PREFACE

The Weinstein transform is an efficient technique to solve the problems of mathematics, mostly in differential equations. It contains complete calculus for getting the solutions to various problems of functional spaces, pseudo-differential operators and Sobolev spaces. It is a generalization of the Fourier and Hankel transforms. The integral representation of the Weinstein transform is the product of an absolutely integrable function and the Weinstein kernel, which can be obtained by multiplying a complex exponential function by a normalized Bessel function of first-kind.
Ultradistributions are a generalization of distributions (generalized functions) used in functional analysis and are particularly useful in the context of solving differential equations and dealing with singularities. They extend the concept of distributions to allow for more rapid growth at infinity. Sobolev spaces are often used to study the properties of solutions to partial differential equations, which can be extended using ultradistributions when dealing with more singular problems.
Pseudo-differential operators generalize partial differential operators, providing a powerful framework for analyzing and solving PDEs. They play a crucial role in microlocal analysis, quantum mechanics, and signal processing by facilitating the study of functions and their transformations in the frequency domain. This thesis consists of seven chapters. The organization of this thesis is as follows:

Chapter 1 is introductory, which provides a brief description and historical background about the Fourier transform, the Hankel transform, and the Weinstein transform. In this chapter, motivation, definitions, and properties of the Weinstein transform on the space $S_\omega(\mathbb{R}^{n+1}_+)$ are given, which are useful for our next subsequent chapters.

Björck [8] and Hörmander [22], investigated the linear partial differential operators and generalized distributions associated with the Fourier transform by utilizing the convolution properties on the spaces of types $D_\omega(\mathbb{R}^n)$, $\mathcal{E}_\omega(\mathbb{R}^n)$, and $S_\omega(\mathbb{R}^n)$.
In chapter 2, motivated by above results, we developed the spaces $D_\omega(\mathbb{R}_+^{n+1})$, and $D'_\omega(\mathbb{R}_+^{n+1})$ associated with the weight function ω and investigated various properties of the Weinstein transform over generalized distributions $D'_\omega(\mathbb{R}_+^{n+1})$ and other spaces.

In chapter 3, the space $\mathcal{E}_\omega(\mathbb{R}_+^{n+1})$ is defined by taking the theory of the Weinstein transform. Further, the authors define convolutions of test functions and generalized distributions and prove that convolution is associative as well as commutative. The authors also examine various properties of the convolution of generalized distributions on $D'_\omega(\mathbb{R}_+^{n+1})$ by exploiting the theory of the Weinstein transform. After that, the space $S_\omega(\mathbb{R}_+^{n+1})$ is developed by utilizing the theory of the Weinstein transform and proved that the space $S_\omega(\mathbb{R}_+^{n+1})$ is topological algebra under point-wise multiplication as well as under convolution.

In chapter 4, the space $G_\omega^{p,s}(\mathbb{R}_+^{n+1})$ is considered, and many properties, including completeness and inclusion, are discussed using the theory of the Weinstein transform. It is shown that the space $S_\omega(\mathbb{R}_+^{n+1})$, is dense in the space $G_\omega^{p,s}(\mathbb{R}_+^{n+1})$. The generalized Hankel potential $\mathcal{H}^k$ associated with the Weinstein transform is introduced, and its various properties are examined. The L^p-space of all such Hankel potentials, denoted by $W_\omega^{m,p}(\mathbb{R}_+^{n+1})$ is defined, and it is proven that the generalized Hankel potential $\mathcal{H}^t$ is an isometry of $W_\omega^{m,p}(\mathbb{R}_+^{n+1})$ onto $W_\omega^{m+t,p}(\mathbb{R}_+^{n+1})$.

Chapter 5 provides the pseudo-differential operator associated with the symbol class S_ω^r on the space $S_\omega(\mathbb{R}_+^{n+1})$ is defined and various properties of the pseudo-differential operator $A(x, D)$ associated with the symbol class $S_M^{r,l}$ are discussed by exploiting the theory of the Weinstein transform. The representation of the pseudo-differential operator $A(x, D)$ associated with the indicatrix $M(t)$ and $\omega(t)$ are obtained. The L^2- norm of pseudo-differential operator $A(x, D)$ with indicatrix $M(t)$ and $\omega(t)$ is investigated.

In the chapter 6, the space $H(A)$ is defined by exploiting the theory of the Weinstein transform and proved that the Weinstein transform $\mathcal{F}_w(\phi)$ is an automorphism on the space $H(A)$. After that, the Banach space-valued test function of Beurling-type ultradistribution $H_\omega^\beta(A)$ is defined by taking the weight function ω and examined various properties. It is shown that the subspace $D_{\mathbb{R}_+^{n+1}}(A)$ is dense in $H_\omega(A)$ and the Weinstein transform $\mathcal{F}_w(\phi)$ is an automorphism on the space $H_\omega(A)$. Further showed that the linear space $\omega D_{\mathbb{R}_+^{n+1}}(A) \oplus (A)$ is dense in $H_\omega(A)$.

In Chapter 7, we have consolidated the findings and conclusions from all previous chapters, providing a comprehensive summary of our work.