# PREFACE

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Gas dynamics is a branch of fluid dynamics that deals specifically with the study of gases in motion. In the $19^{th}$ and $20^{th}$ centuries, the development of the conservation laws of mass, momentum, and energy laid the ground work for gas dynamics. These fundamental laws allowed scientists and engineers to describe the behavior of gases in various flow situations, including compressible and incompressible flows. It has grown with the development of high-speed flight and has become an area of research for physicists, chemists, applied mathematicians, and astrophysicists. The term gas dynamics suggests the idea that the field is exclusively related to the state of gaseous matter. It encompasses the analysis of the behavior of gases under various conditions, including changes in pressure, temperature, density, and velocity.

A wave can be thought as a propagating feature of disturbance. It is defined as any notable feature that is propagated from one medium to another or within the medium with a recognizable speed. It can be any characteristic of the disturbance, such as the formation of trough and crest or sudden change etc., in some physical quantity, provided that it can be clearly noticed and its position at any time can be found. The characteristic feature may contort, be magnified, and change its velocity provided it is still recognizable. Certain types of wave can be formulated mathematically in terms of hyperbolic partial differential equations.

The Riemann problem is a fundamental concept in the study of hyperbolic partial differential equations (PDEs), particularly in the field of gas dynamics and fluid mechanics. It is named after the German mathematician Bernhard Riemann, who introduced this problem in 1860. In the context of gas dynamics, the Riemann problem arises when we consider a one-dimensional gas flow with different initial states on either side of a discontinuity, such as a shock wave or rarefaction wave. The Riemann problem serves as a simple yet crucial model to study the formation and propagation of these waves in the flow of gases. It also gives an exact solution to some complex non-linear equations. A shock wave is a surface of discontinuity across which the flow properties experience a sudden jump. Across a rarefaction wave, the flow properties are continuous. The velocity and pressure are continuous across a contact wave but density, temperature, entropy, etc., experience a sudden change. Shock waves are the most challenging phenomenon occurring in non-linear wave motion; they can develop and propagate, even if the initial data are continuous. The reason is that non-linear partial differential equations do not admit continuous solutions.

In the present thesis, we study the propagation of non-linear waves in various gaseous media governed by quasilinear partial differential equations and embodies the results of research carried out by me at the Department of Mathematical Sciences, Indian Institute of Technology (BHU), Varanasi, during the period January 2019 to September 2023 under the supervision of Prof. L. P. Singh. The present work deals with certain problems associated with the solutions of the Riemann Problem for quasilinear one-dimensional conservative hyperbolic system which occur in many physical phenomena having practical importance in real life. Here, we have solved problems, involving homogeneous and non-homogeneous hyperbolic systems where classical and non-classical situations arise, using various approaches like vanishing pressure limit method, Differential constraints method, Characteristic method, and Wavefront analysis method. We are motivated to solve the problem for non-homogeneous hyperbolic system which is modified into homogeneous hyperbolic system of conservation laws to study the solution of Riemann problem with constant initial data by introducing new variable for the velocity. Also, this thesis concerns with the solutions of the Riemann problem with constant and non-constant initial data for different hyperbolic systems. We introduce the notions of rarefaction waves, shock waves, contact discontinuities and delta shock waves, which play an essential role in the explicit construction of the solution of the Riemann problem. Then, we discuss the local existence and uniqueness of the solution of Riemann problem for a system in the sense that the initial states are sufficiently close. It is also proved that same is true for the dusty gas dynamic equations. We consider the strictly hyperbolic system of conservation laws which describes the background flow carrying dust particles and whose Riemann solution contains classical elementary waves as well as delta shock wave in certain situations. The whole thesis is divided into six chapters as fallows:

**Chapter - 1** is introductory and gives a general idea of when and how a discontinuity appears. Certain terminologies commonly used in the current work have been defined. The mathematical theory and their fundamental properties have also been briefly discussed. The physical properties of hyperbolic systems, equation of state, dusty gas, reacting gas and methods which are used throughout the thesis are briefly reviewed. Some results, which we shall need in subsequent chapters, have also been included in this chapter.

**Chapter - 2** presents an analysis of the of the shock formation in 2-D steady supersonic flow of non-ideal gas with magnetic field for the planar flow and axisymmetric flows. It is shown that the governing equations describing the non-ideal gas flow with magnetic field is hyperbolic in nature. Further, using the theory of the propagation of wavefronts defined by weak shock, we derived the transport equations for shock wave which lead to determination of shock formation distance and provide the conditions of shock formation. It is shown that the formation of shock wave is affected by the presence of magnetic field, non-ideal parameter and upstream flow Mach number with $M_0 > 1$.

The motive of the study in **Chapter - 3** is to investigate the impact of the dust particles on the shock formation in two-dimensional steady supersonic flow of the composition of van der Waals gas and small solid dust particles for the planar, cylindrically symmetric and spherically symmetric cases. It is shown that the governing equations describing the van der Waals gas with dust particles is hyperbolic in nature. Further, by employing the method of wavefront analysis, we derived the transport equations for shock wave which lead to derivation of shock formation distance and also, provide the relations of shock formation. Also, it is determined as how the shock formation distance is affected by the presence of dust particles, parameter of van der Waals gas and upstream flow Mach number ($M_0 > 1$) by using MATHEMATICA 11 software.

In **Chapter - 4**, the solution of the Riemann problem (RP) for the hyperbolic system with the logarithmic equation of state and magnetic field is obtained. The formation of vacuum states and delta shock wave as magnetic field and pressure vanish, have been discussed. Firstly, the Riemann problem for the magnetogasdynamic system is solved. Further, we have constructed the solutions for the pressureless and vanishing magnetic field system (that is transport equations). In the context of vacuum states and delta shocks, it is shown that the solution of RP consisting two shocks converges to the delta shock wave ($\delta$-shock) solution of transport equations, and the Riemann solutions consisting two rarefaction waves converges to the vacuum state solution (that is intermediate state between two-contact discontinuity solution of transport equations). Hence, it is proved that the solutions of system with logarithmic equation of state and magnetic field converge to the solution of corresponding system as magnetic field and pressure vanish, which shows that solution of the RP for the hyperbolic system with the logarithmic equation of state and magnetic field is stable.

In **Chapter - 5**, the phenomena of concentration and cavitation in the Riemann solution for the non-homogeneous hyperbolic system with logarithmic equation of state and magnetic field is analyzed. Firstly, we introduced new state variable for the velocity to modify the non-conservative system into conservative system and solved the Riemann problem for modified system constructively. Further, the Riemann solutions for the transport equations is investigated as pressure and magnetic field vanish. It is proved that the Riemann solution for the non-homogeneous hyperbolic system with logarithmic equation of state and magnetic field having two shock waves converges to the delta shock wave solution of the transport equations as pressure and magnetic field vanish. It is also proved that the Riemann solution for the nonhomogeneous hyperbolic system with logarithmic equation of state and magnetic field having two rarefaction waves converges to the contact discontinuity solution of the transport equations as pressure and magnetic field vanish.

In **Chapter - 6**, we focus on the analytical solution to the Riemann problem (RP) for a 1-D non-ideal flow of dusty gas with external force. Here it is presumed that external force is a continuous function of time. We explicitly obtain the elementary wave curves to 1-D non-ideal flow of dusty gas with external force and determine these wave curves in form of characteristics. Exhaustive calculations were performed for the elementary wave solutions, such as the rarefaction wave, shock wave, and contact discontinuity. We examine the influence of dust particles on density, velocity of flow, and shock speed and their implications on the solution of RP. Here, it was observed the implication of addition of external force is that all solutions are not self similar.

Lastly, in **Chapter - 7**, the work done in the thesis is summarized. Major contributions made in the thesis are briefly discussed followed by a discussion on the future scope.