Preface

The study of singularly perturbed problems has long been considered a cornerstone, offering elegant solutions to various physical and engineering problems. These problems pose a significant challenge because their solutions often exhibit unusual behavior in small regions of the physical domain, mainly due to the appearance of small parameter(s) in the differential equation. When finite difference methods are utilized, it becomes imperative to use a finer mesh in these small regions and use fewer mesh points in the remaining region. This can be achieved through the use of layer-adapted meshes, which are further categorized into a priori and a posteriori meshes.

The purpose of this thesis is to develop and analyze parameter-uniform convergent schemes on layer-adapted meshes for various important classes of singularly perturbed problems. A significant emphasis has been placed on establishing a general error analysis framework that allows to deduce uniform convergence on various layer-adapted meshes conveniently within a single framework. Recognizing the ubiquitous need for error estimates that do not rely on prior knowledge of the exact solution and its derivatives, providing a posteriori error estimates is another vital contribution of this work.

Initially, we consider a class of nonlinear singularly perturbed problems with integral boundary condition. The discretization of the problem consists of a hybrid scheme defined on an arbitrary nonuniform mesh. A general error analysis framework is introduced, and uniform convergence on various layer-adapted meshes is proved. Further, we propose adaptive generation of meshes based on a suitable monitor function and the mesh equidistribution principle.

Moving forward, we investigate a nonlinear singularly perturbed parameterized problem with integral boundary condition. To discretize this problem, we employ the implicit Euler scheme for the nonlinear problem, whereas a composite right rectangle rule is applied to the integral boundary condition. A priori as well as a posteriori error analysis is developed for the proposed scheme. We demonstrate that the scheme achieves optimal first-order uniform convergence on both a priori and a posteriori meshes.

Subsequently, we shift our focus to a first-order linear singularly perturbed delay Volterra integro-differential equation which is characterized by multiple-layer phenomena. The discretization consists of an implicit difference scheme for the derivative term and a composite numerical integration rule for the integral term. A priori and a posteriori error analysis for the proposed discrete scheme is carried out. Additionally, we provide a comparison of uniformly accurate results obtained on these meshes.

Next, we proceed to develop a high-order convergent adaptive numerical method for a system of first-order singularly perturbed nonlinear differential equations with distinct perturbation parameters. The problem is discretized by a hybrid finite difference scheme, for which a posteriori error estimate in the maximum norm is derived. The layer-adapted meshes are generated using the equidistribution of the monitor function, chosen based on the derived a posteriori error estimate.

In the end, our focus shifts to a system of Volterra integro-differential equations. The derivative term in these equations is multiplied with distinct small positive parameters, giving rise to overlapping layers. We introduce a numerical scheme employing a numerical integral rule and subsequently derive both a priori and a posteriori error bounds for this scheme.

Extensive numerical experiments have been conducted to validate the obtained theoretical error estimates. The thesis concludes with a list of references cited throughout the work.