Chapter 1

Introduction

Control systems are a crucial engineering domain in which we study and regulate the behavior of dynamic systems. They comprise a set of components, including sensors, actuators, and controllers, working together to maintain the system's performance within desired limits. The main objective of a control system is to track setpoints, stabilize unstable systems, and reject disturbances to ensure the system operates effectively and efficiently. Control systems can be found in a wide range of applications, such as industrial processes, aerospace, robotics, automotive systems, and household appliances. They play a vital role in enhancing safety, improving productivity, and optimizing performance in complex and interconnected systems.

Generally from the control systems perspective, we usually encounter six different types of problems for a given system. These problems can be classified as follows:

1. Representation problem
2. Estimation and observation problem
3. Solution problem
4. Stability problem
5. Design problem
6. Optimization problem

Firstly, in control systems, the representation problem refers to the challenge of accurately modeling a physical system for effective control. This issue arises due to various complexities such as nonlinearity, uncertainty, time variability, high-dimensionality, and modeling errors. The accurate representation of a system is crucial for developing control strategies that can achieve desired performance, stability, and robustness. Secondly, parameter estimation and observation are essential components of control systems, serving distinct purposes. State estimation involves determining hidden system states based on available measurements and a mathematical model. On the other hand, parameter estimation aims to identify unknown or time-varying parameters in the system model using available data. Thirdly, the solution problem is a fundamental aspect of control theory, involving determining the existence and uniqueness of the system's solution. Fourthly, the stability problem in control systems ensures that the controlled system remains stable under various operating conditions. Stability is critical to prevent undesirable behavior and catastrophic failure. Next, the design problem in control theory involves the systematic development of a control system to achieve specific performance goals and desired control objectives. Key steps include defining control objectives, developing a system model, selecting an appropriate control strategy, designing the control law, and analyzing stability and robustness. Lastly, the optimization problem in a general context refers to the task of finding the best solution or set of solutions from a given set of alternatives, while satisfying certain constraints and optimizing specific objectives.

In this thesis, we only address the first five problems. To be specific, we first introduce the state estimation problem of a nonlinear dynamical system where we achieve the estimation in a predefined time. Next, we consider the parameter estimation problem of a class of uncertain dynamical systems, where system parameters are successfully estimated in some finite time. Additionally, we have addressed the stabilization problem of a dynamical system, where the system states are stabilized within a priori chosen time. Finally, we validate the developed theory in the field of guidance and control of aerial subjects.

The estimation of states for nonlinear dynamic systems from output measurements within a predefined/desired time is an essential task. However, in many practical applications, some key variables cannot be directly measured due to unavailability or expensive sensors. For instance, in nuclear power plants, measurements of precursor density and temperature are crucial physical variables that are challenging to measure directly. State estimation is also essential in other applications, such as tracking the position of an aircraft or estimating the state of a robotic arm. It allows engineers and scientists to make informed decisions based on the estimated state of the system, even when direct measurements of the state are not available. To address this issue, researchers have explored linear and nonlinear observers [1,3] to estimate the physical variables. Various techniques, including Kalman filtering [4], particle filtering [5], and other advanced estimation methods, can be used to estimate the state of a system. These methods rely on mathematical models of the system and measurements of its inputs and outputs to estimate the states of the system over time.

Moreover, effective system identification often requires both state and parameter estimation in many cases. Parameter estimation is a statistical process that determines the values of unknown parameters in a mathematical or statistical model based on observed data [71]. In many real-world applications, certain parameters that govern a system or process are unknown, and estimating these values is critical to understand and predict the system's behavior. This is an essential problem in various fields, including engineering, physics, economics, and biology, and is used for predictions, designing experiments, and optimizing systems [6]. Some early work on parameter estimation is developed on maximum likelihood [49], and Bayesian estimation methods [50]. The choice of the estimation technique depends on the specific application and the model assumptions. Most of the control design techniques available in the literature rely on the knowledge of system parameters. However, these techniques do not give the desired performance, since the real-world system are subjected to various types of uncertainties. For this very reason, the control design problem for these uncertain systems has been a topic of significant interest in the control community. When there is uncertainty in the control affine model, it is possible to represent the system in a polytopic form.

A polytopic system, whether linear or nonlinear, is a dynamic system that can be described by multiple models, each of which is valid over a particular region of the state space. When the system crosses the boundary between these regions, its behavior may change. These systems are also referred to as piecewise-affine systems and are commonly used in control systems, particularly in aerospace and automotive systems, robotics, chemical processes, and other areas where complex nonlinear systems need to be modeled and controlled. The polytopic model is an effective way to characterize the plant uncertainty due to its simpler design. The uncertain parametric systems are described in terms of the parameters of a set of models with a convex structure. The control design and stability analysis problems of linear and nonlinear polytopic systems have been widely discussed in [80–84].

As far as application part of the rated convergence framework is concerned, there are various practical systems where the desired convergence time is of utmost priority such as, aerospace and defense related technologies [110, 111], particularly designing a guidance law that can give the convergence of Line-of-Sight (LOS) rate to zero within a finite/desired time.

A guidance law is a set of rules or algorithms used to calculate the desired trajectory or path for a vehicle or object to follow [96]. These laws are commonly used in aerospace and defense applications, such as missiles, rockets, and spacecraft, to ensure that the vehicle reaches its intended target or destination with a high degree of accuracy [125]. Designing a guidance law involves taking various factors into account, such as the vehicle's dynamics, the target's position and velocity, and any environmental factors that may affect the vehicle's trajectory. The goal is to compute the desired trajectory that will enable the vehicle to reach the target while satisfying any constraints on the vehicle's motion [97]. Proportional navigation (PN) and its variants have been widely used in guidance law design due to their simplicity, efficiency, and ease of implementation [126, 127].

In recent years, designing a guidance law with high accuracy to intercept a target has become a critical problem, especially when the target is performing evasive maneuvers. Control theory plays an important role in precisely intercepting the target in these cases. One of the main objectives in guidance problems is to reduce the LOS rate to zero or a small neighborhood of zero against maneuvering targets [103, 104].

1.1 Literature Review

1.1.1 Rated Convergence

Over the years, significant research has been conducted regarding the rate of convergence in control strategies. The rate of convergence is a crucial attribute that allows a given control strategy to achieve various types of convergence, including exponential, finite-time, fixed-time, predefined-time, prescribed-time, or arbitrary-time convergence to the equilibrium point [21, 26, 35, 36].

Finite-time stability and fixed-time stability have been extensively studied in the literature [19, 20]. Controllers that aim to achieve both fast and accurate responses in dynamical systems were introduced using continuous finite-time differential equations in [22]. Additionally, a rigorous analysis of finite-time stability in continuous autonomous systems is presented in [19]. For a comprehensive understanding of various stability notions, including finite-time and fixed-time stability, please refer to [21].

The most significant application of finite-time convergence lies in the problem of differentiation, which allows the designer to ensure that the controller utilizes correct information after a specific time moment [23]. As a result, the upper bound of the settling time function for any initial conditions becomes crucial. This was the motivation behind introducing the concept of fixed-time convergence [20]. Worth noting is that, for fixed-time stability, the upper bound of the settling time must exhibit a certain uniformity concerning the initial conditions [20].

In recent years, numerous highly significant contributions have been made based on the fixed-time control approach [23, 24]. Notably, some of these contributions introduced the concept of predefined time control [23], which offers an upper bound of the settling time (UBST) [24] as a parameter that users can allocate in advance. Several studies have been conducted concerning the predefined time control algorithm, with a primary focus on autonomous systems.

Later, this phenomenon was further developed for nonautonomous systems and termed as "prescribed time control" [25]. Prescribed time control was designed for applications that involve a predetermined time interval $[t_0,\ t_0 + T)$, such as missile-target engagement, where the control objective must be achieved within the given time frame.

Furthermore, arbitrary time control was designed for nonautonomous systems [37]. This control technique offers users the flexibility to choose UBST (Upper Bound of the Settling Time) a priori and is defined for the entire time horizon. The applications of arbitrary time control can be seen in [38, 39]. Moreover, the nomenclature predefined/prescribed/arbitrary has been used interchangeably due to the similarity among these notions [26].

1.1.2 Sliding Mode Control

Sliding mode control (SMC) finds widespread use in designing controllers for electrical, electronics, mechanical, and aerospace systems [27]. One notable feature of SMC is its ability to achieve finite-time convergence to the designed manifold [28]. However, it should be noted that while the sliding surface is reached in finite time, the convergence of states in finite time is not guaranteed.

To understand the basic structure of SMC let us consider the nonlinear dynamical system:

$$\dot{x} = f(x, u), \;\; x(t_0) = x_0 \tag{1.1}$$

where $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}$ is the control input and $t_0 \in \mathbb{R}_{\geq 0}$ is the initial time.

Sliding mode control law is designed for the system (1.1) as follows:

$$u = -k \; \text{sign}(s)$$

where k is the positive constant and $s \in \mathbb{R}$ is the sliding surface.

Although, SMC has certain advantages over the convectional smooth controllers, however, there are certain limitations, such as control discontinuity and the occurrence of chattering phenomena, leading to undesirable high-frequency oscillations near the sliding surface. To address these limitations, higher-order sliding mode control (HOSMC) has been introduced [140]. Among the HOSMC techniques, the Super-twisting algorithm (STA) stands out as a robust control technique suitable for nonlinear systems, particularly those affected by uncertainties and disturbances [29]. STA operates as a continuous controller, retaining all the primary properties of first-order sliding mode control for systems with smooth matched bounded uncertainties/disturbances and bounded gradients.

To get a more vivid picture, let us consider the following super-twisting control is considered for the system (1.1) as follows:

$$\begin{aligned} u &= -\,\alpha|s|^{1/2}\text{sign}(s) + v \\ \dot{v} &= -\,\frac{\beta}{2}\text{sign}(s). \end{aligned}$$

where α and β are the positive constants. v is a dummy variable.

Moreover, STA faces a challenge as it does not allow the gains to decrease, potentially leading to gain overestimation and chattering during system implementation. To overcome these challenges associated with gain tuning and avoid over-estimation, adaptive super-twisting algorithm (AST) have been frequently explored in the literature [74], enhancing the performance of various SMC-based control laws.

We consider the following adaptive super-twisting (AST) control for the system (1.1) to get the underlying idea:

$$\begin{aligned} u =& - \alpha(t)|s|^{1/2}\text{sign}(s) + v \\ \dot{v} =& - \frac{\beta(t)}{2}\text{sign}(s). \end{aligned}$$

where $\alpha(t)$ and $\beta(t)$ are the adaptive gains.

Some recent works in this direction include the presentation of a barrier function-based variable gain STA in [116].

1.1.3 Parameter and State Estimation

In control theory, estimating the states of nonlinear systems within a predefined time is a significant and challenging task. In literature several designs of nonlinear observers can be found (see [3] and references therein). Nonlinear dynamics can be expressed as a sum of linear and nonlinear parts, and high-gain observers are designed by taking system gains large enough to ensure that the linear part dominates over the nonlinear part. These observers have a triangular structure and are derived from the properties of uniform observability of nonlinear systems [12]. Further, by utilizing the concept of triangular structure, an observer has been developed for a certain class of nonlinear systems in [7].

In recent years, observer design is a highly demanded field due to the advanced theory of nonlinear observability. Some fundamental works have established the relation between the nonlinear observability and the nonlinear observers [8, 12, 13, 15, 31, 42–44]. These findings are utilized to design an observer for nonlinear systems using various approaches, such as the Lyapunov approach-based observer [13], canonical form approach-based observer [15], and high-gain observers [2, 8, 9, 12, 31, 42]. Switched controller has been discussed in [147]. Recent research has focused on observers for nonlinear systems with finite-time and fixed-time convergence [30, 32, 33, 40, 45]. Although, the notion of predefined-time convergence and stability has been developed recently, the concept has been given very little attention from the perspective of observation problem [90, 146].

The system identification problem not only comprises of the state estimation but also the parameter estimation. The problem of parameter estimation for uncertain systems is equally crucial in control theory. Many control design techniques rely on the assumption that the precise model of the system is known. However, exact modeling of real systems is not possible. To address this, system identification and parameter estimation techniques have been developed. This is also important as it improves the stability and robustness of the closed-loop adaptive systems [46]. Early work on parameter estimation mostly involved least-squares [47] and gradient descent algorithms [48]. These methods guarantee convergence of the estimation error if the regressor vector or matrix satisfies the Persistent Excitation (PE) condition [51]. However, the PE condition is difficult to judge in practical applications, and these techniques are not robust and have unpredictable transient responses [52, 53]. In [54], parameter estimators were developed based on dynamic regressor extension and mixing. The parameter estimates converge to the actual parameters using the min-max algorithm in [55]. Optimization techniques are also used for parameter identification, typically involving evolutionary algorithms to solve a multi-dimensional optimization problem [56, 57]. The approach of adaptive parameter estimation has been considered often [58–60, 77, 78]. An exact estimation of the uncertain parameters is achieved in finite time using this technique in [61] but it depends on the online computation of the matrix inverses and determinants. Further, the adaptation method is independent with respect to online computation of the matrix in [62]. Recently, an adaptive approach is used to estimate the states for time-delay systems in [63]. The problem of estimating unknown parameters have been solved by using stability theory of dynamic systems [64, 65]. These designs include the construction of an identical system having the same response as the original system whose parameters are required to be estimated [66]. SMC is one of the most effective control design techniques which shows robustness against model uncertainties [67, 76, 114]. An SMC based parameter estimation has been done in [59, 68–70]. Based on this, a parameter estimation technique that is robust against model uncertainties has been proposed in [71]. This method has been used for observer design for nonlinear systems [72]. State estimation problem has been designed in [148]. Some work solve the problem of parameter estimation by including it as a part of state observer design [61, 73]. Although most of approaches can accurately estimate unknown parameters, the cost of computation is quite high, and approximate ranges of the unknown parameters are required before they can be applied to real systems, making it difficult for them to be directly applicable. Once the system identification problem is completely addressed, the next step is the stabilization of the dynamical systems.

1.1.4 Polytopic Systems

The control communities have devoted significant attention to the control design problem of uncertain systems. When a control affine model is affected by uncertainty, it can be represented as a polytopic form, which has a versatile modeling structure [128, 141–143]. The control design and stability analysis problems of linear/nonlinear polytopic systems have been extensively discussed in previous studies [80–84, 128]. The parameter-independent Lyapunov function provides more conservative results with fast-varying uncertain parameters and less conservative results with slow-varying or constant uncertain parameters [85]. To obtain less conservative results, many researchers have explored the parameter-dependent Lyapunov function [80, 81, 91], with many results involving linear matrix inequality (LMI) formulations. In the control design problem of linear polytopic systems, it is generally assumed that the uncertain parameters are explicitly present in the state part, or both the state and control parts [86].

In practice, nonlinearity is exhibited by most physical systems, and as a result, the study of nonlinear polytopic systems is highly valued today. The asymptotic stabilization of SISO nonlinear polytopic systems is addressed in [87], and this approach is extended to MIMO nonlinear polytopic systems in [88]. Although the asymptotic stability of nonlinear polytopic systems has been extensively studied in the literature, limited results are available for the finite-time stability of these systems [82, 83]. The control Lyapunov function is used to study fixed-time and finite-time stability in [10] and [11], respectively. Despite its importance, there has been little focus on predefined-time stability, as noted by [37].

1.1.5 Guidance Law

Let's delve into the concept of rated convergence in the field of aerospace and defense related technologies, particularly in the context of designing guidance laws for missiles. One example of this is ensuring that the Line-of-Sight (LOS) rate is to be made zero within a finite time. Further, the same problem has been explored to the predefined time.

The design of guidance laws for missile-target engagement is one of the main challenges in developing various guidance strategies [97, 99]. It is often necessary to achieve the desired convergence of the LOS rate, which plays a major role in the successful operation of the guidance system in the terminal phase [96]. An other various techniques are used to design the guidance laws [107, 117–124]. Proportional Navigation (PN) guidance law is one of the most used guidance laws that can be easily implemented and performs satisfactorily for non-maneuvering targets [98,105]. However, it is not effective for maneuvering targets or weakly maneuvering targets. In general situations, information about the target is unknown, demanding a guidance law capable of operating in the presence of target uncertainties. Many researchers have used Sliding Mode Control (SMC) to design guidance laws against maneuvering targets, as in [18, 99–104, 112, 125]. SMC-based guidance laws are frequently employed to handle maneuvering targets due to SMC's efficient performance for systems with matched disturbances [114]. However, SMC has its limitations, especially when dealing with chattering phenomena, characterized by undesirable high-frequency oscillations near the sliding surface. To address this, higher-order sliding modes, differentiation, and output-feedback control techniques have been developed [140]. One such advancement is STA based guidance law, designed to drive the LOS rate to converge to zero within a finite time. The STA was proposed as a solution to overcome the shortcomings of conventional SMC and to provide an even higher level of robustness and control accuracy. It has been used in the problem of differentiator design in [115]. Finite-time and fixed-time differentiators have been utilized in guidance problems using adaptive algorithms to avoid gain over-estimation in [129]. Due to the difficulty associated with tuning gain parameters and to avoid over-estimation, adaptive laws have been explored frequently in the literature, enhancing the performance of various SMC-based control laws. Recent works in this direction include a barrier function-based variable gain STA presented in [116] and Adaptive Super-Twisting (AST) laws based on the work of [74]. An event-triggered based approach has been discussed, as noted in [130–133, 137–139]. In [134, 135], researchers investigated event-triggered-based SMC for uncertain systems. In [136], researchers developed an adaptive event-triggered super-twisting control (AET-STC) technique for multi-variable second-order nonlinear systems with unknown external disturbances.

Most of the guidance laws in recent years have been based on finite-time and fixed-time convergence. Some significant works on obtaining finite-time convergence include [16, 102–104, 106, 108, 123]. A guidance law based on the Lyapunov method has been designed to achieve finite-time convergence in three-dimensional space [125]. Fixed-time stability has been discussed in adaptive terminal angle constraint interception against maneuvering targets [109, 111]. In finite-time convergence, the settling time function depends on the initial conditions, while in fixed-time convergence, the convergence time is invariant with respect to initial conditions but depends on the system parameters. To address this issue, predefined-time stability has been discussed in [37]. In this regard, no work has been reported such that the LOS rate converges to zero within a predefined time, where the settling time of the LOS rate can be chosen at the designer's discretion.

1.2 Motivation

As discussed in previous studies, finite-time and fixed-time stability have their drawbacks. The limitation of the fixed-time control is that the settling time function depends on the system parameters, requiring adjustment of the system parameters to choose different convergence times. Many modern applications demand that the desired convergence time be chosen at the beginning. Therefore, the settling time function with minimal dependence on system parameters is gaining importance. These limitations have prompted a few questions: Is it possible to estimate the unavailable states within a predefined time? Is it possible to stabilize nonlinear polytopic systems within a predefined time? Can we design a guidance law such that the LOS rate converges to zero within a predefined time?

In the early stages of parameter estimation, the primary reliance was on least-squares and gradient descent algorithms. However, these techniques' robustness has been a recurring issue, particularly when confronted with model uncertainties. To address this challenge, researchers proposed an SMC-based parameter estimation technique known for its ability to handle model uncertainties effectively. Despite the advantages of the SMC approach, conventional implementations often suffer from the undesirable chattering phenomenon, which adversely affects most systems. To circumvent this problem, HOSM based schemes have been devised as a solution. The STA is a second-order SMC used to estimate the parameters for a class of uncertain nonlinear systems. Nevertheless, using the STA approach presents its own difficulties, notably in the process of tuning gain parameters and overcoming the overestimation problem. To tackle these challenges, researchers have turned to adaptive laws based on STA to prevent overestimation and enhance parameter estimation accuracy. Estimating parameters amidst uncertainties remains an exceedingly arduous task. However, our inspiration and motivation lie in the notion of creating an identifier input based on adaptive super-twisting, which could represent a substantial contribution to the field of parameter estimation. Consequently, we have developed an identifier input using this innovative concept.

In the context of designing guidance laws, the Proportional Navigation (PN) guidance law has been an early contribution that is widely used due to its simplicity, efficiency, and ease of implementation. After that, many researchers have developed guidance laws based on SMC, STA, and AST, etc. Although most guidance laws rely on advanced sensors to obtain target acceleration information, measuring acceleration using sensors requires significant effort to achieve accuracy in the presence of unavoidable measurement noises, which can significantly increase the overall cost of system design. However, we were inspired by the potential value of developing an extended state observer-based guidance law for maneuvering targets, which would be a significant contribution. As a result, we developed an extended state observer-based guidance law.

In practical systems, the control signal is frequently used, but minimizing bandwidth and resource consumption is essential. Consequently, researchers have developed event-triggered approaches that update the control signal only during specific events, rather than continuously. Our motivation was to design guidance laws based on the adaptive event-triggered super-twisting algorithm, which we deemed as a significant contribution. This motivation led us to develop a guidance law.

After reviewing most of the existing literature, we have found that researchers have primarily focused on finite-time and fixed-time convergence. As discussed in previous studies, finite-time and fixed-time convergence have their drawbacks. Only a few researchers have discussed predefined-time convergence, where the convergence time is independent of initial conditions and system parameters. Motivated by this, we have explored the concept of predefined-time convergence in the design of a guidance law that ensures the LOS rate converges to zero within a predefined time.

1.3 Objectives

The objectives of the work is stated as follows:

- To improve the estimation of unknown states in terms of convergence time, i.e., predefined time/desired time.
- To enhance robustness against parametric uncertainty within a predefined time.
- To estimate the unknown target acceleration and utilize this information in designing a guidance law that ensures precise convergence.
- Further, to design a guidance law using event-triggering approach that minimizes control effort against unknown target acceleration with a known upper bound, while ensuring the LOS rate converges to zero.
- Finally, to design a guidance law that ensures achieving convergence within a predefined time, even in the presence of unknown target acceleration.

1.4 Organization of the Thesis

This thesis is comprised of seventh chapters. This chapter introduces a brief idea of the work in the thesis. It provides the motivation behind the work, detailed literature survey to show the research gap. Further, the main objectives of the proposed work are outlined.

In continuation, the second chapter outlines the preliminaries which includes the important notions and definitions that are used throughout the thesis.

In the third chapter, a switched high-gain observer for nonlinear systems is proposed with the desired convergence time. A switching structure is introduced that plays an essential role, resulting in desired convergence. Moreover, the convergence time can be chosen at the will of the designer. Finally, two practical examples are presented through simulation.

In the fourth chapter, parameter estimation problem for a class of uncertain nonlinear systems is discussed. The unknown parameters are estimated using an adaptive super-twisting algorithm in the presence of uncertainties. The sufficient condition for the Lyapunov stability is discussed. An explanatory example is presented. Simulation results establish the proposed estimator's satisfactory performance even under the presence of uncertainties.

The fifth chapter deals with predefined-time control for nonlinear polytopic systems. With such a control, the settling time function is uniform with respect to initial conditions of the system and can be chosen by the designer. Moreover, the closed-loop nonlinear polytopic system has a robust control Lyapunov function (RCLF) for all possible parametric uncertainties. Finally, a practical system shows the efficacy of the proposed approach.

In the sixth chapter, an adaptive super-twisting algorithm based guidance law is designed for planar geometry of missile-target engagement. Further, the unknown target acceleration is estimated by using an extended state observer. The estimated information of the uncertain target acceleration is utilized in the design. Simulation results for a practical example are presented which show satisfactory performance of the designed guidance law.

Further, an event-triggered adaptive super-twisting algorithm (ETASTA) based guidance law for a planar relative motion is introduced. Further, a triggering condition is provided using an event-based approach that uses the minimum amount of control while meeting the stability requirements. Finally, a practical example demonstrates the efficacy of the proposed guidance law.

After that, a guidance law is introduced with predefined time for planar motion. The proposed scheme is based on a switched control action which results into a desired convergence of the LOS rate to zero or a small neighborhood of zero. The sufficient condition for the Lyapunov stability is discussed. The method is illustrated by considering a practical example which shows the effectiveness of the proposed technique.

Finally, the seventh chapter concludes the overall work carried out in this thesis and provides the future work direction.