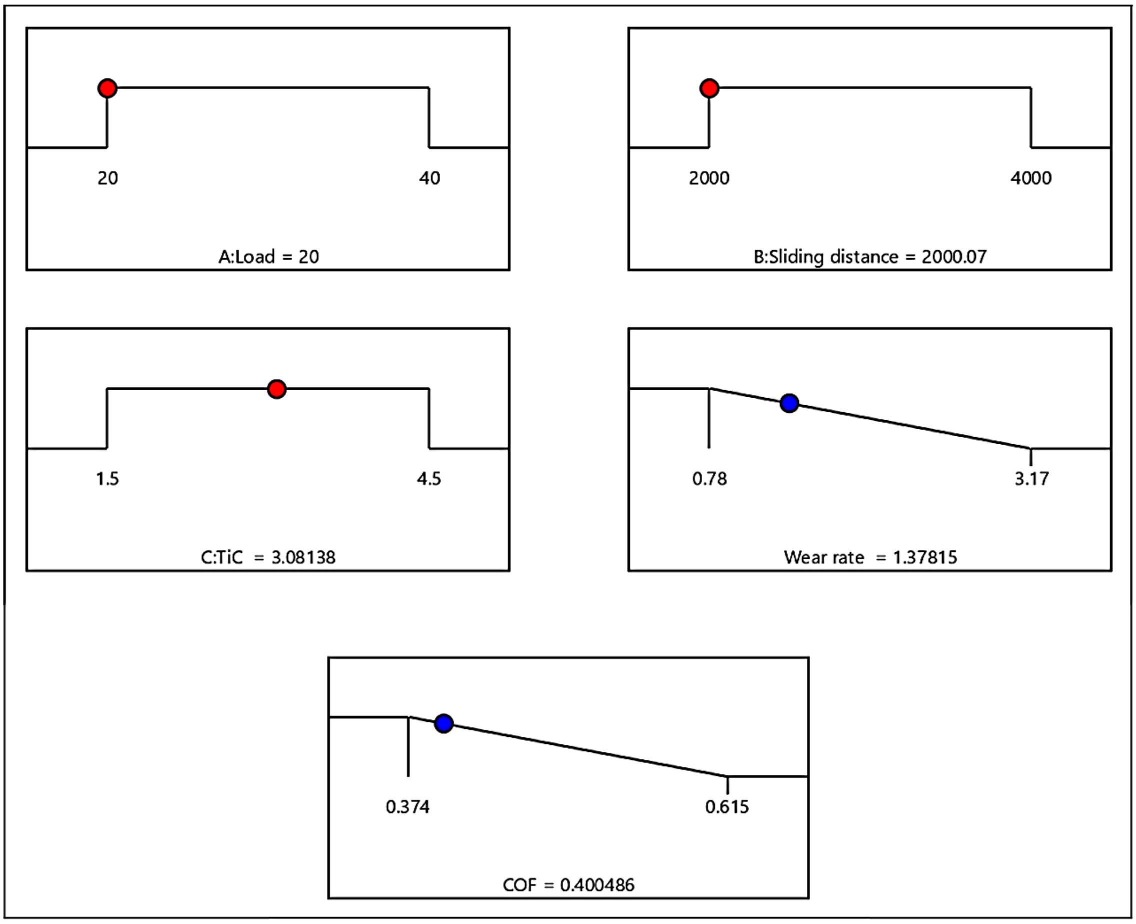

**Fig. 6.8** Influence of individual factors on wear rate and COF

The obtained experimental values and predicted values of wear rate and COF at the optimized input parameters are given in Table 6.7.

**Table 6.7** Experimental and predicted values of the model for optimal wear rate and COF

| **Parameters** | **Optimum parameter** | **Predicted value** | | **Experimental value** | | **% Error** | |
|---|---|---|---|---|---|---|---|
| | | **WR* $10^{-4}$ $mm^3/m$** | **COF** | **WR* $10^{-4}$ $mm^3/m$** | **COF** | **WR** | **COF** |
| **Load (N)** | 20 | 1.3781 | 0.4004 | 1.2714 | 0.396 | 8 | 1.1 |
| **Sliding distance (m)** | 3000 | | | | | | |
| **TiC (wt.%)** | 3.08 (~3) | | | | | | |

The comparison of experimental and predicted values of wear rate and COF depicted ~8%error and ~1.1% error, respectively. Both experimental and predicted values show close tolerance implying the developed model is significant with high efficacy. The model can be considered for prediction and optimization of tribological behaviour of the composites [Kartheesan et al., 2022].

## 6.3 STATISTICAL MODELLING OF WEAR RATE AND COF IN LUBRICATING SLIDING CONDITION

### 6.3.1 Central composite design (CCD)

Using the Design Expert-13 software, CCD of experiments was utilized to understand the impact of input factors on wear rate and COF of composites in the lubricating sliding condition. The three factors along with their levels that were employed are shown in Table 6.8.

**Table 6.8** Input factors and levels for the composites under lubricating sliding condition

| Factor | Unit | -α | +α | Low (-1) | High (+1) |
|---|---|---|---|---|---|
| A: Load | N | 30 | 70 | 30 | 70 |
| B: Sliding distance | M | 4000 | 8000 | 4000 | 8000 |
| C: TiC content | Weight % | 1.5 | 4.5 | 1.5 | 4.5 |

The Table 6.9 displays the design matrix of input factors (load, sliding distance and TiC content) with their different levels as obtained by CCD. The respective experimental results (wear rate and COF) with 20 iterations were also established. Additionally, the developed model was subjected to an ANOVA test to determine the model's significance for the responses wear rate and COF.

**Table 6.9** Design of experiment with input factors and responses

| Std | Run | Factor 1 | Factor 2 | Factor 3 | Response 1 | Response 2 |
|---|---|---|---|---|---|---|
|  |  | A: Load N | B: Sliding distance m | C: TiC wt. % | Wear rate $\times 10^{-4}$ mm$^3$.m$^{-1}$ | COF |
| 20 | 1 | 50 | 6000 | 3 | 0.96 | 0.34 |
| 14 | 2 | 50 | 6000 | 4.5 | 0.81 | 0.312 |
| 5 | 3 | 30 | 4000 | 4.5 | 0.68 | 0.24 |
| 9 | 4 | 30 | 6000 | 3 | 0.66 | 0.28 |
| 16 | 5 | 50 | 6000 | 3 | 0.98 | 0.337 |
| 6 | 6 | 70 | 4000 | 4.5 | 1.2 | 0.34 |
| 10 | 7 | 70 | 6000 | 3 | 1.27 | 0.39 |
| 18 | 8 | 50 | 6000 | 3 | 0.99 | 0.336 |
| 13 | 9 | 50 | 6000 | 1.5 | 1.12 | 0.39 |
| 1 | 10 | 30 | 4000 | 1.5 | 1.09 | 0.29 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 3 | 11 | 30 | 8000 | 1.5 | 0.65 | 0.35 |
| 8 | 12 | 70 | 8000 | 4.5 | 0.98 | 0.37 |
| 12 | 13 | 50 | 8000 | 3 | 0.75 | 0.362 |
| 17 | 14 | 50 | 6000 | 3 | 0.99 | 0.34 |
| 2 | 15 | 70 | 4000 | 1.5 | 1.58 | 0.408 |
| 15 | 16 | 50 | 6000 | 3 | 1.03 | 0.34 |
| 4 | 17 | 70 | 8000 | 1.5 | 1.22 | 0.473 |
| 7 | 18 | 30 | 8000 | 4.5 | 0.44 | 0.268 |
| 9 | 19 | 50 | 6000 | 3 | 1.01 | 0.34 |
| 11 | 20 | 50 | 4000 | 3 | 1.13 | 0.317 |

### 6.3.2 Quadratic model and analysis of variance

RSM was utilized for the determining the mathematical correlation between input factors and responses. The statistical model summary indicated that the quadratic model is crucial for assessing and evaluating both wear rate and COF. From the ANOVA, the $R^2$ regression coefficient was determined for gauging the accuracy and level of fit of the developed model. A $R^2$ value closer to 1 implies model is suitable with the experimental outcomes.

**Table 6.10** Statistics model summary of wear rate

| **Source** | **Std. Dev.** | **$R^2$** | **Pred. $R^2$** | **Adj. $R^2$** | **PRESS** | |
|---|---|---|---|---|---|---|
| Linear | 0.0421 | 0.9779 | 0.9573 | 0.9738 | 0.0547 | |
| 2FI | 0.0342 | 0.9909 | 0.9747 | 0.9856 | 0.0643 | |
| **Quadratic** | 0.**0312** | **0.9902** | **0.9499** | **0.9827** | **0.0325** | **Suggested** |
| Cubic | 0.0343 | 0.9945 | -2.9085 | 0.9827 | 5.01 | Aliased |

**Table 6.11** Statistics model summary of COF

| Source | Std. Dev. | $R^2$ | Pred. $R^2$ | Adj. $R^2$ | PRESS | |
|---|---|---|---|---|---|---|
| Linear | 0.0083 | 0.9787 | 0.9560 | 0.9746 | 0.0023 | |
| 2FI | 0.0051 | 0.9934 | 0.9880 | 0.9903 | 0.0006 | |
| **Quadratic** | 0.0018 | **0.9993** | **0.9976** | **0.9988** | **0.0001** | **Suggested** |
| Cubic | 0.0022 | 0.9994 | 0.6888 | 0.9982 | 0.0160 | Aliased |

From the Tables 6.10 and 6.11, we observe that developed model in the study exhibits high goodness-of-fit, with $R^2$ values of 0.9902 for wear rate and 0.9993 for COF, both tending to unity. Additionally, the adjusted $R^2$ and predicted $R^2$ closely align. These findings indicate that the model is desirable and have a good fit.

Table 6.12 and Table 6.13 shows the ANOVA results for optimization of the wear rate and COF corresponding to the input factors. The tables illustrate the significance of the model through the inclusion of the p-value. Additionally, p-value is useful for identifying the significant factors ($p\text{-value} < 0.05$) and insignificant factors (($p\text{-value} > 0.05$), respectively. From both the ANOVA tables of wear rate and COF, it was revealed that significance of the model is acceptable as their $p\text{-value} <0.0001$ which is less than 0.05.

The F-value can be employed to assess the contribution of each factor in the statistical model. However, the higher F- value suggests higher contribution of a factor in wear rate and COF calculation. In both the cases, the factors A, B and C i.e., load, sliding distance and wt.% of TiC are significant factors that affect wear rate and COF since their p-value is lower than 0.0001. Load (A) is having maximum F- value as 636.41 implying it is dominant factor for wear rate followed by TiC (C): 230.15 and Sliding distance (B): 229.67, respectively. Similar, in case of COF, the F- value is maximum for load as 9146.62 implying it is dominant factor for COF followed by TiC (C): 4341.71 and Sliding distance (B): 1554.82. Additionally, the absence of a significant "Lack of Fit" is favourable, suggesting that the model fits well for both wear rate and COF. This implies that the developed model is suitable for prediction and optimization of the wear rate and COF, respectively.

**Table 6.12** ANOVA for optimisation of wear rate

| **Source** | **$d_f$** | **Sum of Squares** | **Mean Square** | **F-value** | **p-value** | |
|---|---|---|---|---|---|---|
| **Model** | 9 | 1.27 | 0.1412 | 120.6 | < 0.0001 | **significant** |
| A-Load | 1.0 | 0.7453 | 0.7453 | 636.41 | < 0.0001 | |
| B-Sliding distance | 1.0 | 0.2690 | 0.2690 | 229.67 | < 0.0001 | |
| C-TiC | 1.0 | 0.2402 | 0.2402 | 230.15 | < 0.0001 | |
| AB | 1.0 | 0.0012 | 0.0012 | 1.07 | 0.3259 | |
| AC | 1.0 | 0.0000 | 0.0000 | 0.0000 | 1.0000 | |
| BC | 1.0 | 0.0144 | 0.0144 | 12.34 | 0.0056 | |
| $A^2$ | 1.0 | 0.0001 | 0.0001 | 0.1092 | 0.7479 | |
| $B^2$ | 1.0 | 0.0009 | 0.0009 | 0.7763 | 0.3990 | |
| $C^2$ | 1.0 | 0.0001 | 0.0001 | 0.1092 | 0.7479 | |
| Residual | 10.0 | 0.0117 | 0.0012 | | | |
| **Lack of Fit** | 5.0 | 0.0088 | 0.0018 | 2.99 | 0.1271 | **not significant** |
| Pure Error | 5.0 | 0.0029 | 0.0006 | | | |
| Cor Total | 19.0 | 1.28 | | | | |

**Table 6.13** ANOVA for optimisation of COF

| **Source** | **$d_f$** | **Sum of Squares** | **Mean Square** | **F-value** | **p-value** | |
|---|---|---|---|---|---|---|
| **Model** | 9 | 0.0514 | 0.0057 | 1706.81 | < 0.0001 | **significant** |
| A-Load | 1.0 | 0.0306 | 0.0306 | 9146.62 | < 0.0001 | |
| B-Sliding distance | 1.0 | 0.0052 | 0.0052 | 1554.82 | < 0.0001 | |
| C-TiC | 1.0 | 0.0145 | 0.0145 | 4341.71 | < 0.0001 | |
| AB | 1.0 | 6.125E-06 | 6.125E-06 | 1.83 | 0.2057 | |
| AC | 1.0 | 0.0002 | 0.0002 | 56.87 | < 0.0001 | |
| BC | 1.0 | 0.0006 | 0.0006 | 167.83 | < 0.0001 | |
| $A^2$ | 1.0 | 0.0001 | 0.0001 | 27.84 | 0.0004 | |

| | | | | | | |
|---|---|---|---|---|---|---|
| $B^2$ | 1.0 | 4.778E-06 | 4.778E-06 | 1.43 | 0.2595 | |
| $C^2$ | 1.0 | 0.0003 | 0.0003 | 85.27 | < 0.0001 | |
| Residual | 10.0 | 0.0000 | 3.43E-06 | | | |
| **Lack of Fit** | 5.0 | 0.0000 | 3.320E-06 | 0.9862 | 0.5059 | **not significant** |
| Pure Error | 5.0 | 0.0000 | 3.367E-06 | | | |
| Cor Total | 19.0 | 0.0514 | | | | |

### 6.3.3 Regression equation for wear rate and COF

The input factors were used to establish a mathematical relation for wear rate and COF. The regression Eqns. (6.3) and (6.4) were obtained by "Design Expert 13" software to develop the relationship between the input factors such as load (A), sliding distance (B), and wt.% of TiC (C) and wear rate (WR) and coefficient of friction (COF).

***Wear rate*** $=+1.35377 + 0.010070A - 0.000086B - 0.206515C + 3.125e^{-07} A*B - 3.95802e^{-18} A*C + 0.000014 B*C + 0.000017 A^2 - 4.54545e^{-09} B^2 + 0.003030C^2$ *(Eqn. 6.3)*

***COF*** $=+0.133615 + 0.004576A + 0.000023B - 0.027677C + 2.18750e^{-08} A*B - 0.000163 A*C - 2.79167e^{-06} B*C - 0.000015 A^2 - 3.29545e^{-10} B^2 + 0.004525C^2$ *(Eqn. 6.4)*

Figure 6.9 (a-c) illustrates surface and contour plots (2D and 3D) showing influence of load and sliding distance on wear rate of TiC reinforced composites with varying weight % of TiC content (1.5-4.5 wt.%). These visual representations are employed to predict wear properties, providing valuable insights into how changes in input factors influence the wear rate. It is seen that the wear rate value rises as load is increased for all compositions [M. Freschi et al., 2022]. The higher load causes more deformation that leads to closure of surface pores which reduce the lubricant retention ability of composites and hence increase in wear rate. Further slight decrement in wear rate is observed with the increase of sliding distance.

Figure 6.10 (a) and (b) depict the combined effect of reinforcement content and load at fixed sliding distance 6000 m and influence of reinforcement and sliding distance at fixed 50 N load, respectively. A significant increase in wear rate is observed at maximum value of load of 70 N owing to more deformation of matrix whereas wear rate decreases slightly with the rise in TiC content because of increase in lubricant retention capacity of 4.5 wt.% TiC reinforced composite.

Figure 6.11 provides 2D and 3D surface and contour plots showing the impact of input factors on the coefficient of friction (COF). Figure 6.11 (a-c) depicts the contour plots showing influence of load and sliding distance on COF with varying TiC wt.%. The surge in applied load results in the swift removal of lubricant layer and thereby increasing the COF. However, when TiC wt.% is increased, the porosity percentage rises which enables the composites to accumulate more lubricant oil and thus helping in the reduction of the overall COF. Figure 6.11 (a-c) we observe a gradual reduction in COF at different load and sliding distance.

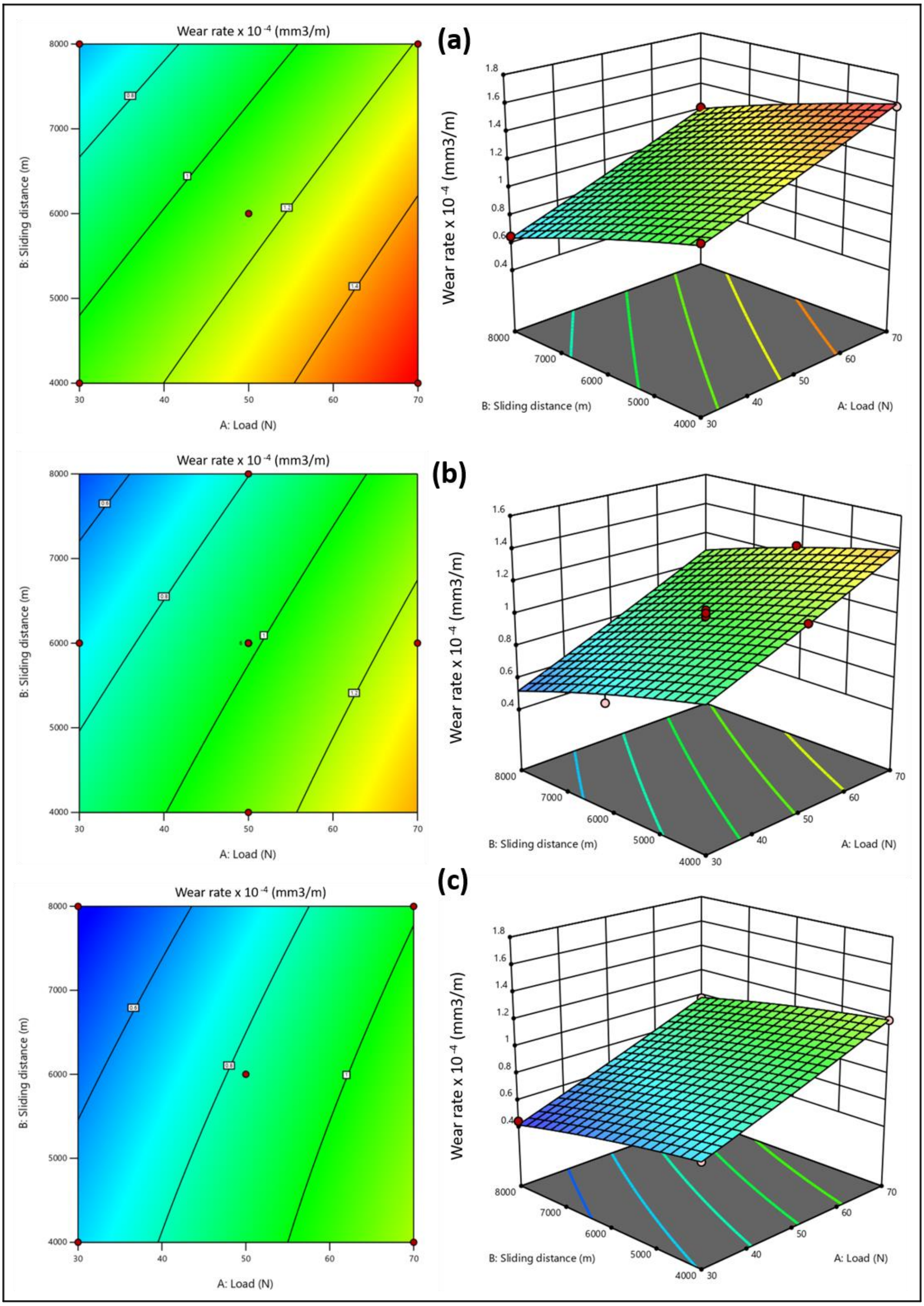

**Fig. 6.9** 2D and 3D- surface and contour plots showing wear rate variation with applied load and sliding distance for (a) T1.5 (b) T3.0 (c) T4.5

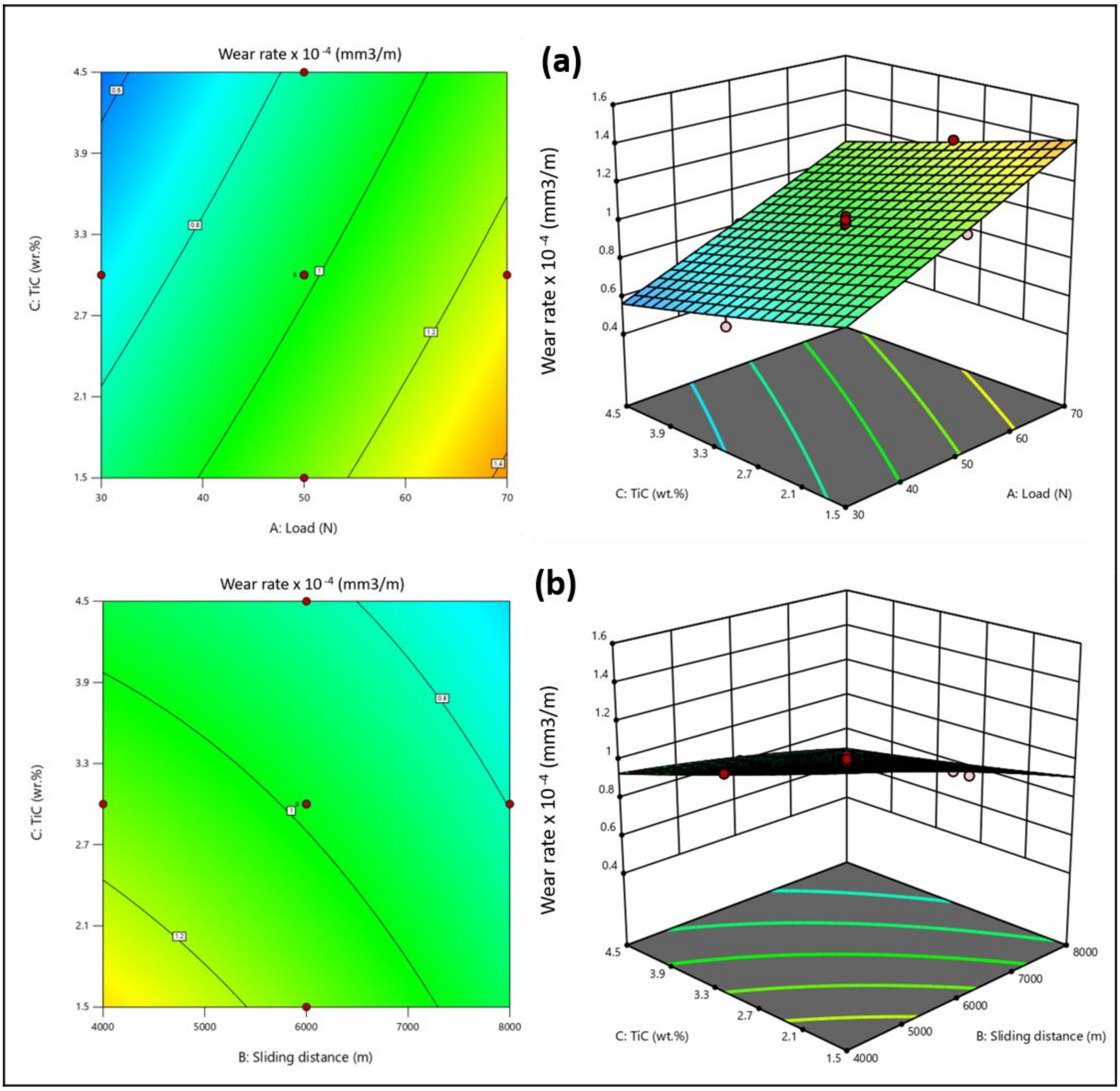

**Fig. 6.10** 2D and 3D- surface and contour plots for wear rate (a) applied load vs wt.% of TiC and (b) sliding distance vs wt.% of TiC

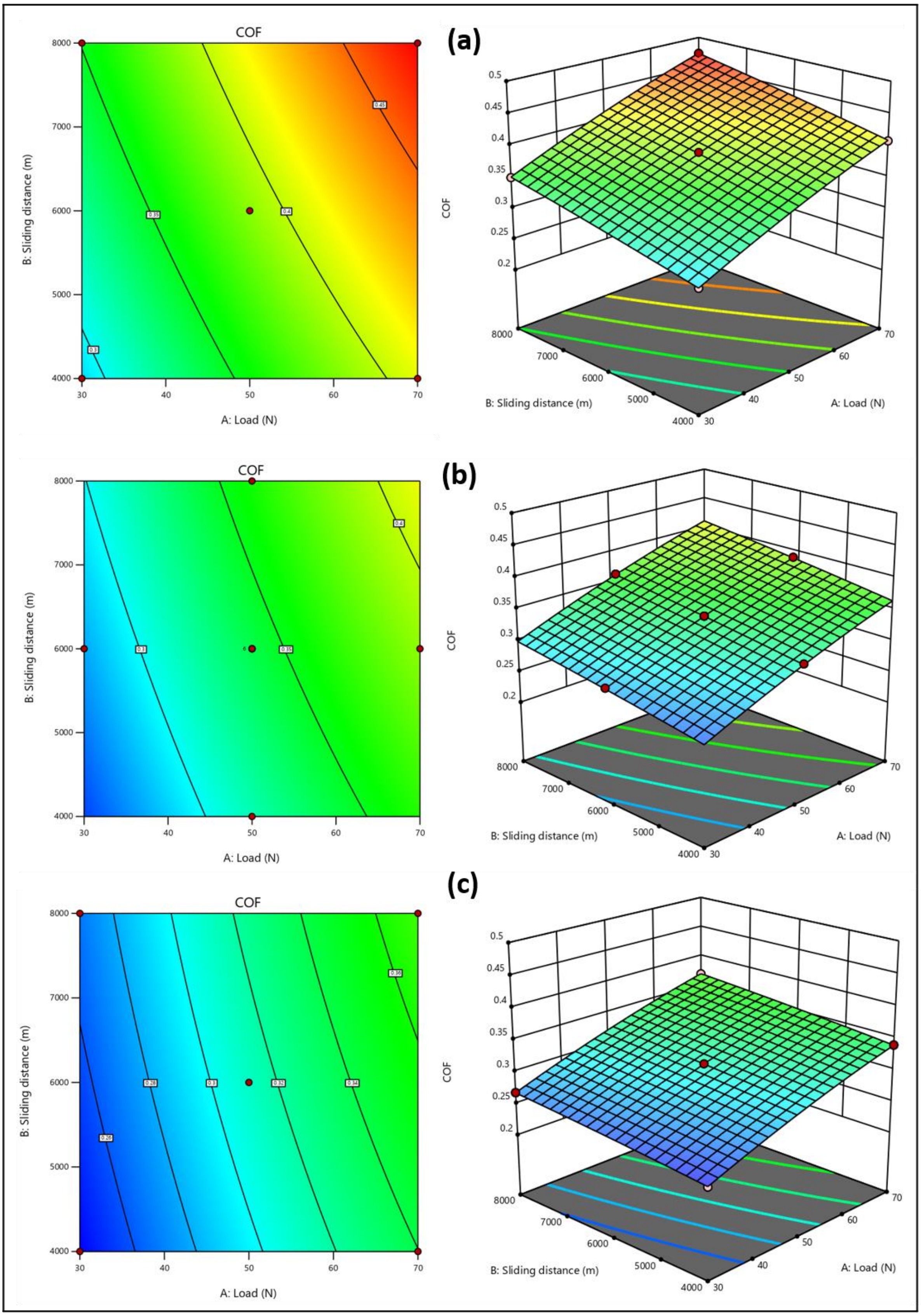

**Fig. 6.11** 2D and 3D- surface and contour plots showing COF variation with applied load and sliding distance for (a) T1.5 (b) T3.0 (c) T4.5

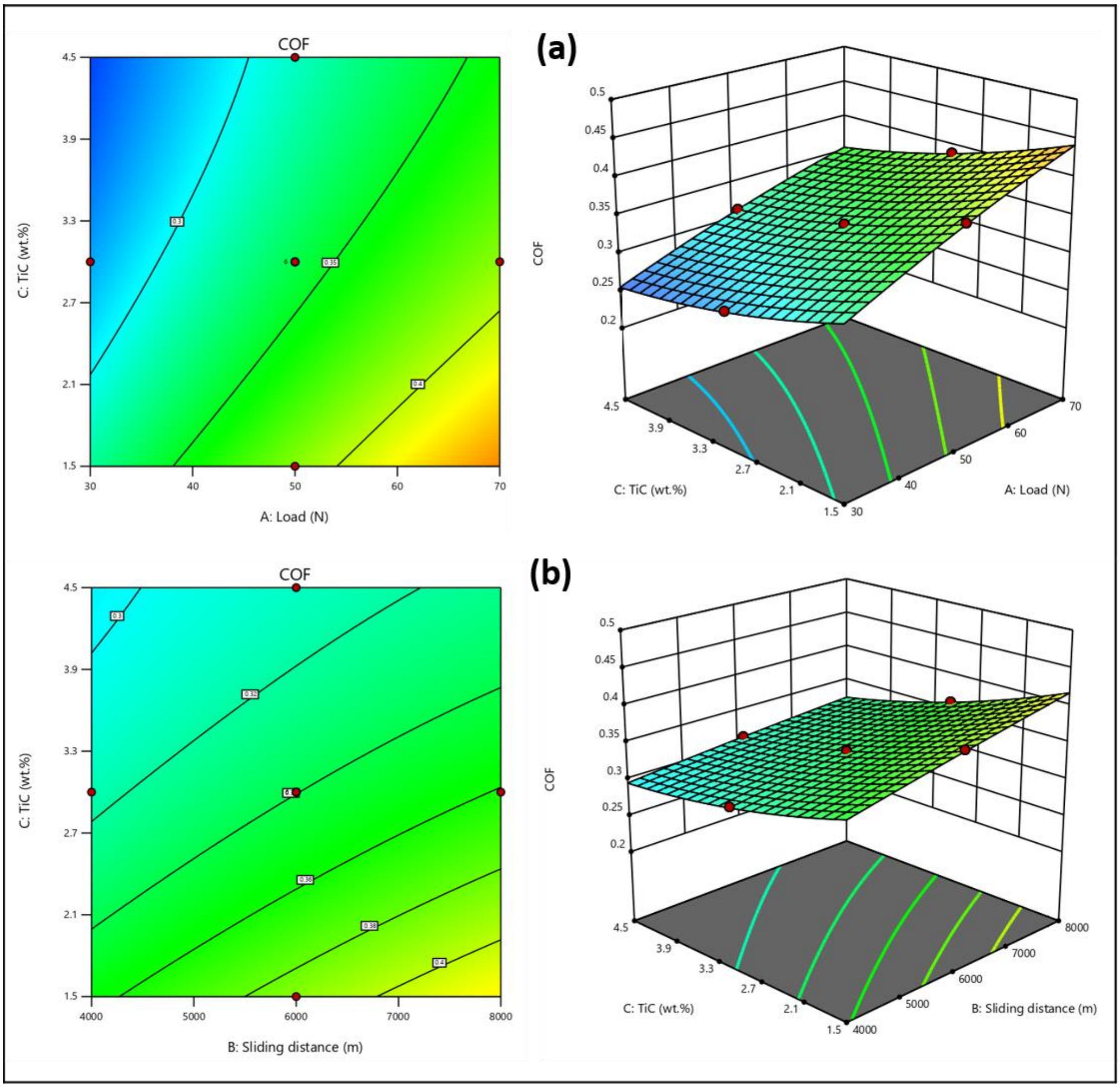

**Fig. 6.12** 2D and 3D- surface and contour plots for COF (a) applied load vs wt.% of TiC and (b) sliding distance vs wt.% of TiC

Figure 6.12 (a) and Figure 6.12 (b) displays the combined effect of reinforcement content and load at fixed sliding distance 6000 m and influence of reinforcement and sliding distance at fixed 50 N load, respectively. The COF value rises with the load due to increased interaction of asperities of matrix with the counter surface. However maximum COF value is observed at 70 N load and 1.5 wt.% TiC reinforced composite. From Fig. 6.12 (b), an increase in sliding distance results in increment of COF since more fragmentation of asperities of the matrix occurs at larger distances. However minimum value of COF is observed at 4000 m sliding distance and 4.5 wt.% TiC reinforced composite.

Figures 6.13 and 6.14 depict the correlation between the observed (experimental) values and the predicted values obtained from the developed model. Additionally, they also illustrate the normal probability distribution for the residuals associated with both wear rate and coefficient of friction (COF), respectively.

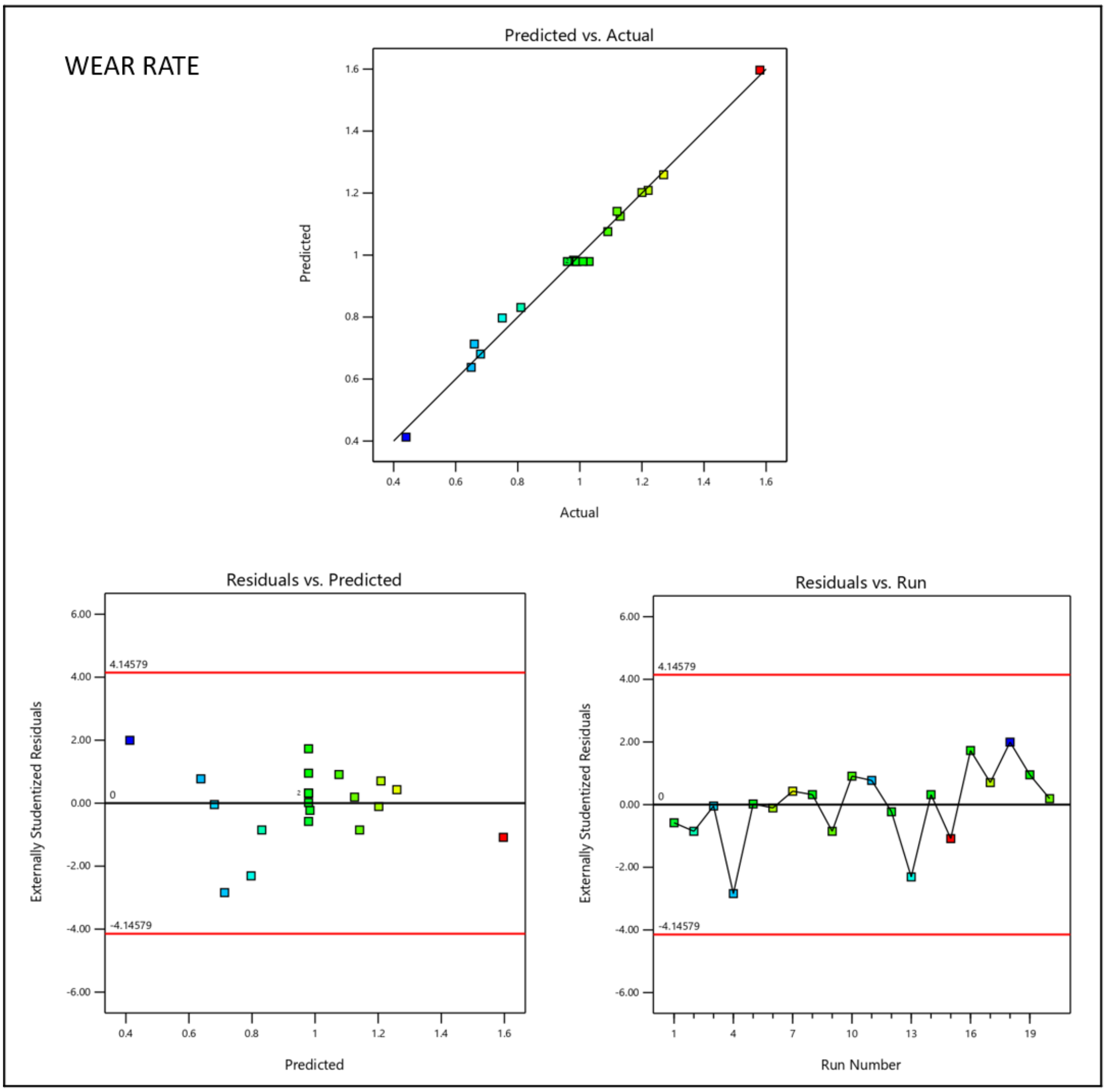

**Fig. 6.13** Variation of experimental and predicted values and normal probability for residuals of wear rate

It is inferred that there exists very little difference among predicted and actual values of wear rate and COF in the lubricating condition. So, the model can be successfully employed for prediction of wear rate and COF of the composites. From the normal probability plots showing residual vs. predicted and residuals against run order, it is inferred that the residuals are uniformly dispersed and are within limits. Therefore, it is concluded that the developed statistical model can be satisfactorily used for describing the data.

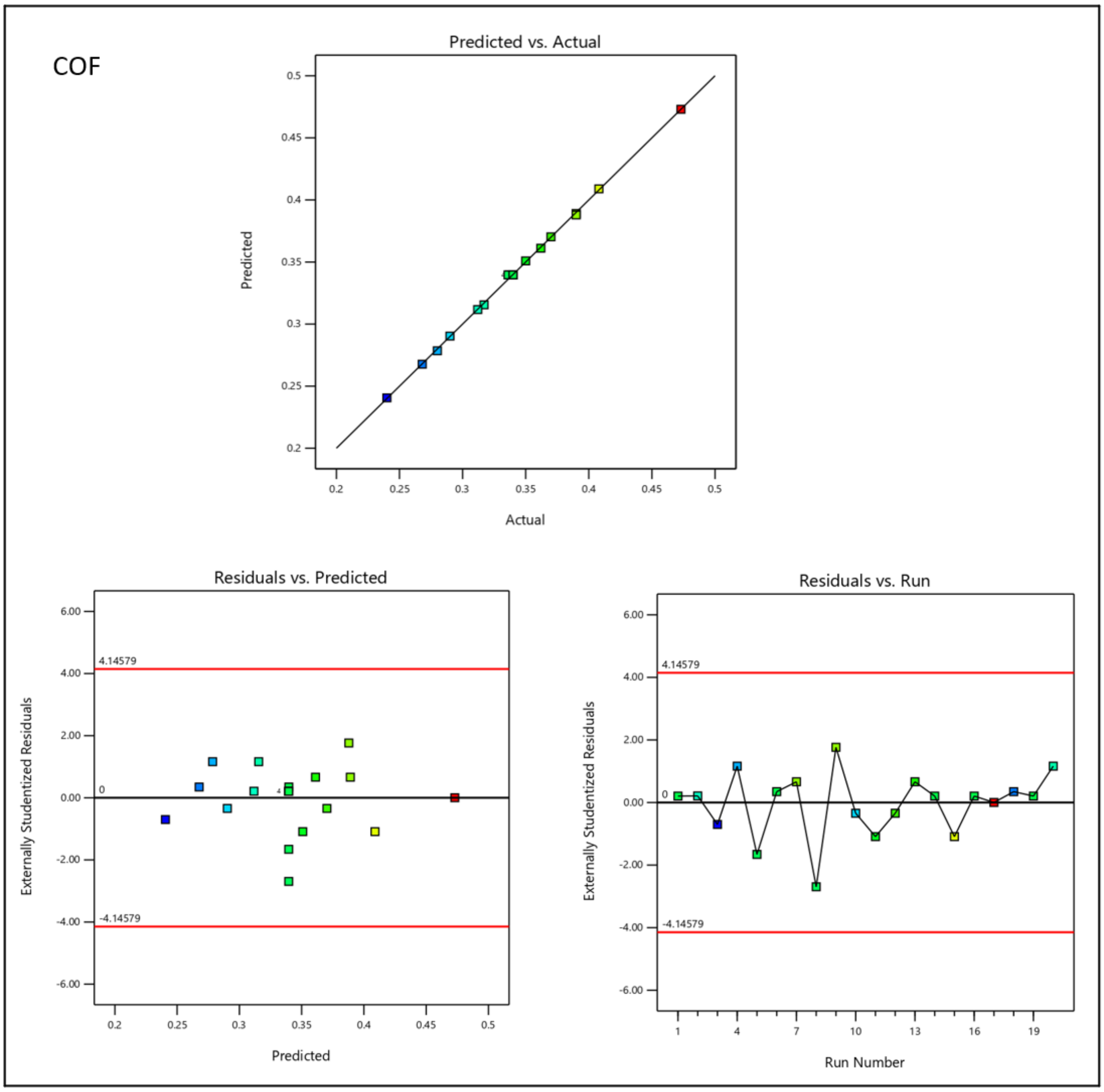

**Fig. 6.14** Variation of experimental and predicted values and normal probability for residuals of COF

Figure 6.15 (a) and (b) demonstrates the perturbation plots that depicts the influence of individual factors on wear rate and COF, respectively. a reference point was selected in order to plot the perturbation graph. The reference point selected was the central level of all individual factors (Load-50 N, sliding distance-6000 m and 3 wt.% TiC content). It was inferred that wear rate of the composites increases with rise in load while wear rate decreases with rise in reinforcement content and sliding distance. Also, COF of composites decreases with TiC content but enhances with the rise in load and with increase of sliding distance.

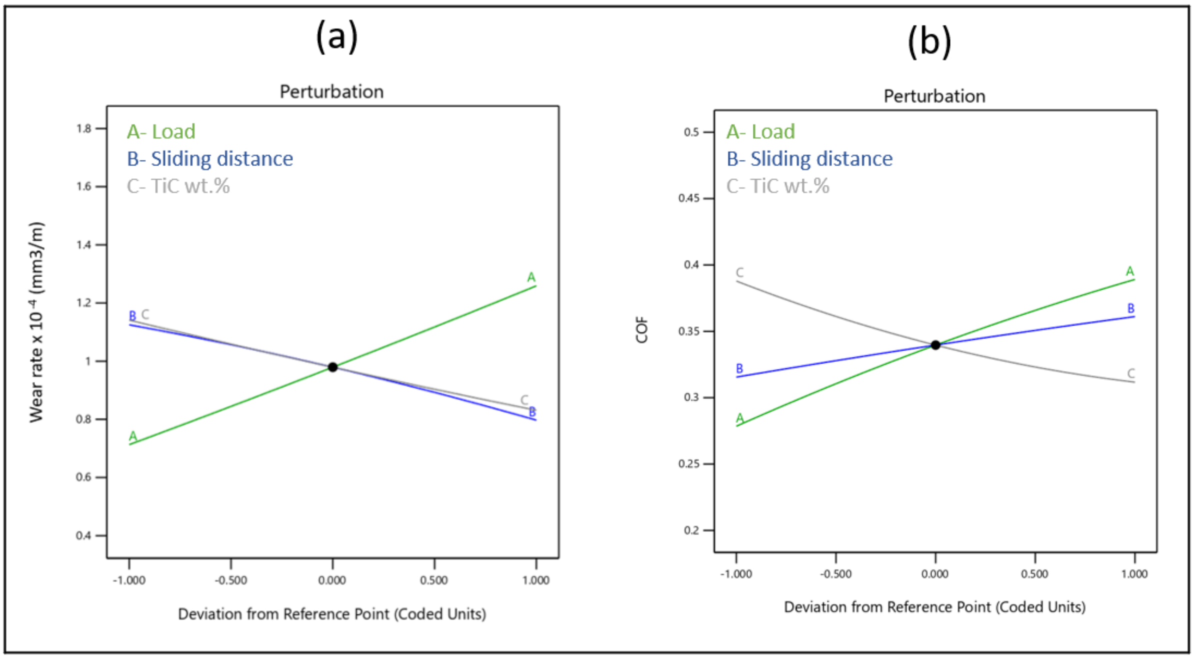

**Fig. 6.15** Influence of individual factors on the wear rate and COF

### 6.3.4 Validation of model and optimization of tribological parameters

The aim of optimization is to find out those input factors that lead to the best wear rate and coefficient of friction (COF). It becomes difficult when multiple input variables are affecting the responses. The desirability-based method is an extensively used technique for parameter optimization where the desirability values range from 0 to 1, and when its value is near to 1, it means closeness to the target. However, in RSM a range of variables was developed to optimize both wear rate and COF. Among the several sets of input variables, those having highest desirability were preferred as the optimal conditions as depicted in Figure 6.16 for both the wear rate and COF.

From Figure 6.16, the optimum values of wear rate and COF are $0.44\times10^{-4}$ $mm^3/m$ and 0.2658, respectively. Optimized predicted values of wear rate and COF occur at input factors when A-30 N load; B-7674 m sliding distance and C-4.99 wt.% TiC. The relation among the input parameters at minimum load, 7674 m of sliding distance, and maximum weight percent of TiC (4.49 wt.%) are the most optimized parameters for minimum wear rate and COF.

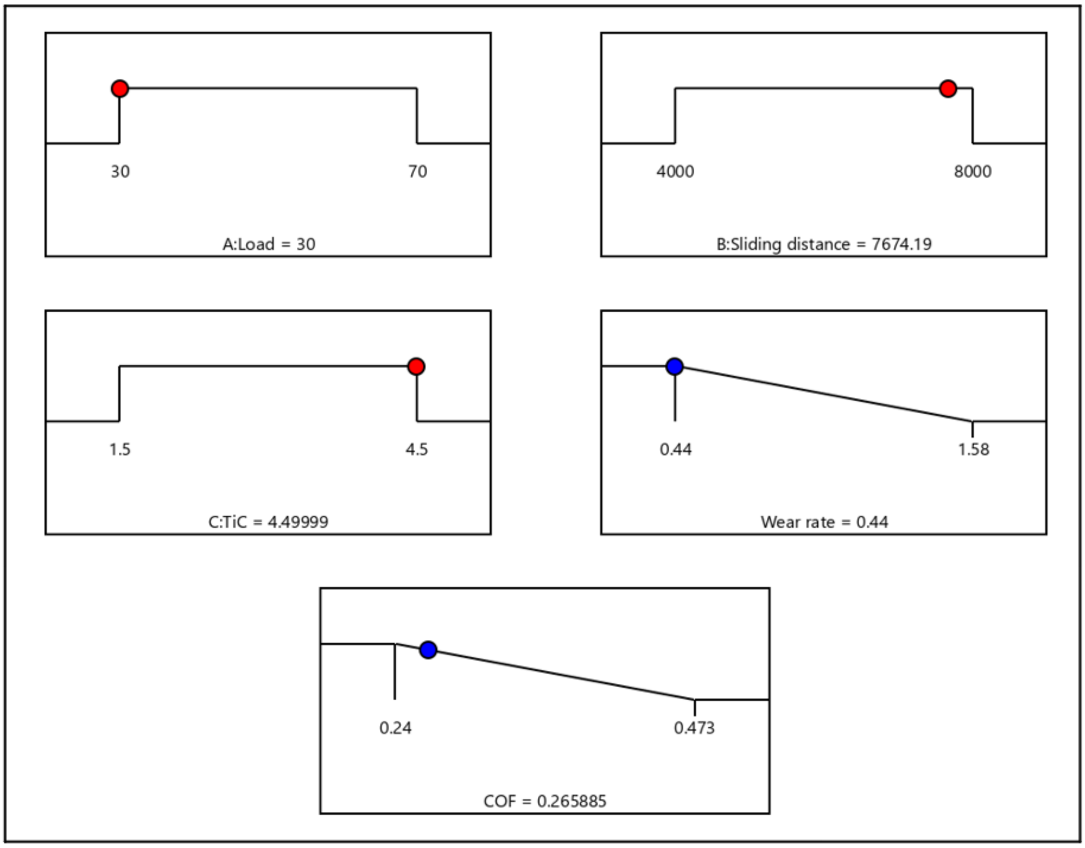

**Fig. 6.16** Influence of individual factors on wear rate and COF

Table 6.14 depicts the obtained experimental values and predicted values at the optimized input parameters in lubricating sliding condition. The comparison of experimental and predicted values of wear rate and COF depicted ~2% error and ~1.2% error, respectively.

**Table 6.14** Experimental and predicted values of the model for optimal wear rate and COF

| Parameters | Optimum parameter | Predicted value | | Experimental value | | % Error | |
|---|---|---|---|---|---|---|---|
| | | **WR* $10^{-4}$ $mm^3/m$** | **COF** | **WR* $10^{-4}$ $mm^3/m$** | **COF** | **WR** | **COF** |
| **Load (N)** | 30 | 0.44 | 0.2658 | 0.45 | 0.269 | 2 | 1.2 |
| **Sliding distance (m)** | 7674 | | | | | | |
| **TiC (wt.%)** | 4.49 (~4.5) | | | | | | |

Both experimental and predicted values show close tolerance implying the developed model is significant with high efficacy. The model is suitable for predicting and optimizing the tribological performance of the composites.

## 6.4 CONCLUSIONS

To minimize the number of experiments and reduce time and resource consumption, optimizing the wear rate and COF with varying input parameters is crucial. The following conclusions can be derived from this chapter:

- The central composite design of Response Surface Methodology (RSM) can be efficiently employed to construct a mathematical model capable of predicting wear rate and COF. This approach proves effective in minimizing the required number of experiments.
- From the result of ANOVA, applied load is the major factor affecting both the wear rate and COF of composites under dry sliding conditions followed by TiC wt.% and sliding distance.

- From the result of ANOVA under lubricating conditions, applied load majorly affect the wear rate whereas TiC wt.% and sliding distance contribute equally in the wear rate. However, COF of composites is affected by the load, followed by TiC content and then the sliding distance.
- The parameters for least wear rate and COF as observed from RSM are found to be 3.08 wt.% of TiC, 20 N applied load and 2000 m sliding distance for dry sliding condition.
- The parameters for least wear rate and COF as observed from RSM are found to be 4.49 wt.% of TiC, 30 N applied load and 7674 m sliding distance for lubricating sliding condition.
- The comparison of experimental and predicted values of wear rate and COF depicted ~8% error and ~1.11% error in dry sliding condition whereas, error of ~2% for wear rate and ~1.2% for COF of composites under lubricating condition helped in the validation of developed regression model and signify that model can be employed for prediction and optimization.
- The present chapter suggests that RSM statistical technique can be utilized for the prediction of wear behaviour of composite in an efficient way. Thus, it will help in time reduction and less resource consumption for the fabrication of composites.