

5 Applicability of reliability concept in square-base conic shaped Tensile Membrane Structure

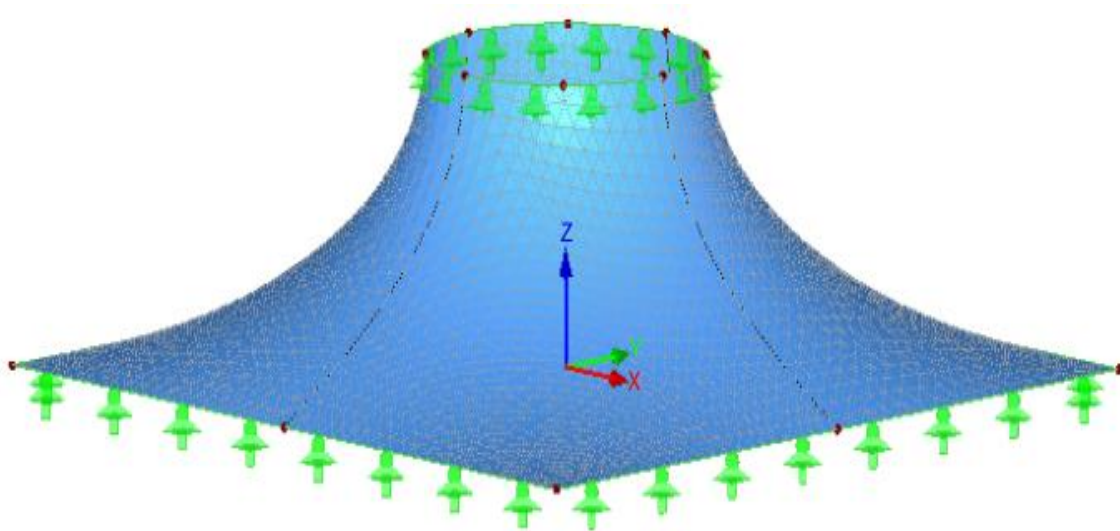


Figure 29 An isometric view of a Square-Base Conic Shaped TMS.

5.1 General

Each tensile membrane structure is an archetype itself and not the duplicate one recreated following a fixed procedure. Due to their recent history in design and analysis, the researchers have not been able to achieve a consolidated design pattern. Hence due to no proper design code, the TMS are designed based on experience, pragmatism and engineering judgment of the structure. There are two primary design requirements for the general use of these type of structures more efficiently, and these factors are the need of skilled and expert personal for the design. The other is the availability of the suitable material which can be handled easily and have high longevity (Burger 2012).

In this chapter, firstly there is an attempt to bring all the statistical and structural information for square-base conic shaped TMS together. A non-linear finite element methodology is applied to contain the unusual behavior of the conic shaped TMS. Besides the demonstration of a finite element based design methodology a reliability assessment through the calculation of reliability index based on the various limit states parameters is

accomplished and verified with the help of existing design approaches and researches in the domain.

The form finding is the most basic and most important step to begin with the design process for a membrane structures. The initial surface stresses are induced and manipulated to obtain a desired and an equilibrium form. The surface of a fabric membrane is comprised of the orthogonally woven yarns in warp and fill/weft directions. The non-rigidity in the TMS is the main reason of the highly unconventional behavior of these type of structures. The slightest variation in the membrane surface may cause a big difference in its structural behavior. As explained earlier in the literature study, the membrane structures are commonly classified in 3 basic shapes: - the hyper, the conic and the barrel vault. The common most characteristics in these three TMS shapes is that, they inherently acquire a double curvature geometry in order to withstand the loads and stresses provided by the specified loading environment. In addition to resistance to the external loads, the double curves are often shaped to give an aesthetic excellence to the TMS.

Considering the weightage of loading and the double curvature into the design and analysis estimation of the TMS, the form-finding of square-base conic shaped membrane structure is undertaken by inducing isotropic and anisotropic pre-stressing. To consider the effects of curvature in obtaining the structural responses there is another variability included in this work, the height of conic TMS is varied over a range of 3.0m to 5.0 m. This variation of height obtains the required stresses and deformations as a response to different load and load combinations for each height case of the structure.

The inherent non-linearity in the behavior of membrane structures makes design and analysis procedure not so easy. In the subsequent part of this chapter a design approach involving permissible stress method by reducing the original material strength characteristics using the reduction factor adopted from the IASS recommendations for TMS design. This reduction factor is implemented to take in to account of the uncertainties and stochasticity involved with the input parameters to obtain the desired structural responses. The principles of BS EN 1990:2002 “Eurocode - Basis for Structural Design” are applied to a square-base conic membrane structure and the implications of design and analysis are explained. The key feature to this approach is the prediction of safety of conic TMS through the application of statistical mathematical tools as an

explicit function of inherent uncertainty involved in the loading environment, material characteristics and the geometry of the structure. This new approach to membrane structure analysis is demonstrated through implementing it to a square-base conic membrane structure, which show the application of reliability analysis in achieving the objectives of “Eurocode - Basis for Structural Design” and highlight the further work that is required before this approach can be fully utilized by industry.

5.2 Description of the TMS

The tensile membrane structure considered for two case studies here is a simple, real conic structure with a rigidly supported square base and a circular head ring at the top center. For the first case study, the details (plan dimension and elevation) of the structure is shown in Figures 3.1(b) and (c). This TMS is adopted from a recent round-robin exercise on the analysis and design of membrane structures [Gosling et al., 2013b], which is considered as a benchmark problem for the present study. The following information has been provided for the case studies:

The description of the Square-base conic TMS case is given below:

1. The membrane yarn (warp and fill) directions are in the principal directions: warp along radial direction and fill along circumferential direction.
2. Initial prestress is applied in the warp and fill directions.
3. The warp and fill directions coincides with the element local x and y directions respectively.
4. Material properties: Modulus of elasticity, $E = 600.0 \text{ kN/m}$; Poisson's ratio, $\nu = 0.4$; Yield stress, $f_y = 40.0 \text{ kN/m}$.
5. Thickness of the membrane is 1 mm.
6. Geometry: Square base– $14 \times 14 \text{ m}$; Head ring– 5 m above base, 4 m diameter for the first case study and an isotropic pre-stress of 4 kN/m in both warp and weft direction is applied.
7. In the second case study, the anisotropic pre-stress of Pre-stress: warp = 4 kN/m ; fill = 2kN/m

The TMS surface is discretized with the CST finite elements and the form-finding analysis begins with the initially defined surface topology. As the thickness of the fabric membranes used in the construction of TMS are negligible as compared to the overall dimension of the structure, hence the stresses along the thickness of the membrane are considered as negligible. Therefore it should be noted that the stress values are expressed in kN/m, force per unit length of the membrane. There are three DOFs per node (u_x, u_y, u_z) along the global X, Y, Z, coordinates. The edges of the structure are lined using the stainless steel cable. The supports are hinged with the boundary condition $u_x = 0, u_y = 0, u_z = 0$. A detailed description in elevation and plan view of the hyper TMS is shown in the Figure 30.

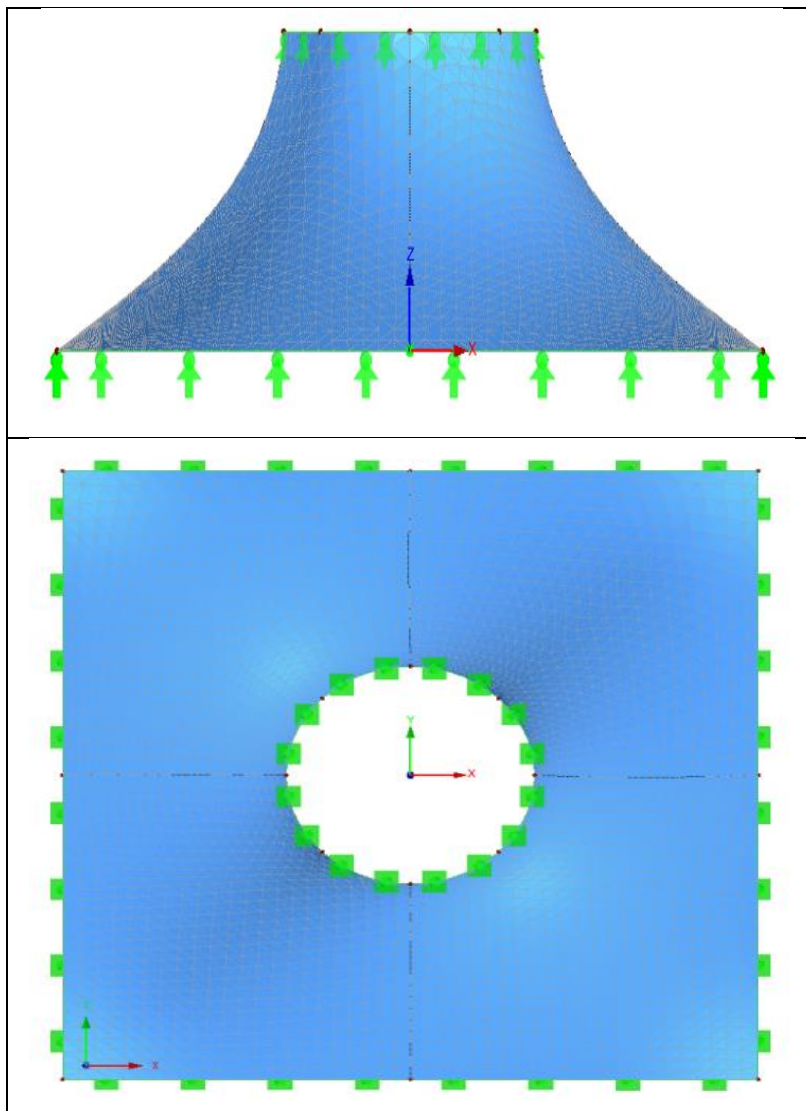


Figure 30 The elevation plan view of Square-Base Conic TMS

5.3 Current design philosophy

Knowing the fact that, the surface pre-stress is responsible for carrying principal loads within the surface for a TMS with the help of a supporting structure, unlike steel, concrete or timber building elements which react to the external forces through bending. The Fabrics used in TMS possess negligible compression and bending stiffness, which indicates that the membrane structures must be designed with sufficient curvature to facilitate the structure in resisting the environmental loads through in-plane tensile and shear forces.

The shape and orientation of the TMS is vital to its ability to withstand the applied external loads predominantly in stressed form. To cope up with both the uplift forces and the downward forces, imparted mainly due to wind and snow respectively, the surface membrane must possess a doubly curved orientation and pre-stresses. To this extent it can be concluded that the detailed shape of a doubly curved surface structure is critical to its engineering and architectural performance. The curved geometry coupled with the initially induced pre-stress governs the distribution and magnitude of the stresses and the deformations subjected to applied load cases.

The fundamental basis of most of the computational systems implemented for the purpose of design of fabric membrane structures is itself a form of equilibrium modeling. The structure is discretized in small finite elements mesh with fixed topology, but with exception of fixed points, rather approximate nodal coordinates in local axis. Based on the type of finite element selection, the internal and external forces are determined and observed at the nodal points. Once external loads are applied, the out-of-balance forces at the nodes can be estimated. The equivalent nodal coordinate values can be obtained through a directed perturbation of the finite element mesh based on these residual forces. One of the main advantages of using such a computational approach to form finding is that the subsequent prediction of member forces under both pre-stress and applied load is facilitated.

The hyperparaboloid TMS chosen for this study is analyzed for the following cases:

- (a) Pre-stress: warp = fill = 4kN/m;
- (b) Pre-stress: warp = 4kN/m and fill = 2kN/m

The main focus of this study is to analyze the TMS behavior in terms of deformation and the plane stresses under the influence of pre-stress and other loads. However, one grade of irregularity is also introduced to have a broader range of design possibilities. It is proven from a past case study of the Seoul Southwestern Baseball Stadium dome and the Jeju World Cup Stadium dome that the geometry of a TMS is the most influential factor. The geometry of a square-base cone shaped membrane structure can be easily molded by changing the curvature observed between the fixed head ring of the membrane and the base of the structure. Hence the curvature of the TMS is varied over a range between $H = 3.0\text{m}$ to 5.0m , a detailed visualization of how the curvature is changing with height of the opposite high points is presented in the Figure 31 below.

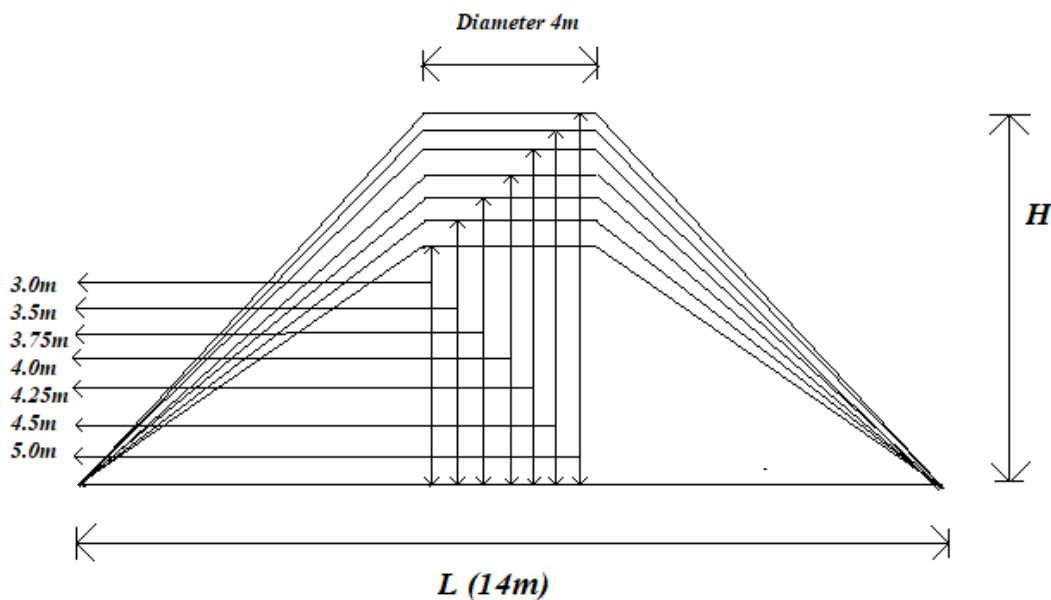


Figure 31 The height variation profiles for Conic TMS adopted in this case study

The external loads apart from pre-stresses are wind load and snow load and their combinations, the combination and classification of load cases considered for this study are implemented according to standard: ASCE 7-16 (minimum design loads and associated criteria for buildings and other structures), and the different load combinations are :

- (f) Pre-stress
- (g) Pre-stress + LC1(wind-uplift)
- (h) Pre-stress + LC2(snow-load)

- (i) Pre-stress + CO1(1.6*LC2)
- (j) Pre-stress + CO2(0.5*LC1+1.6LC2)

For this study, the two square-base conic models are designed investigating the curvature parameter as mentioned in previous section. After imparting a fixed geometry case-wise, the analysis procedure starts with the form-finding analysis, the proper orientation and placement of the coordinates of the model are obtained in response to a corresponding balanced initial stress. The capacity of the curvature shape to achieve a proper balanced stress is also contemplated as an important factor for the design limitations.

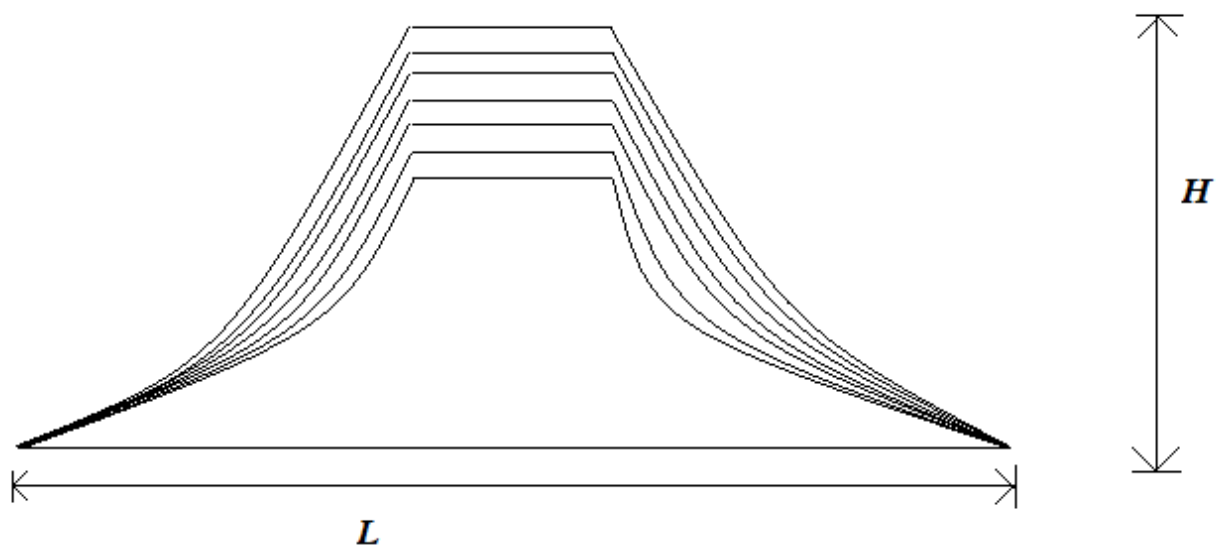


Figure 32 Basic form-found curvature profile of Square-base conic TMS after application of initial Pre-stressing.

After obtaining the form found square-base conic models, the stress deformation analysis under the above mentioned loading conditions is performed. From the results, the two design based limitations are identified :- (a) the excess of the maximum allowed stress, which signifies that the model is not safe, and (b) the development of negative stresses after the loading analysis, this indicates wrinkling on the surface of the membrane and thus the possibility of ponding due to down-ward force due to snow loading.

5.4 Identification of the Limit State Functions

An allowable strength of 6.667kN/m is implemented, which resembles a PVC coated polyester membrane with ultimate tensile fabric strength 40kN/m subjected to a stress reduction factor 6. The consideration of load cases and combination is implemented

according to ASCE 7-16 (minimum design loads and associated criteria for buildings and other structures).

The limit state for this structure can be defined as:

- Limit state of fabric failure : $G_1 = \sigma_{permissible} - \sigma_{max}^w$.
- Limit state of wrinkling failure: $G_2 = \sigma_{minimum}^p - \sigma_{permissible}^p$
- Limit state of deflection failure: $G_3 = D_{allowable} - D_{Maximum}$

The allowable deflection for the study is arbitrarily chosen as $L/60$, i.e. 100mm, due to lack of proper guidelines the main reason for such a limit state for deflection is mainly centred to address the issues like, overlapping of membrane with supporting steel work and ponding due to precipitation. $\sigma_{permissible}$ is calculated as, $\sigma_{permissible} = \sigma_{ultimate}/6$ and $\sigma_{permissible}^p = 0$. For this study the random variables considered are supposed to be normally distributed and a standard deviation of 10% (COV = 0.10) of the mean value for each variable is set initially.

In this TMS example the probability of failure of a pre-stressed hyperbolic paraboloid made up of poly vinyl chloride coated fabric membrane is considered. A combination of both probabilistic (statistically defined) and deterministic parameter is implemented. The selection of the values of a statistical parameter is decided based on the available statistical information.

5.5 Results and Discussions

The results of the analysis have been recorded on the detailed forms which appeals the values of maximum warp stress and minimum fill stresses. Also the reactions at for each in terms of maximum membrane deformation along the z-axis direction, for each height case for different loading environment. A number of assumptions are made in order to facilitate the design analysis process. The minimum tensile strength in fill direction is considered to define one of the limit state of design as mentioned in the previous section.

The obtained results from the analysis are presented in two stages for both isotropic and anisotropic pre-stressing cases. The form-finding analysis is performed with the help of obtainment of deformation and stress responses for the pre-stressing loading case only. The effect of height variation is also depicted in the form finding procedure and the

results of deformation and stress variations in warp and fill directions are demonstrated with the help of graphs and simulation models.

The next stage is stress-deformation analysis, the square-base conic TMS is subjected to different kind of loadings applied as per ASCE 7-16 standards. The main objective of this practice is to find the structural responses of the conic TMS under wind load, snow load and their combinations. The results are organized in terms of stresses (warp/fill) and deformations. The findings of this exercise shows very interesting correlation with the earlier findings of Gosling et.al. (2013), and Dutta et.al. (2017).

5.5.1 Form-finding analysis bound for different height of square base conic shaped TMS in the isotropic pre-stressing scenario.

The application of initial pre-stressing of 4.0kN/m along both warp and fill/weft directions for a square-base conic TMS, have resulted in obtainment of a minimal surface area orientation as the observed surface stresses are almost equivalent to the applied stresses, i.e. 4.0 kN/m. As it can be seen from table below, the maximum warp stresses and minimum fill/weft stresses are both very close to the magnitude of the applied pre-stress. The deformation results for form finding is found to be exceeding the limit state value of $L/60$, which is equivalent to 233.33 mm. ($L=14.0m.$). This shows that the deformation limit state is can only be taken as a serviceability criteria fulfillment unlike ultimate limit states. The outcome of form finding in terms of warp and fill stresses and the maximum deformation along the z-axis is described in Table 30.

Table 30 Description of deformation and stresses for form-finding of Conic TMS (Isotropic pre-stress)

Height in m.	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
Warp stress, n_x (kN/m)	4.002	4.003	4.002	4.003	4.003	4.002	4.005
Fill stress, n_y (kN/m)	3.98	3.98	3.97	3.97	3.96	3.96	3.97
Deformation along z-axis (mm)	838.0	978.6	1051.6	1125.2	1205.1	1291.0	1490.0

There is a continuous increase in the deformation along the z-axis experienced by the membrane surface. The range of the deformation in magnitude lies between 800 mm. to

1500 mm, for the variation of the height from 3.0m to 5.0m. graphical representation of variation of deformation with height is depicted in the Figure 33.

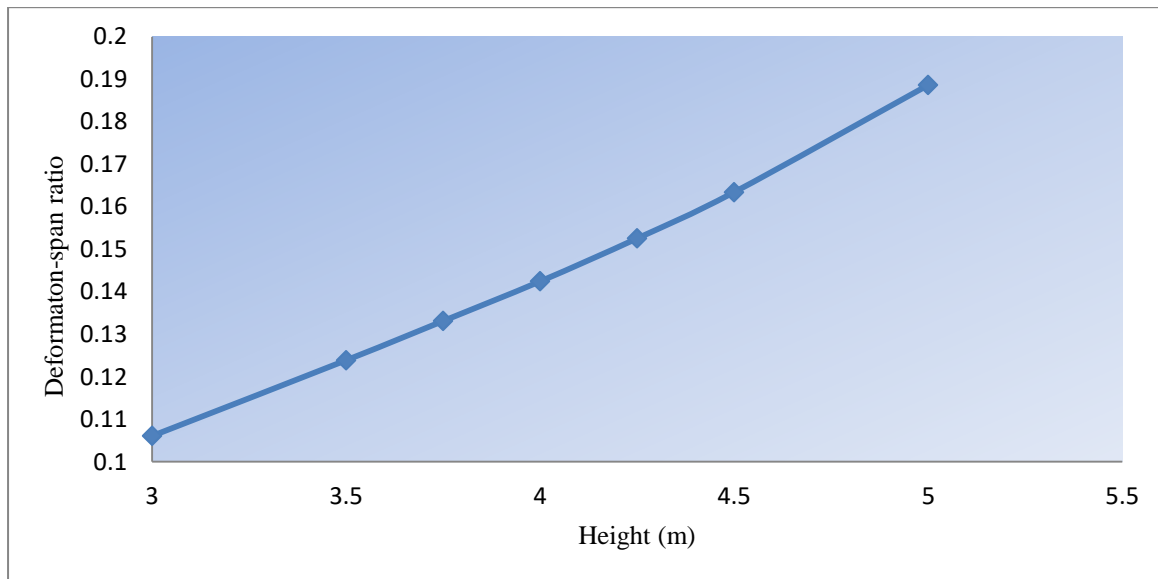


Figure 33 Deformation-span ratio for deformation along z-axis for form-finding of conic TMS (Isotropic pre-stress)

Given below in Figure 34 are the representation of the deformation contours for each height of the conic TMS. The obtained results in the form of simulation models suggest that there is an uneven form of the conic membrane is found with the regions of high deformation concentrated within a smaller area. The high deformation are mainly experienced along the diagonal line connecting top right corner (see figure 35a to 35f) to the head ring of the structure. It is also observed that with increase in height, the area of high deformation gradually starts increasing and the development of high higher deformation profiles were evidenced along the membrane edges bisecting line. The rest of the surface of membrane remains at the minimum deformation around the limiting value adopted for this study. This indicates the high uncertainty in the design of a conic TMS with isotropic pre-stressing, while taking deflection limit state as a primary design parameter for membrane failure.

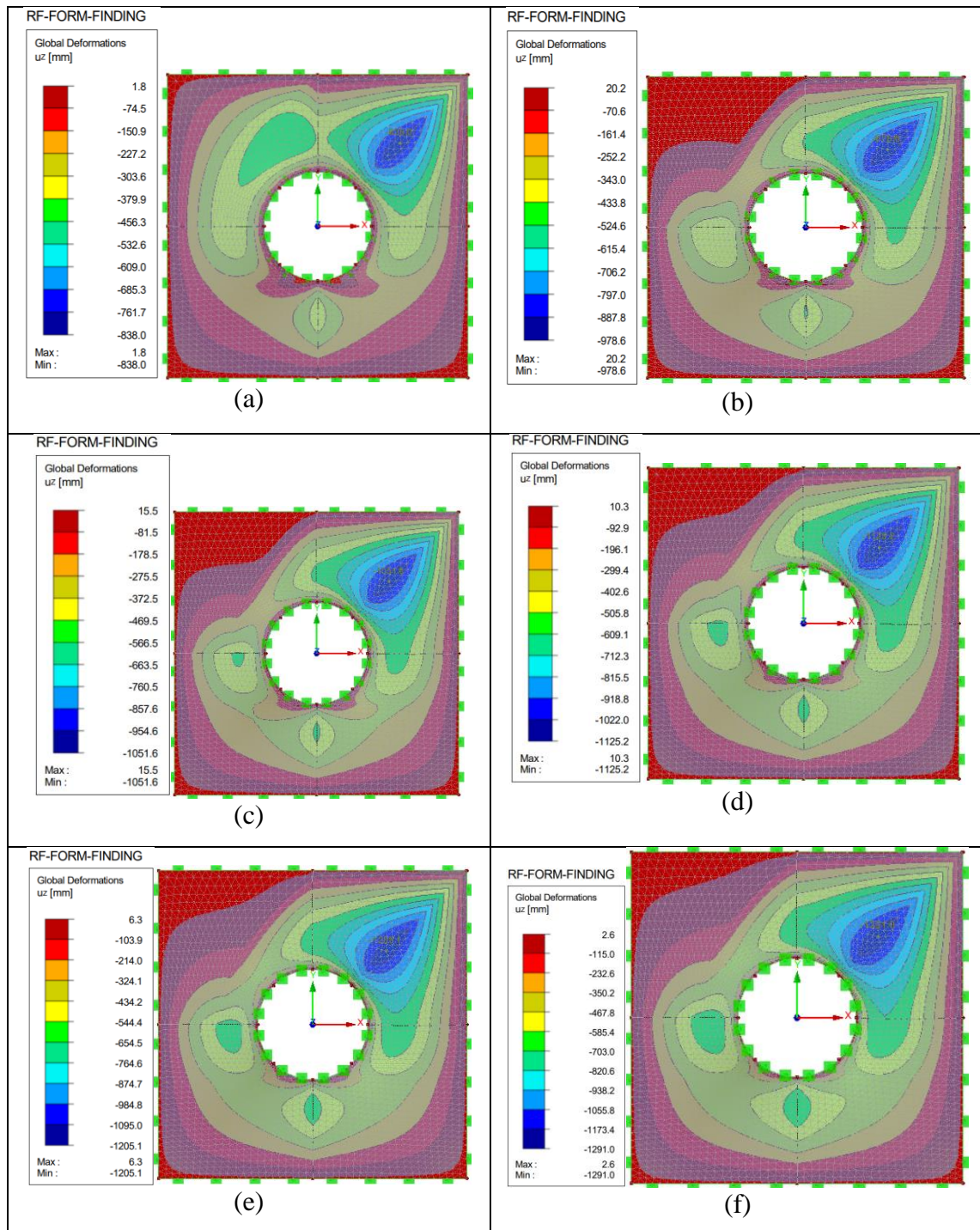


Figure 34 Deformation contours of form-finding process for conic TMS (Isotropic pre-stress)

The stress development contours for the height case 5.0m is displayed in the Figure 3436 below. Looking into the obtained models the form of the conic membrane is found to be very near to the close to minimal surface geometry. Both warp and fill stress contours show a variation of 3.97 kN/m to 4.005kN/m stresses, which seems to be approximately equal to the applied pre-stress which is 4.0kN/m. It can be said that adopting the limit

state design criteria using the ultimate strength of fabric, is a better design approach to form-find the initial shape of the square-base conic shaped membrane structures.

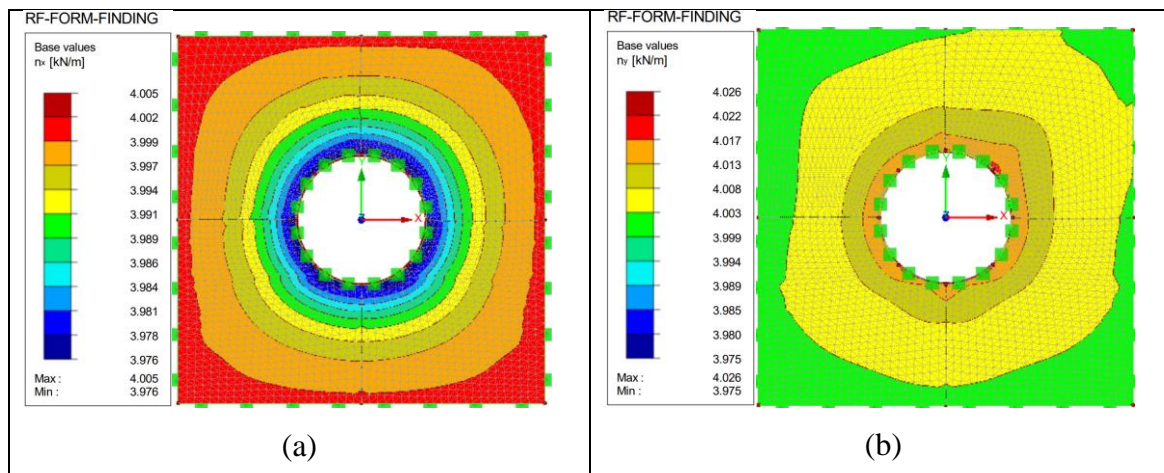


Figure 35 The Warp stress (a) and fill stress (b) obtained after form-finding of the conic TMS(Isotropic pre-stress) with height $H = 5.0$ m.

5.5.2 Form-finding analysis bound for different height of square base conic shaped TMS in the Anisotropic pre-stressing scenario.

The membrane surface responses after application of an anisotropic pre-stress of 5kN/m in warp direction and 2.0kN/ in fill/weft direction are tabulated in Table 31 below. The stress magnitude suggests that an acceptable form is found for the prescribed case of conic TMS. The values of maximum warping stresses and minimum fill stresses are found to be closer to the applied initial pre-stressing. This indicates the attainment of close to minimal surface area geometry of the membrane surface.

Table 31 Description of deformation and stresses for form-finding of Conic TMS (Anisotropic pre-stress)

Height in m.	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
Warp stress, n_x (kN/m)	5.79	5.77	5.75	5.75	5.73	5.71	5.68
Fill stress, n_y (kN/m)	1.92	1.93	1.93	1.93	1.93	1.94	1.94
Deformation along z-axis (mm)	1240.1	1342.3	1445	1548.98	1656.6	1666.5	1885.1

The deformation results in the other hand shows very high amplitude deflection experienced by membrane surface, during form-finding analysis. The minimum deformation observed at height 1.5m is itself high enough to be counted closer to the

allowable deflection limit. The range of deformation for this TMS case varies from 1.0m to 2.0m with change in the height of the conic TMS. The deformation contours for different height is demonstrated in the Figure 36 below

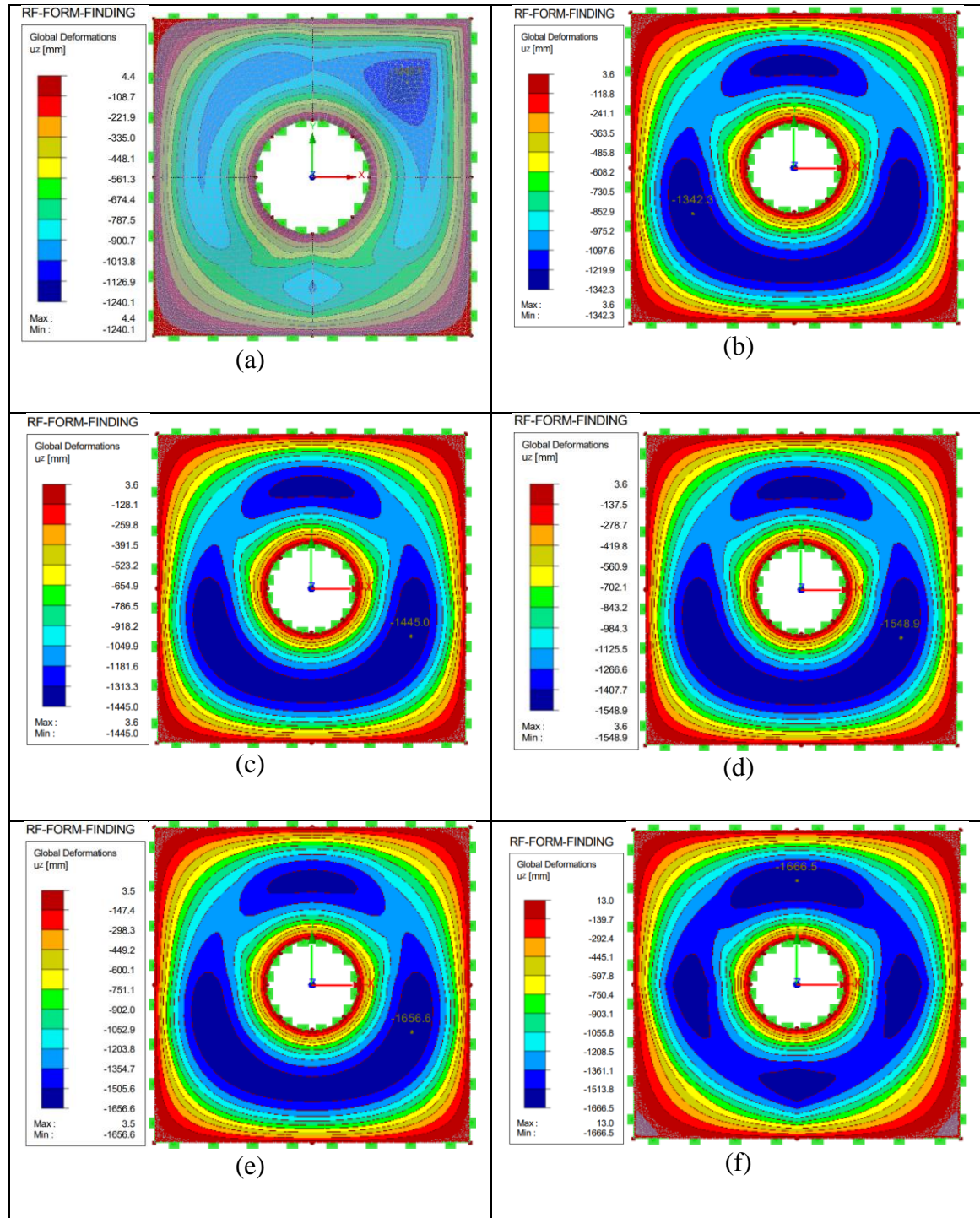


Figure 36 Deformation contours of form-finding process for conic TMS (Anisotropic pre-stress)

The deformation contours shows that there is the development of high deformation zone around the head ring and the constrained boundaries of the conic TMS. The increase in the height of the structure has resulted in the development of a neck like orientation with the maximum deformation of 1885.1 mm for the structure.

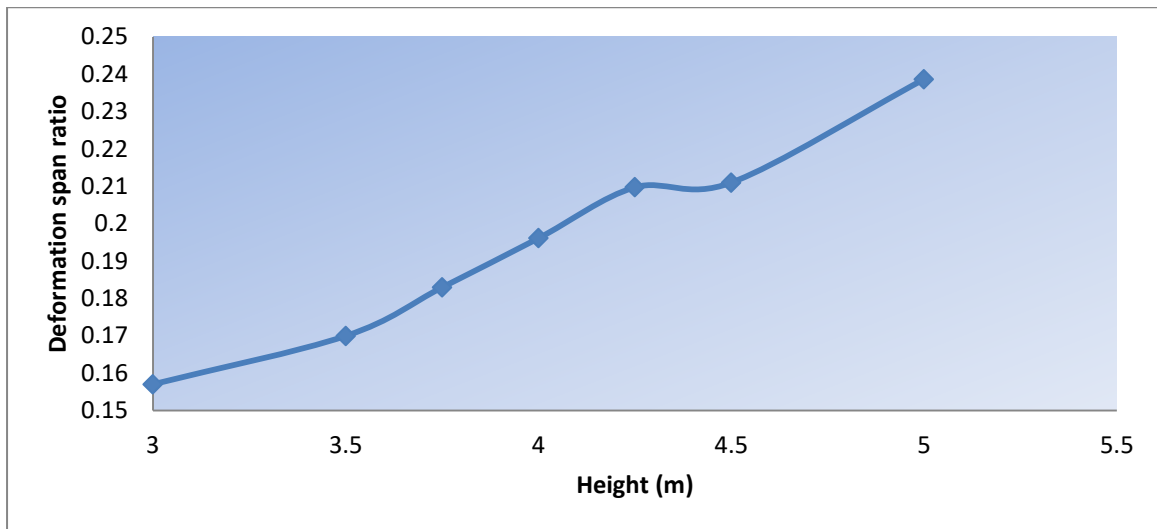


Figure 37 Deformation-span ratio for deflection along z-axis for form-finding of conic TMS (Anisotropic pre-stress)

The variation of height have resulted in the continuous increase in the magnitude of deformation along z-axis. As the values obtained for deformations are very high in magnitude, the deflection limit state for design in this case of TMS is again found to be limited to the serviceability criteria fulfillment of the structure.

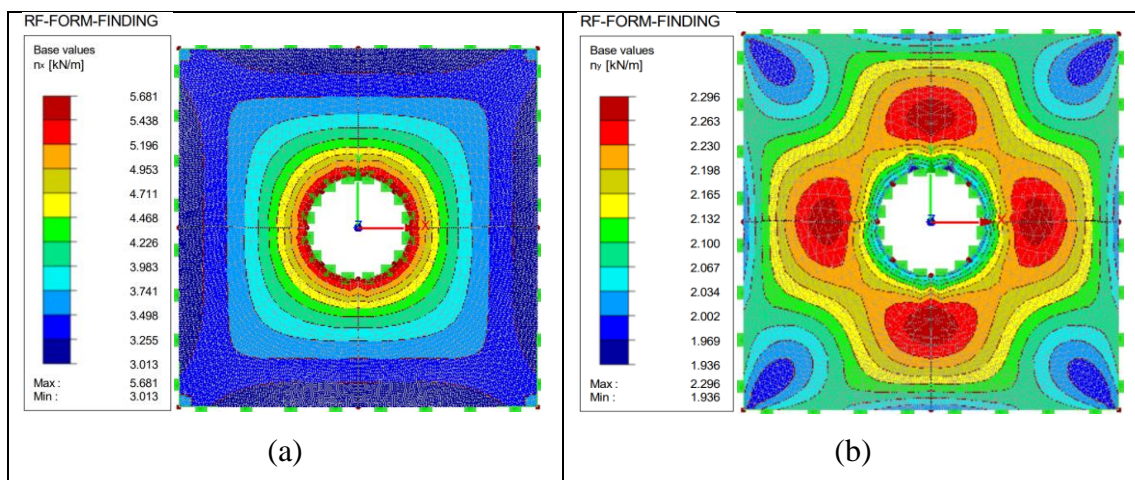


Figure 38 The Warp stress (a) and fill stress (b) obtained after form-finding of the conic TMS(Anisotropic pre-stress) with height $H = 5.0$ m.

The warp direction stresses and fill direction stresses for the height case 5.0m are displayed in the Figure 38 above. The development of surface stresses show an elegant and systematic development of surface stresses in response to the external loads.

5.5.3 Stress-deformation analysis of square-base conic TMS for various loads and load combinations with isotropic pre-stressing

The form-finding procedure has so far been a successfully applied for both the type of pre-stressing in conic TMS. The load analysis for various external loads primarily comprising of wind load, snow load and their combination as per ASCE 7-16 national annex is accomplished as a part of stress deformation analysis for conic TMS. The principles of surface response collection as similar to those explain in section 4.5.3. The stress deformation analysis is important aspect to carry out in order to decode the highly unconventional/non-linear behavior of the non-rigid membrane structures. The results of maximum warp stresses (along x-axis) and minimum fill/weft stresses (along y-axis) for the various loads and load combinations are displayed in Table 32 and Table 33.

Table 32 The maximum warp stress responses for various load cases and different height for isotropic pre-stressing of Conic TMS.

Load cases as per ASCE 7-16	Maximum warp stress kN/m for variation of height H						
	3.0 m	2.0m	2.5m	2.75m	3.0m	3.25m	3.5m
Pre-stress	4.00	4.00	4.00	4.00	4.00	4.00	4.00
LC1(wind-uplift)	4.45	4.47	4.50	4.55	4.53	4.55	4.56
LC2(snow-load)	4.31	4.32	4.31	4.30	4.30	4.30	4.30
CO1(1.6*LC2)	4.40	4.43	4.46	4.25	4.44	4.25	4.19
CO2(0.5*LC1+1.6LC2)	4.26	4.26	4.25	4.25	4.24	4.24	4.24

The Application of initial isotropic pre-stress of 4.0kN/m in both warp and fill direction have resulted in a very sustaining outcomes in terms of maximum warping stress. After a perfect form-finding analysis with the attainment of minimal surface area of the membrane surface, the stress-deformation analysis shows equally satisfying results. The maximum warping stress is found to be lying very close to the applied initial pre-stress.

As can be seen from the Table 32, most of the warp stress outcome are either equal to or above the value initially applied pre-stress. All the load cases corresponding to the different height of conic TMS have achieved the maximum warping stress value in the range of 4.00 kN/m to 4.56kN/m, for form-finding case of each height and for wind-uplift case for the height 3.50m respectively.

Table 33 The minimum fill stress responses for various load cases and different height for isotropic pre-stressing of Conic TMS.

Load cases as per ASCE 7-16	Minimum fill stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
Pre-stress	3.98	3.98	3.97	3.97	3.97	3.96	3.97
LC1(wind-uplift)	2.75	2.43	2.13	1.49	1.75	1.49	0.81
LC2(snow-load)	3.77	3.75	3.75	3.74	3.75	3.76	3.81
CO1(1.6*LC2)	3.63	3.48	3.32	3.75	3.67	3.75	3.80
CO2(0.5*LC1+1.6LC2)	3.77	3.74	3.74	3.73	3.73	3.75	3.75

The results of minimum fill/weft stresses are tabulated in Table 33, it can be interpreted from the minimum stress results is that except wind load case for large height of conic structure, all the other loads and load combinations show very good consistency with corresponding warp stresses. All the values of obtained minimum fill stresses are found to be closer to the initially applied pre-stress. The wind load in exception show a descending trend in terms of minimum fill stress. The minima of the minimum fill stress is found to be 0.81 kN/m for the load case wind uplift corresponding to height 5.0 m. This implies that the possibility of membrane fabric wrinkling or slackening is predicted to take place when the height of a conic TMS with square base is increased further.

The warp and fill stress development contours during the stress-deformation analysis of the conic TMS is performed for height 5.0m, are displayed in the Figure 39.

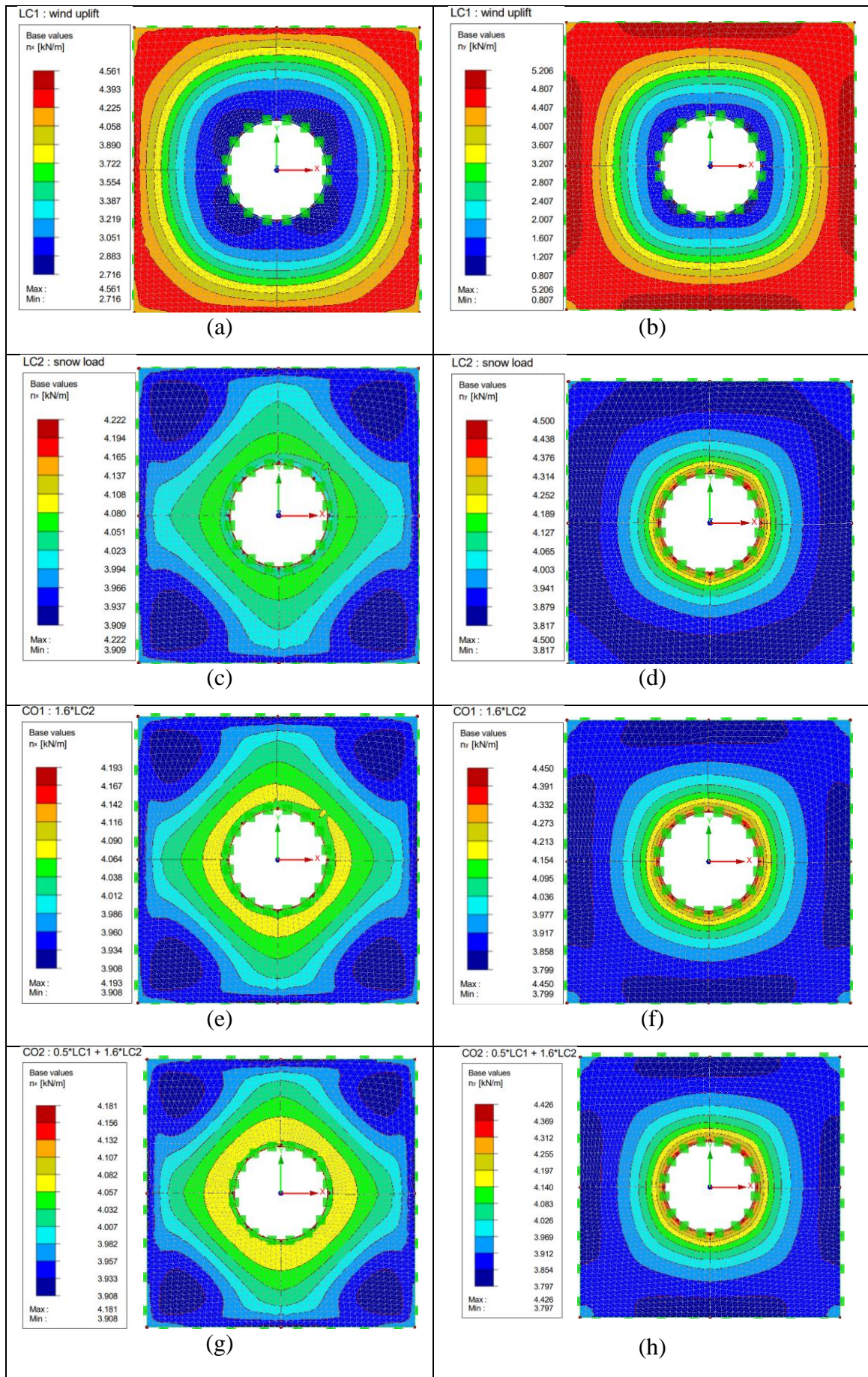


Figure 39 The warp and fill stresses development contours after stress-deformation analysis for isotropic pre-stressing of Conic TMS

5.5.3.1 Maximum deformation of TMS due to change in the coefficient of curvature for Isotropic pre-stressing of square-base conic shaped TMS.

The obtained deformations along z-axis shows a very unusual behavior, the extent of deformation is observed to be very high with the maximum value of above 3.5m for the wind-uplift load case for height 5.0m. The magnitude of maximum deformations along z-axis are tabulated in Table 34.

Table 34 The maximum deformation results various loads corresponding to Isotropic pre-stressing of Conic TMS.

Load cases as per	Variation of the height (H) in m.						
	3.0	3.5	3.75	4.0	4.25	4.5	5.0
ASCE 7-16							
Pre-stress	838	978.6	1051.6	1125.2	1205.1	1291	1490
LC1(wind-uplift)	2098.2	2356.7	2661.6	2816.1	3059.3	3310.7	3792.3
LC2(snow-load)	1133.4	1290.2	1371.2	1461.7	1563.1	1707.4	1810.2
CO1(1.6*LC2)	1522.0	1927.7	2228.2	2546.1	2395.6	1386.8	1224.5
CO2(0.5*LC1+1.6LC	888.3	1020.3	1089.2	1162.8	1241.5	1336.2	1425.

2)

The deformation results suggest that the Conic TMS in isotropic pre-stressing case is highly unstable, but then looking into the stresses obtained in the previous section of the chapter it seems that there are good chances of the current methodology to perform the overall analysis of similar type of structures.

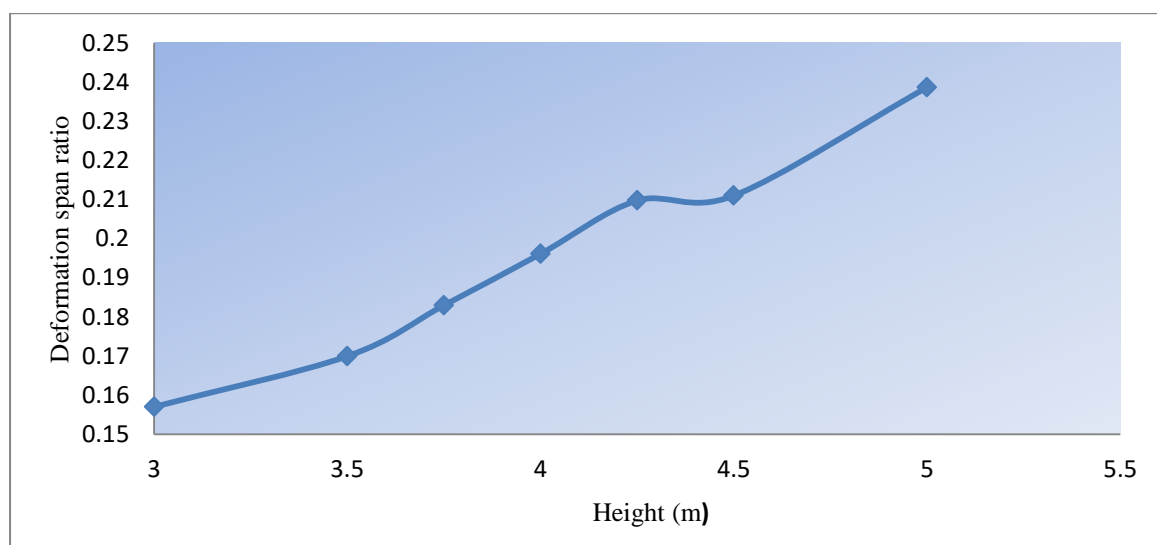


Figure 40 The deformation span ratio under wind-uplift for deflection along z-axis for Conic TMS (Isotropic pre-stress)

5.5.3.2 The demonstration of deformation contours for conic TMS (Isotropic Pre-stress)

The deformation contour models for stress-deformation analysis were generated and presented in this section.

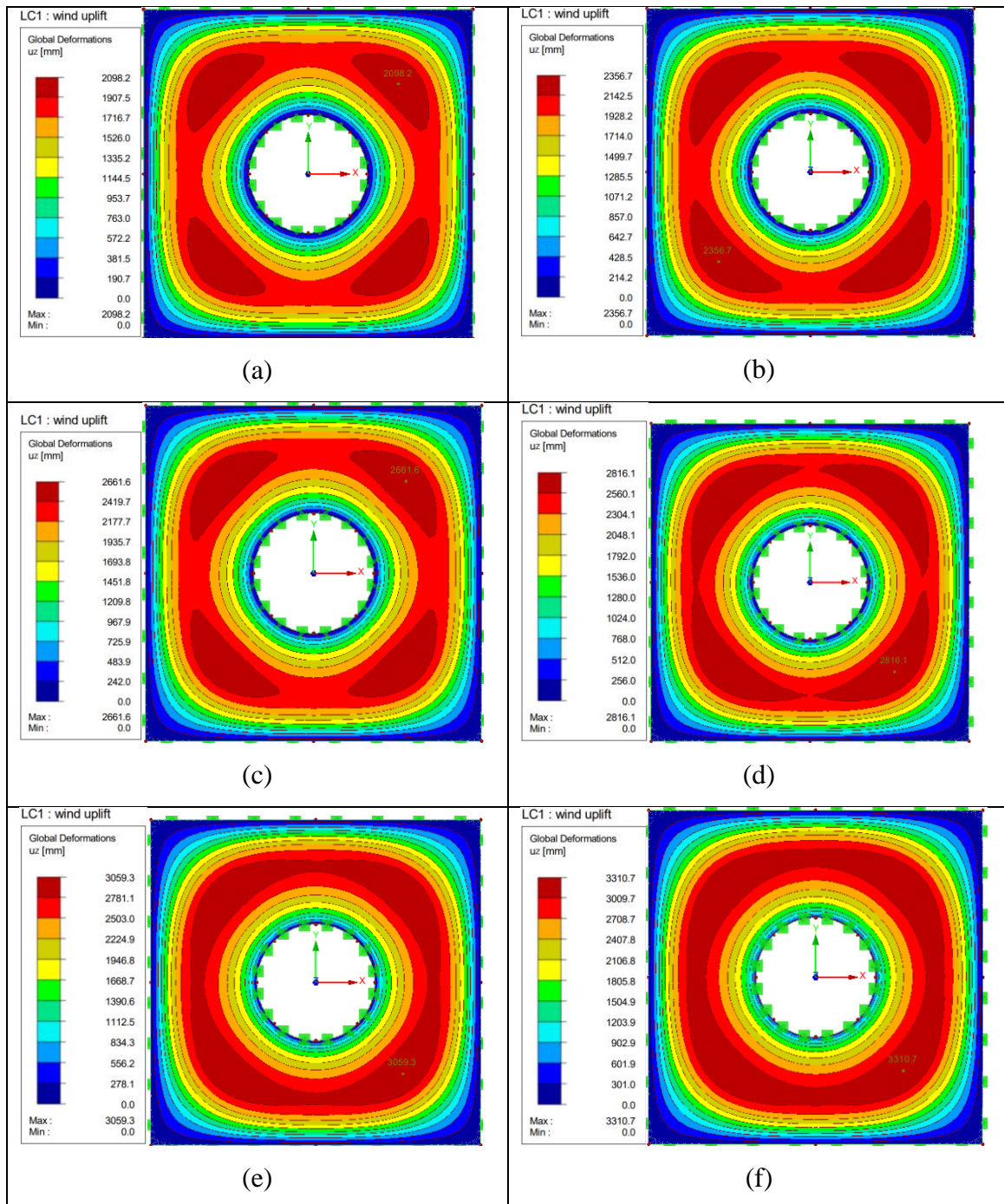


Figure 41 The deformation contour models for wind uplift load for Conic TMS (Isotropic pre-stress)

The wind-uplift induced deformations take very high values and may require a different approach to be explained correctly, the current limiting value criteria obtains very small value of allowable deformation which is $L/60$, where L is the distance between the circumference of the head-ring diameter to the nearest corner of the square base conic TMS. This signifies that in order to implement the deformation criteria to define the limit state design point of a conic TMS, the limiting value of the deformation should be chosen wisely.

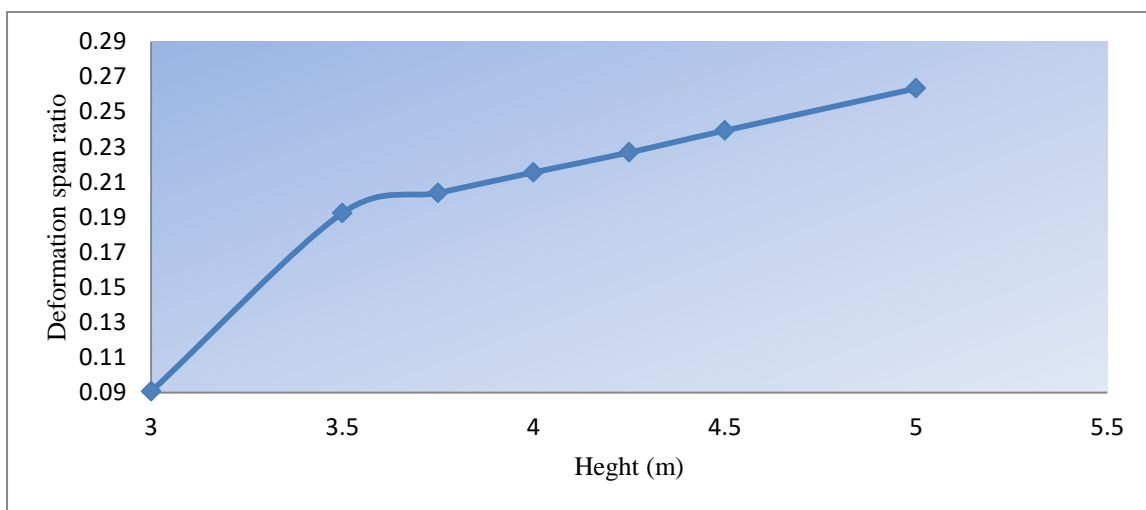


Figure 42 The deformation span ratio under snow load for deflection along z-axis for Conic TMS (Isotropic pre-stress)

The snow load of magnitude 0.6kN/m^2 in the vertically downward direction have also resulted in the high value of deformation, but not as high as in the case of wind-uplift. There is an increasing trend in deformation values with increase in the height of the structure.

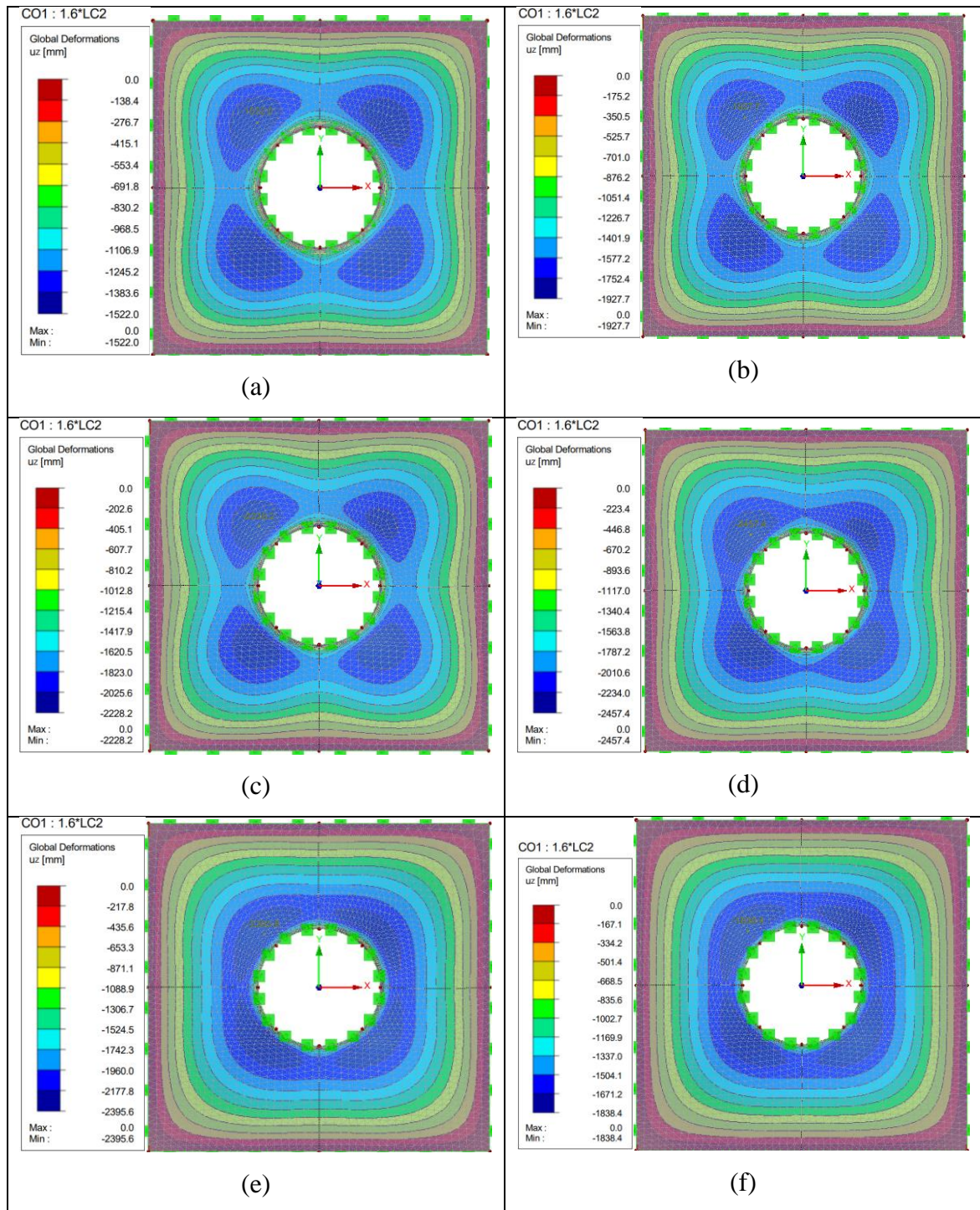


Figure 43 Deformation contour results of Conic TMS (Isotropic Pre-stress) for load case CO1

The development of deformation contours are displayed in Figure 43, the load case *CO1* manifests the situation of heavy snowfall. The development of of deformation contours takes place with the semicircular localized regions of high deformations for height 3.0m along the line connecting corner point of the square base. The localized high deformation regions starts increasing their influence area as the height of the structures increased. And

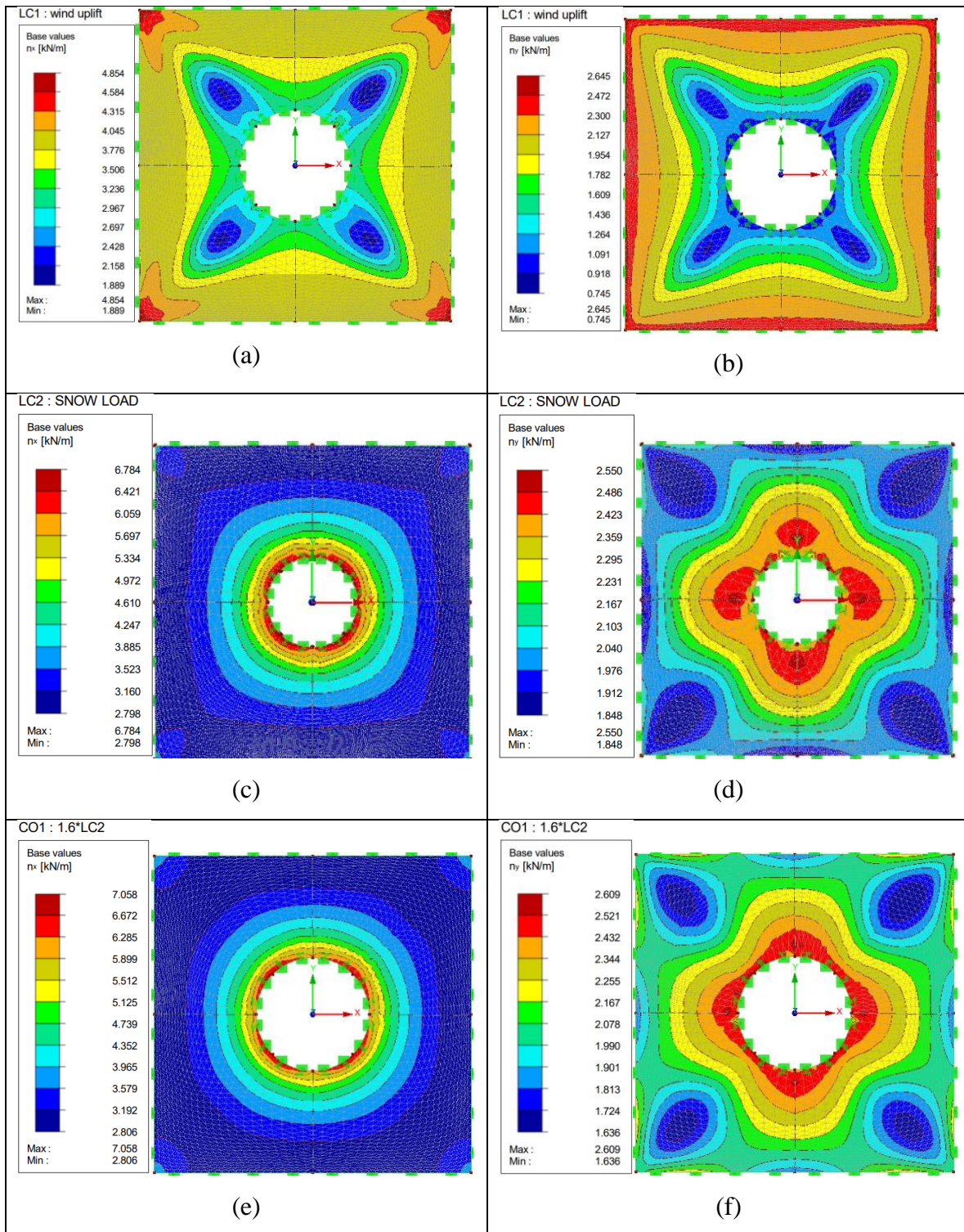
Pre-stress	5.79	5.77	5.75	5.75	5.73	5.71	5.68
LC1(wind-uplift)	4.96	4.84	4.79	4.86	4.86	4.83	4.85
LC2(snow-load)	6.85	6.86	6.85	6.87	6.85	6.84	6.78
CO1(1.6*LC2)	7.10	7.06	7.20	7.21	7.19	7.22	7.06
CO2(0.5*LC1+1.6LC2)	6.66	6.68	6.68	6.70	6.69	6.68	6.64

The minimum fill stresses response are tabulated in Table 36, the results suggests that the fill stress values vary uniformly from the range of 0.74kN/m for height 5.0 m for the load case wind-uplift to 1.94 kN/m for the initial pre-stressing of structure height 4.5m and 5.0m against the initially applied pre-stressing of 2.0 kN/m in the fill direction.

Table 36 The minimum fill stress responses for various load cases and different height for Anisotropic pre-stressing of Conic TMS

Load cases as per ASCE 7-16	Minimum fill stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
Pre-stress	1.92	1.93	1.93	1.93	1.93	1.94	1.94
LC1(wind-uplift)	1.18	1.19	1.31	1.02	1.02	1.00	0.74
LC2(snow-load)	1.87	1.87	1.86	1.86	1.86	1.85	1.85
CO1(1.6*LC2)	1.56	1.70	1.46	1.46	1.46	1.35	1.63
CO2(0.5*LC1+1.6LC2)	1.91	1.90	1.89	1.90	1.89	1.88	1.87

The results of warp and fill stresses suggest that the anisotropic implementation of the initial pre-stress will result in the stable form-finding and will facilitate the serviceability criteria of the conic shaped TMS. It was also observed that this scenario of pre-stressing is more sensitive to wind load as compared to snow load and their combinations. Given below are the warp and fill stress contour development profiles for the structure height 5.0m.



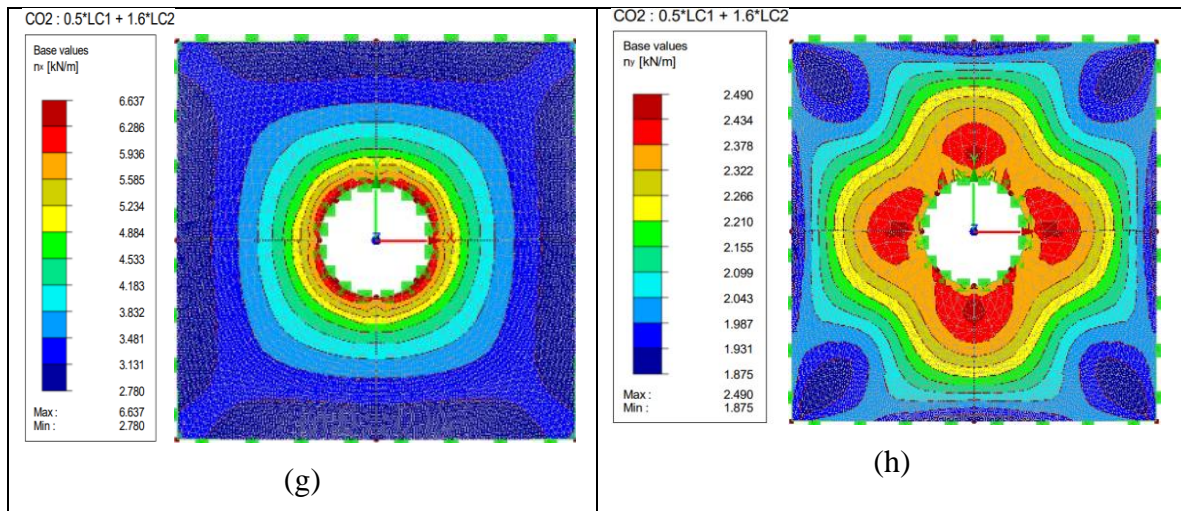


Figure 45 The warp and fill stresses development contours after stress-deformation analysis for Anisotropic pre-stressing of Conic TMS

5.5.4.1 Maximum deformation of TMS due to change in height for Anisotropic pre-stressing of square-base conic shaped TMS.

The maximum deformation values for various load cases with varying structure height are listed in the Table 37. As discussed earlier the deformation results obtained from the stress-deformation exercise are beyond the containment of the current design approach. Still a fair commentary can be made in the increasing pattern of the maximum deformations for each load case including initial pre-stressing. The maximum value of deformation have taken the magnitude even more than 3.5m similar to the case of isotropically pre-stressed conic TMS discussed in section 5.5.3.1.

Table 37 Deformation results for conic TMS (Anisotropic pre-stress)

Load cases as per	Variation of the height (H)						
	3.0	3.5	3.75	4.0	4.25	4.5	5.0
ASCE 7-16							
Pre-stress	1240.1	1342.3	1445	1548.9	1656.6	1666.5	1885.1
LC1(wind-uplift)	2720.2	2937.1	2843.1	3247.1	3373.7	3518.4	3898.4
LC2(snow-load)	716.0	1519.1	1609.6	1702.3	1791.8	1890.1	2080.0
CO1(1.6*LC2)	2067.7	2040.3	2535.5	2631.0	2718.8	3203.6	2705.8
CO2(0.5*LC1+1.6LC	900.1	1178.4	1260.4	1346.4	1427.7	1514.5	1686.9

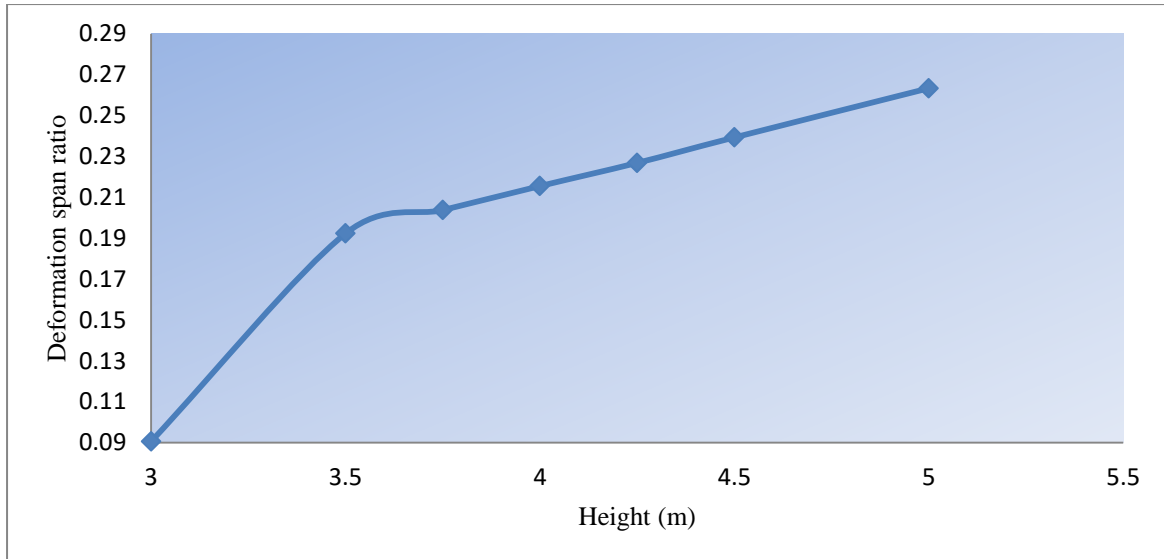
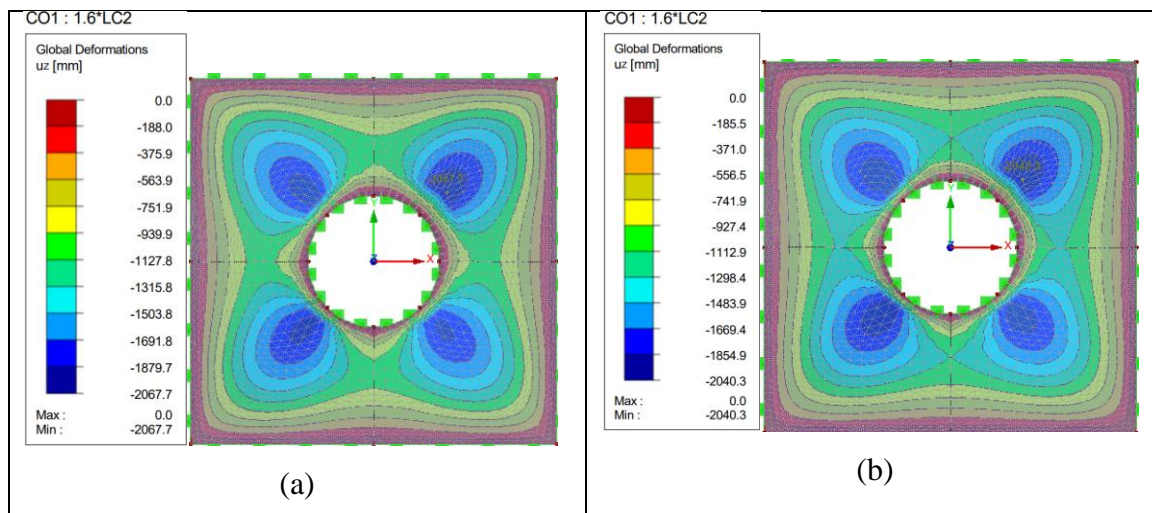


Figure 46 The deformation span ratio under snow load for deflection along z-axis for Conic TMS (Anisotropic pre-stress)

The deformation profile for snow load is displayed in the Figure 46 above, it shows that the deformation between 3.0m height to 3.5m height increases in the fast rate and the gradient slope of the deformation curve becomes low with the increase in the height of the structure.



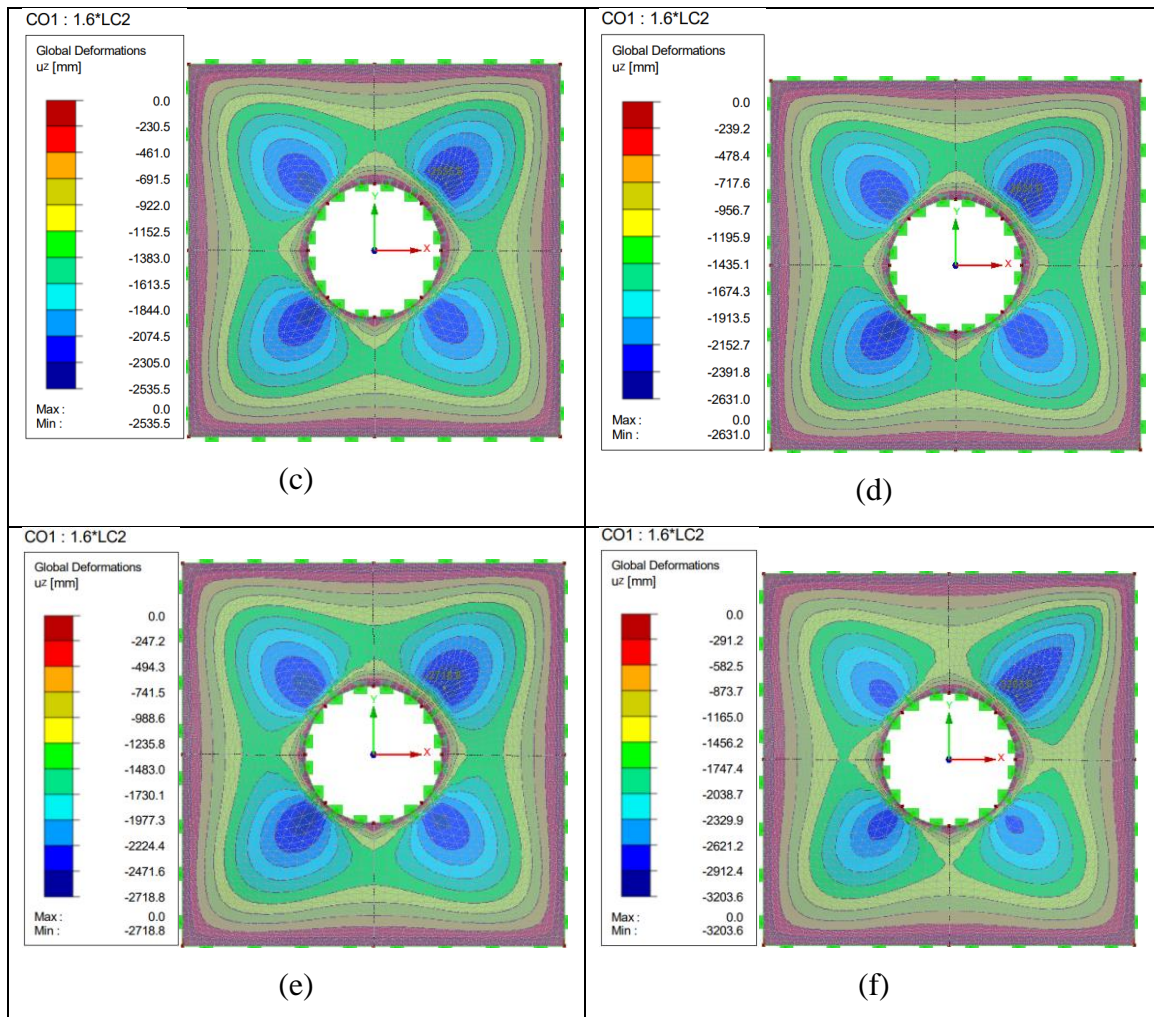


Figure 47 Deformation contours of Conic TMS (Anisotropic pre-stress) for load case CO1

The deformation contour development for load case CO1 is demonstrated in the Figure 47, the results shows localized high deformation regions being developed along the surface curve between head ring and nearest square base corner of the conic TMS.

The results of the deformations suggests that the value of maximum deformation responses increases with the increment in the magnitude of height of the structure. The overall stress-deformation analysis of the TMS enables the closer view of the behavioral change in the terms of stress and deformations when the wind load and snow load are applied to a square-base conic TMS with both equal and variable pre-stressing in both the directions

5.5.5 Reliability Analysis of conic TMS

In the procedure of reliability analysis implementing FORM, a deterministic analysis approach is usually adopted in order to calculate the values of the limit state functions.

The derivatives of performance functions are estimated using numerical and analytical methods, this results to an approach which can contain the uncertainties in establishing an effective sensitivity analysis of the performance of the structures. The FORM reliability analysis is therefore most efficient among the methods like Monte-Carlo simulation although both the methods makes the use of prevailing deterministic analysis. There are several past researches which consider FORM reliability analysis as a computational tool which manifests a standard analysis. Its versatility can be measured by knowing the fact that it can be bring to use with implementing standard deterministic codes/approaches.

In this chapter FORM is adopted for reliability assessment of a square-base conic shaped TMS. It is a realistic and a benchmark structure in the domain of TMS adopted from a recent research (Gosling et.al, 2013), focusing on a round-robin exercise for analysis and design of fabric structures. A deterministic approach involving non-linear finite element formulations is used and the design strains in the finite elements are obtained using Newton-Raphson method. The limit state functions identified are derived using an assumed equilibrium state, these state are controlled by the uncertainties in the design parameters like permissible stress, allowable deflections etc. Therefore limit states are depicted in terms of stresses and displacements. The stresses are indirectly non-linear function of the displacements, because displacements can be written in terms of strains and strains are directly associated to stresses. The description of limit states considered for this case study are expressed in detail in section 5.4. The three limit states contributing to ultimate failure criteria and serviceability criteria of the membrane surface are taken into account. A detailed design philosophy is discussed in section 5.3.

5.5.5.1 Reliability indices for the ultimate fabric failure limit state in case of isotropic pre-stressing of Conic TMS

The reliability estimation of the conic TMS starts with the comparison of obtained stresses and deformations with the corresponding limit state parameter. For this case of membrane structure, the height of the structure is varied from 3.0m to 5.0m for the load cases adopted in chapter 4 in the case of hypar TMS. The isotropic pre-stressing of conic TMS have implemented the initial pre-stress of 4.0kN/m in both the directions. The warp direction surface stresses are used to define the ultimate fabric failure limit state in reliability index calculation. The standard deviation for the statistical operations is varied

according to $COV = 0.10$ to 0.25 and the reliability results for corresponding case are demonstrated in the tables below.

Table 38 Reliability indices for the fabric failure limit state for each load case with $COV = 0.10$ corresponding to different height of Conic TMS (Isotropic pre-stress)

Load cases as per ASCE 7-16	Maximum warp stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
Pre-stress	3.44	3.44	3.44	3.44	3.44	3.44	3.44
LC1(wind-uplift)	2.77	2.73	2.68	2.61	2.65	2.61	2.61
LC2(snow-load)	2.97	2.95	2.97	2.98	2.98	2.98	2.98
CO1(1.6*LC2)	2.83	2.79	2.75	3.05	2.78	3.05	3.14
CO2(0.5*LC1+1.6LC2)	3.04	3.04	3.05	3.05	3.07	3.07	3.07

Table 39 Reliability indices for the fabric failure limit state for each load case with $COV = 0.15$ corresponding to different height of Conic TMS (Isotropic pre-stress)

Load cases as per ASCE 7-16	Maximum warp stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
Pre-stress	2.28	2.28	2.28	2.28	2.28	2.28	2.28
LC1(wind-uplift)	1.84	1.82	1.79	1.75	1.78	1.75	1.74
LC2(snow-load)	1.99	1.98	1.99	1.99	1.99	1.99	1.99
CO1(1.6*LC2)	1.90	1.86	1.83	2.03	1.85	2.03	2.10
CO2(0.5*LC1+1.6LC2)	2.03	2.03	2.03	2.03	2.05	2.05	2.05

Table 40 Reliability indices for the fabric failure limit state for each load case with $COV = 0.20$ corresponding to different height of Conic TMS (Isotropic pre-stress)

Load cases as per ASCE 7-16	Maximum warp stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
Pre-stress	1.71	1.71	1.71	1.71	1.71	1.71	1.71
LC1(wind-uplift)	1.38	1.37	1.35	1.31	1.33	1.31	1.30
LC2(snow-load)	1.49	1.48	1.49	1.49	1.49	1.49	1.49
CO1(1.6*LC2)	1.42	1.40	1.38	1.53	1.39	1.53	1.57
CO2(0.5*LC1+1.6LC2)	1.52	1.52	1.53	1.53	1.54	1.54	1.54

Table 41 Reliability indices for the fabric failure limit state for each load case with COV = 0.25 corresponding to different height of Conic TMS (Isotropic pre-stress)

Load cases as per ASCE	Maximum warp stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
7-16							
Pre-stress	1.37	1.37	1.37	1.37	1.37	1.37	1.37
LC1(wind-uplift)	1.10	1.09	1.08	1.05	1.06	1.05	1.04
LC2(snow-load)	1.19	1.18	1.19	1.20	1.20	1.20	1.20
CO1(1.6*LC2)	1.13	1.20	1.10	1.22	1.11	1.22	1.26
CO2(0.5*LC1+1.6LC2)	1.21	1.21	1.24	1.21	1.23	1.23	1.23

The results shows that 3.44 value (Table 38) for reliability indices which is below the Eurocode requirements for RC2 type building (see Table 1) which is 3.8 for for COV = 0.10, hence it lacks to fulfill ultimate fabric failure criteria for RC2 type buildings. It rather is a safe design with form-finding perspective if it is designed as per the purpose of building type RC3 (see Table 2) where the reliability index requirement for ultimate failure of fabric membrane is 3.3.

5.5.5.2 Reliability indices for the wrinkling failure limit state in case of isotropic pre-stressing of Conic TMS

The wrinkling failure criteria is gaged for each height of the conic TMS, Then minimum permissible fill stress is implemented to obtain the reliability indices with the standard deviation variation of 10 % to 25%. The results shows good agreement with the standards of Eurocode and are displayed from Table 42, to Table 45 and are discussed in the further section.

Table 42 Reliability indices for the wrinkling failure limit state for each load case with COV = 0.10 corresponding to different height of Conic TMS (Isotropic pre-stress)

Load cases as per ASCE	Minimum fill stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
7-16							
Pre-stress	2.94	2.94	2.94	2.94	2.94	2.94	2.94
LC1(wind-uplift)	2.17	1.98	1.45	0.88	1.02	0.88	0.45
LC2(snow-load)	2.37	2.35	2.35	2.36	2.35	2.33	2.41

CO1(1.6*LC2)	2.46	2.63	2.71	2.35	2.49	2.35	2.39
CO2(0.5*LC1+1.6LC2)	2.37	2.36	2.36	2.38	2.38	2.35	2.35

Table 43 Reliability indices for the wrinkling failure limit state for each load case with COV = 0.15 corresponding to different height of Conic TMS (Isotropic pre-stress)

Load cases as per ASCE 7-16	Minimum fill stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
Pre-stress	1.73	1.73	1.73	1.73	1.73	1.73	1.73
LC1(wind-uplift)	1.91	1.76	1.21	0.67	0.90	0.67	0.26
LC2(snow-load)	1.16	1.14	1.14	1.15	1.14	1.12	1.20
CO1(1.6*LC2)	1.25	1.42	1.50	1.15	1.28	1.15	1.18
CO2(0.5*LC1+1.6LC2)	1.16	1.15	1.15	1.17	1.17	1.14	1.14

Table 44 Reliability indices for the wrinkling failure limit state for each load case with COV = 0.20 corresponding to different height of Conic TMS (Isotropic pre-stress)

Load cases as per ASCE 7-16	Minimum fill stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
Pre-stress	1.03	1.03	1.03	1.03	1.03	1.03	1.03
LC1(wind-uplift)	0.98	0.86	0.62	0.34	0.45	0.34	0.11
LC2(snow-load)	0.56	0.54	0.54	0.55	0.54	0.52	0.60
CO1(1.6*LC2)	0.65	0.82	0.91	0.55	0.68	0.55	0.48
CO2(0.5*LC1+1.6LC2)	0.56	0.55	0.55	0.57	0.57	0.54	0.54

Table 45 Reliability indices for the wrinkling failure limit state for each load case with COV = 0.25 corresponding to different height of Conic TMS (Isotropic pre-stress)

Load cases as per ASCE 7-16	Minimum fill stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
Pre-stress	0.63	0.63	0.63	0.63	0.63	0.63	0.63
LC1(wind-uplift)	0.49	0.45	0.29	0.18	0.26	0.18	0.07
LC2(snow-load)	0.35	0.33	0.33	0.34	0.33	0.31	0.39
CO1(1.6*LC2)	0.45	0.62	0.70	0.34	0.47	0.34	0.27

CO2(0.5*LC1+1.6LC2)	0.35	0.34	0.34	0.36	0.36	0.33	0.33
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The reliability index value against the wrinkling failure criteria according to Eurocode design standards is 1.5, the reliability index values in Table 42 proves that the square-base conic TMS safely designed for wrinkling failure criteria of RC2 type buildings as most of the values are above 1.5 except few cases of larger heights for wind uplift load.

The overall analysis of fabric ultimate failure and wrinkling failure suggests that the current model fits better in the RC1 category building type rather than RC2 type for ultimate fabric failure. The wrinkling failure criteria is fulfilled for the RC2 type building using the prescribed design method for adopted load cases.

5.5.5.3 Reliability indices for the fabric failure limit state in case of Anisotropic prestressing of Conic TMS

On application of different level of pre-stressing to the square-base conic TMS with the magnitude 4kN/m in warp direction and 2 kN/m in fill direction, the membrane have shown a considerable amount of consistency in terms of warp stresses and fill stresses for different loading scenario and height of the structure as discussed in section 5.5.4. The reliability indices for both ultimate and serviceability failure using FORM methodology for fabric failure limit state are displayed from Table 46 to Table 49, and are discussed in the further section of this chapter.

Table 46 Reliability indices for the fabric failure limit state for each load case with COV = 0.10 corresponding to different height of Conic TMS (Anisotropic pre-stress)

Load cases as per ASCE 7-16	Maximum warp stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
Pre-stress	2.23	2.26	2.28	2.28	2.30	2.32	2.36
LC1(wind-uplift)	3.23	3.38	3.44	3.35	3.35	3.39	3.37
LC2(snow-load)	1.09	1.07	1.09	1.06	1.09	1.10	1.16
CO1(1.6*LC2)	0.84	0.88	0.74	0.72	0.75	0.70	0.88
CO2(0.5*LC1+1.6LC2)	1.29	1.27	1.27	1.24	1.25	1.27	1.31

Table 47 Reliability indices for the fabric failure limit state for each load case with COV = 0.15 corresponding to different height of Conic TMS (Anisotropic pre-stress)

Load cases as per ASCE 7-16	Maximum warp stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
Pre-stress	1.49	1.50	1.52	1.52	1.54	1.55	1.58
LC1(wind-uplift)	2.15	2.25	2.29	2.24	2.24	2.27	2.23
LC2(snow-load)	0.73	0.72	0.73	0.71	0.71	0.75	0.77
CO1(1.6*LC2)	0.56	0.59	0.49	0.48	0.51	0.46	0.59
CO2(0.5*LC1+1.6LC2)	0.86	0.84	0.84	0.82	0.83	0.82	0.89

Table 48 Reliability indices for the fabric failure limit state for each load case with COV = 0.20 corresponding to different height of Conic TMS (Anisotropic pre-stress)

Load cases as per ASCE 7-16	Maximum warp stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
Pre-stress	1.12	1.13	1.14	1.14	1.15	1.16	1.18
LC1(wind-uplift)	1.61	1.69	1.72	1.67	1.67	1.70	1.68
LC2(snow-load)	0.54	0.53	0.54	0.51	0.54	0.53	0.58
CO1(1.6*LC2)	0.42	0.44	0.37	0.36	0.38	0.36	0.44
CO2(0.5*LC1+1.6LC2)	0.64	0.63	0.63	0.61	0.62	0.63	0.65

Table 49 Reliability indices for the fabric failure limit state for each load case with COV = 0.10 corresponding to different height of Conic TMS (Anisotropic pre-stress)

Load cases as per ASCE 7-16	Maximum warp stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
Pre-stress	0.89	0.90	0.91	0.91	0.92	0.93	0.94
LC1(wind-uplift)	1.29	1.35	1.38	1.33	1.33	1.37	1.34
LC2(snow-load)	0.43	0.42	0.43	0.41	0.43	0.44	0.43
CO1(1.6*LC2)	0.34	0.35	0.29	0.29	0.30	0.27	0.35
CO2(0.5*LC1+1.6LC2)	0.51	0.51	0.51	0.50	0.50	0.50	0.53

Against the fabric strength of 6.667kN/m the warp stresses obtained were found to be nearing the allowable stress limit in most of the load cases, and it is rather exceeding in the load case *CO1* which is considered as a heavy snow scenario. The wind load case obtains the reliability closer to 3.0 and rest other indices fall below it for COV 10%. This indicates that the proposed design approach is incompetent if containing the fabric failure limit criteria for a conic TMS (anisotropic pre-stress) under prescribed loading and geometry.

5.5.5.4 Reliability indices for the wrinkling failure limit state in case of Anisotropic prestressing of Conic TMS

The reliability indices corresponding to wrinkling failure criteria for anisotropic-ally prestressed are enumerated from Table 50 to Table 53. The results are compared with the standards and are discussed further.

Table 50 Reliability indices for the wrinkling failure limit state for each load case with COV = 0.10 corresponding to different height of Conic TMS (Anisotropic pre-stress)

Load cases as per ASCE 7-16	Minimum fill stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
Pre-stress	1.62	1.63	1.63	1.63	1.63	1.64	1.64
LC1(wind-uplift)	1.08	1.09	1.21	0.92	0.92	0.90	0.64
LC2(snow-load)	1.56	1.56	1.55	1.55	1.55	1.54	1.54
CO1(1.6*LC2)	1.26	1.39	1.15	1.15	1.15	0.24	1.31
CO2(0.5*LC1+1.6LC2)	1.60	1.59	1.58	1.59	1.58	1.57	1.56

Table 51 Reliability indices for the wrinkling failure limit state for each load case with COV = 0.15 corresponding to different height of Conic TMS (Anisotropic pre-stress)

Load cases as per ASCE 7-16	Minimum fill stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
Pre-stress	1.41	1.42	1.42	1.42	1.42	1.43	1.43
LC1(wind-uplift)	0.98	0.99	1.01	0.81	0.82	0.79	0.52
LC2(snow-load)	1.35	1.35	1.34	1.34	1.34	1.33	1.33
CO1(1.6*LC2)	1.06	1.18	0.94	0.94	0.94	0.12	0.10

CO2(0.5*LC1+1.6LC)	1.39	1.38	1.37	1.38	1.37	1.36	1.35
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Table 52 Reliability indices for the wrinkling failure limit state for each load case with COV = 0.20 corresponding to different height of Conic TMS (Anisotropic pre-stress)

Load cases as per ASCE	Minimum fill stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
7-16							
Pre-stress	1.01	1.01	1.01	1.01	1.01	1.02	1.02
LC1(wind-uplift)	0.76	0.77	0.79	0.59	0.60	0.57	0.34
LC2(snow-load)	0.95	0.95	0.94	0.94	0.94	0.97	0.97
CO1(1.6*LC2)	0.75	0.78	0.56	0.56	0.56	0.08	0.06
CO2(0.5*LC1+1.6LC2)	0.99	0.92	0.93	0.92	0.93	0.94	0.95

Table 53 Reliability indices for the wrinkling failure limit state for each load case with COV = 0.25 corresponding to different height of Conic TMS (Anisotropic pre-stress)

Load cases as per ASCE	Minimum fill stress kN/m for variation of height H						
	3.0m	3.5m	3.75m	4.0m	4.25m	4.5m	5.0m
ASCE 7-16							
Pre-stress	0.81	0.81	0.81	0.81	0.81	0.81	0.82
LC1(wind-uplift)	0.41	0.43	0.44	0.43	0.27	0.21	0.04
LC2(snow-load)	0.74	0.74	0.73	0.73	0.73	0.76	0.76
CO1(1.6*LC2)	0.54	0.57	0.35	0.35	0.35	0.03	0.02
CO2(0.5*LC1+1.6LC2)	0.68	0.61	0.62	0.61	0.62	0.63	0.64

In the wrinkling failure criteria assessment for conic shape TMS (anisotropic pre-stress) it was observed that for load cases form-finding, snow load and CO2 the reliability index values for almost all the height of structure is found to be above 1.5 for standard deviation of 10%. The load case wind load and CO1 in the other hand falls short below 1.5 value of reliability index. This indicates that for load cases with magnitude 1.0 kN/m² and higher are more likely to encounter the wrinkling of the membrane surface.

5.6 Summary

An overall analysis of the square-base conic shaped is done using the adopted methodology. Based on the analysis related to form-finding it was observed that the initial pre-stresses are very well estimated for both the types of pre-stressing of a square-base conic shaped TMS. As for the best form-finding outcome, a membrane surface should attain the minimal surface area geometry. Both the pre-stressing cases of conic TMS have evidenced very close observations of the in-plane stresses in warp and fill both directions where the TMS is found to attain the minimal surface area orientation.

The stress-deformation analysis using prescribed approach for the conic membrane structure reveals the nature of the structure in presence of external environmental loads. The wind load and snow load are two major concerned forces which are considered as the design loads for this type of structures. According to the structural responses perceived from the analysis, it is evident that the conic TMS (both isotropic and anisotropic pre-stress) are pretty much sensitive towards wind load rather than snow load and any other combination of loads.

The reliability analysis using a robust approach of FORM gives a better way to standardize the structural responses of the conic TMS. The obtained results of stresses and deformation are statistical inputs for reliability assessment. The conic membrane (isotropic pre-stress) is found to be lacking behind the fabric failure limit state for RC2 type buildings and rather fulfilling the form-finding reliability criteria for RC1 type buildings according to Euro-codes. The variable initial pre-stressing of conic membrane is also found to be drifting away from the fabric failure limit. In contrary to that both the pre-stressing scenarios of conic TMS are well capable of meeting the wrinkling failure limit state.