Calibration Requirements for EoR Observations - Effect of Time and Frequency Correlated Gains

Thesis submitted in partial fulfillment
for the Award of Degree

DOCTOR OF PHILOSOPHY

by

JAIS KUMAR

DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY
(BANARAS HINDU UNIVERSITY)
VARANASI - 221005

ROLL NUMBER	YEAR OF SUBMISSION
16171010	2022

Certificate

It is certified that the work contained in the thesis titled **"Calibration Requirements for EoR Observations - Effect of Time and Frequency Correlated Gains"** by **Jais Kumar** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. Degree.

Date: 20/06/22

Place: Varanasi

Supervisor Prasun Dutta

Dr. Prasun Dutta
Assistant Professor
Department of Physics
IIT (BHU) Varanasi

Dr. Prasun Dutta
Assistant Professor
Department of Physics
IIT (BHU), Varanasi
Varanasi, PIN 221005

Declaration

I, **Jais Kumar**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Dr. Prasun Dutta** from August 2016 to June 2022 at the **Department of Physics**, Indian Institute of Technology (BHU), Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers whenever and wherever their works have been cited in my work in this thesis. I further declare that I have not wilfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

Date: 20/06/2022

Place: Varanasi

Signature of the Student

Jais Kumar

Certificate by the Supervisor

It is certified that the above statement made by the student is correct to the best of my knowledge.

Prasun Dutta

Supervisor 20/06/22

Dr. Prasun Dutta

Dr. Prasun Dutta
Assistant Professor
Department of Physics
IIT (BHU), Varanasi
Varanasi, PIN 221005

20/6/22

Head of the Department

HEAD/विभागाध्यक्ष
भौतिकी विभाग/Deptt. of Physics
भा०प्रौ०सं०/(का०हि०वि०)/IIT (BHU)
वाराणसी/Varanasi-221005

Copyright Transfer Certificate

Title of the Thesis: **Calibration Requirements for EoR Observations - Effect of Time and Frequency Correlated Gains**

Name of the Student : **Jais Kumar**

Copyright Transfer

The undersigned hereby assigns to the Indian Institute of Technology (Banaras Hindu University) Varanasi all rights under copyright that may exist in and for the above thesis submitted for the award of the ***Doctor of Philosophy***.

Date: 20/06/2022

Place: Varanasi

Signature of the Student

Jais Kumar

To *The person who taught me*
to draw the first symbols,
my grandfather, and my family.

Life isn't about finding yourself. Life is about creating yourself.

- George Bernard Shaw

Acknowledgements

It has been an incredible journey at IIT (BHU) Varanasi towards working on this thesis. This Ph.D. has changed me so much in so many ways that it can hardly be described. There is a long list of people without whom the work in this thesis would not have been possible or at least much less enjoyable. I want to thank all the people who supervised, helped, inspired, and supported me during this period.

The first one to thank would definitely be my supervisor, Dr. Prasun Dutta, who has always been there truly as a "Guide" in this journey. It is impossible to express my gratitude toward you in a small paragraph like this; still, I would like to express my heartfelt gratitude towards you for mentoring me on my academic path and helping me navigate this large academic landscape. I got to learn so many things from you, not only the things "To Do" but also the things "Not To Do" which I believe will always be helpful for me in academics as well as in everyday life. Thanks for mentoring me, tolerating me for so long, and forgiving me for the stupid mistakes I made during the research work. Although I have missed many deadlines, you were always patient and believed more than me that the thesis would be finished someday. This thesis would never have been possible without the unimaginable patience and constant encouragement from Prasun, whose role as my thesis guide kept propelling me forward toward reaching this end.

I want to express my sincere thanks to my research progress evaluation committee (RPEC) members, Dr. Somak Bhattacharyya, and Dr. Abhishek Kumar Srivastava for the invaluable inspiration and numerous insightful suggestions during the entire course of this research. I wish to express my deep regards to my collaborators Dr. Nirpuam Roy and Dr. Samir Choudhury, for their scientific discussion and invaluable suggestions during my research work.

I offer my special thanks to the Head of the Department of Physics, the DPGC chairman, and all other faculty members of the Department for their support, guidance, and motivation. It would be worth mentioning the name of Prof. Bhola Nath Dwivedi for his unique and philosophical approach towards the subject. I extend my thanks to all the research colleagues and staff of the Department of Physics, IIT (BHU), for their kind help whenever I required it. I want to thank UGC, the Government of India, for providing me the financial support through the fellowship scheme UGC-JRF. The support and the resources provided by the 'PARAM Shivay Facility' under the National Supercomputing Mission, Government of India, at the Indian Institute of Technology, Varanasi, are gratefully acknowledged. I would also like to acknowledge the computing facility provided by the Department of Physics and CIC IIT (BHU).

I want to thank all my labmates, Pavan, Meera, Urvashi, and Saumyadeep, for their help and great company in the lab. Heartly thanks to my beloved friends Kanchan, and Sudheer for their constant support and encouragement throughout the whole journey. The joyful life shared with Samir, Ajay, Jay Prakash, Manish, and other friends in the hostel is unforgettable, be the games of badminton, cricket or carrom, random talks or anything. The list of friends from campus also includes but is not limited to Pragati, Mahima, Vandna, Yamini, Bharat, and Anupam. Every moment spent with them in the campus has been so cheerful. Memories shared with you will always cherish in my nervous system.

This journey started not precisely from the date of registration at IIT (BHU) but well before that. There are a bunch of people who always supported and encouraged me to go after my dream. The impact of Ashok Pathak sir, Sugata Pratik Khastgir Sir, and Somnath Bharadwaj sir on my academic life can't be erased. The cooperation and moral support received from Nidhi, and the insightful discussions with Mohit Pal on any topic out of any world require special mention. I sincerely acknowledge the constant support and help received from Abinash Kumar Shaw during any technical or other related difficulties since

my MSc days. The list is neither complete nor ends here, along with these I also want to thank all other known or unknown persons who came into my life, played their part, and supported me in any way.

My entire family has been the biggest source of strength, whose love, and affection gave me inspiration at every step of my life. My gratitude to my grandparents Late Mr. Chhote Lal & Mrs. Ram Sakhi, parents Mr. Shiv Babu & Mrs. Amrawati Devi, uncle Late Mr. Mahesh Chandra & Mr. Shiv Murat, brothers Mr. Rajesh Kumar, Mr. Awadhesh Kumar, Mr. Dharmendra Kumar, and sister Kalpana, for the faith they have posed, the patience they have demonstrated, and the support they have been unquestioningly extending throughout my life. I dedicate this thesis to my family.

Finally, I bow with reverence and gratitude to thank the Almighty, who has enriched me with such an excellent opportunity and infused the power in my mind to fulfill the work assigned to me. Without the blessings of Lord Shiva, it would not have been possible to travel so far on this journey. Har Har Mahadev

Jais Kumar

Department of Physics

IIT (BHU) Varanasi

Table of contents

List of figures

List of tables

List of Symbols

Symbols	Definition
$\tilde{g}_A$	Complex gain for antenna A
δ_A	Residual amplitude gain of antenna A
ϕ_A	Residual phase gain of antenna A
δ_{AR}	Real part of residual gain of antenna A
δ_{AI}	Imaginary part pf residual gain of antenna A
σ_δ^2	Variance of residual amplitude gain
σ_ϕ^2	Variance of residual phase gain
σ_R^2	Variance of real part of residual gain
σ_I^2	Variance imaginary part pf residual gain
$\Delta\tau$	Integration time
T_D	Time taken by an antenna pair to cross a grid
$\tilde{b}_A(\nu)$	Bandpass response of antenna A
b_{AR}	Real part of bandpass response of antenna A
b_{AI}	Imaginary part pf bandpass response of antenna A
σ_b^2	Variance of bandpass response
$\xi(\tau)$	Normalized two-point correlation function of time-correlated gain errors
$\xi(\Delta\nu)$	Normalized two-point correlation function of frequency correlated bandpass errors
T_{Corr}	Correlation time (sec)
χ	Average effect of time-correlation over a grid
$\Delta\nu$	Frequency separation between two channels
$\Delta\nu_c$	Frequency channel width (MHz)
ν_c	Frequency correlation length

Symbols	**Definition**
n_i	Baseline pair fractions of Type i (used in Chapter 5)
n_i'	Baseline pair fractions of Type i (used in Chapter 6)
σ_g	Residual gain error for the case $\sigma_R = \sigma_I$
N	Total number of antennae of an array
N_d	Number of days of observation
N_b	Number of baselines in a grid
N_g	Number of baseline pairs in a grid
N_B	Number of baseline pairs in an annulus at baseline U
N_G	Number of grids in an annulus at baseline U
$\tilde{N}_i(\mathbf{U}_i)$	System noise in the i^{th} visibility measurement
σ_N	Noise rms in real or imaginary part of the visibility
P_{HI}	HI 21-cm power spectrum (Jy^2)
P_F	Foreground power spectrum (Jy^2)
$\mathscr{B}_P$	Bias of the H I 21- cm power spectrum
σ_P^2	Variance of the H I 21- cm power spectrum
$C_l(\Delta\nu)$	Multifrequency angular power spectrum of foregrounds at frequency separation $\Delta\nu$ $(\mathrm{m}K^2)$
$I_l(\nu_1,\nu_2)$	Foreground frequency decorrelation function
α	Mean spectral index
ζ	Foreground frequency decorrelation factor
β	power law index for foreground power spectrum
$\mathscr{B}_{Cl}$	Bias of the H I 21- cm angular power spectrum estimate
σ_{Cl}^2	Variance of the H I 21- cm power spectrum
ν_e	Rest frame frequency of H I 21- cm signal
r_ν	Comoving distance corresponding to ν
r_ν'	Derivative of r_ν

Symbols	Definition
H_0	Hubble parameter at present(Km/s/Mpc)
$H(z)$	Hubble parameter at redshift z (Km/s/Mpc)
h	$H_0/100$ km/s/Mpc
h_p	Planck's constant
Ω_b	Baryon density parameter
Ω_m	Matter density parameter
T_s	Spin temperature of hydrogen gas
τ_e	CMB electron scattering optical depth
T_γ	CMB temperature (K)
$\delta T_b(\mathbf{n},z)$	Excess 21- cm brightness temperature along a direction $\hat{n}$
x_{HI}	Neutral hydrogen fraction
$\bar{x}_{HI}(z)$	Spatially averaged HI fraction
ρ_{HI}	Neutral Hydrogen Density
$\Delta\rho_{HI}$	Fluctuation in the neutral hydrogen density
$\bar{\rho}_H$	Mean Hydrogen Density
$\eta_{HI}(\mathbf{n},z)$	21-cm radiation efficiency in redshift space
$\eta_{HI}(\mathbf{k},z)$	Fourier transform of $\eta_{HI}(\mathbf{n},z)(Mpc^{-3})$
δ	Dark matter density contrast
$P(z,k)$	Dark matter power spectrum (Mpc^{-3})
$\delta_{21}(\mathbf{x}$	Fractional Perturbation to the H I brightness temperature
$\delta_{21}(\mathbf{k}$	Fourier Transform of $\delta_{21}(\mathbf{x}$
$\delta_D(x)$	Dirac delta function
T_γ	CMB temperature (K)

Abstract

In the cosmic history of the matter, at the end of the Epoch of recombination, tiny fluctuations in the matter densities were present. With the growth of these tiny fluctuations due to gravitational instabilities, the first luminous objects in the universe emerged. The era during which these first objects formed is termed as the Cosmic Dawn. The radiation from these luminous objects started ionizing the surrounding neutral hydrogen in the universe. As time progressed, the bubbles of ionized hydrogen increasingly overlapped, resulting in an entirely ionized universe. This process of phase transition of the universe from a neutral state to an almost completely ionized state is known as reionization. The Epoch of Re-ionization is one of the most important but less understood eras in the history of the universe's evolution. As neutral hydrogen was the dominant component of the universe during Cosmic Dawn, EoR, and post-EoR era, the radiation emerging from the forbidden spin-flip transition of neutral hydrogen provides excellent prob of these important stages of the universe. This radiation resulting from the line transition of the neutral hydrogen peaked at a frequency of 1420 MHz or equivalently at the 21-cm in wavelength units, hence getting its famous name "the 21- cm signal". Observing this 21- cm signal at redshifted frequencies can solve many mysteries, including our understanding of structure formation, the evolution, and the radiation mechanism of these first luminous objects. The direct imaging of the redshifted 21- cm signal is quite difficult due to the presence of very strong radiation coming from the foregrounds in similar observational frequency ranges, collectively known as foregrounds. Apart from the presence of orders of magnitude larger foregrounds in the observed frequency range, the instrumental effects of the interferometers, combined with the ionospheric effects, present a considerable challenge in the extraction of 21-cm signals from strong foregrounds. The combination of these instrumental and ionospheric effects, which is termed as "gain", is complex in nature and depends both on time and frequency.

The gains are estimated during the data preparation using various calibration procedures. Even after investing a good amount of effort, there are some residual errors in the gain. Although these residual gain errors are small in magnitude, these gain errors become important in high dynamic range observations with the presence of strong foregrounds. The systematic effects of these time and frequency-correlated residual gain errors originating from the measurement process introduce a bias and enhance the variance of the power spectrum measurements. Our work studies the effect of time and frequency-correlated residual gain errors in estimating the power spectrum of the sky brightness distribution in high dynamic range observations, such as EoR observations.

In the first project included in this thesis, we study the effect of residual gain errors in primary calibration. Modeling the residual gain errors as time-independent Gaussian random variables with zero mean, we analytically calculated the amplitude and variance of the visibility. Calculations showed that the visibility amplitude decreases with the phase gain error. Also, the variance of the visibility increases with both amplitude and phase gain error. For a single point source at the center of the field of view, we study the statistics of simulated visibilities and find that the simulated visibilities show similar behavior as predicted by analytical results.

In the next project, we discuss a methodology to estimate the bias in the power spectrum estimates of the redshifted 21-cm signal from neutral hydrogen in the presence of strong foregrounds. Considering time correlation in residual gain, we model the two-point correlation function of the residual gain errors using a power law and study its effect on the power spectrum. Through simulated observations using GMRT baseline configuration, we show that the presence of time-correlated residual gain errors introduces a bias in the power spectrum estimator that uses the correlation of visibilities in nearby baselines. We consider the contributions from various types of baseline pairs involved in the visibility measurement to be used in visibility correlation. We find that for the visibility correlation-

based power spectrum estimator, the bias in the power spectrum arises mainly from those types of baseline pairs that have at least one antenna in common. The bias also depends on the foreground model and the correlation properties of the gain errors. We define the ratio of the bias to the foreground power spectrum as the "Power spectral gain". This quantity was later used to calculate the bias for the foreground model, including the bright point sources and the galactic diffuse synchrotron emission. Our results show that more than 10,000 hours of observation will be required using GMRT baseline configuration for the calibration accuracy of 1% for the full foreground. In this work, we do not consider the frequency dependence of the gain. Also, to check the effect of gain errors explicitly, we do not consider the thermal noise in our visibility measurements.

In the next project, we included this measurement noise in the estimation of bias and variance in the power spectrum with residual gain errors. We propagate errors in the presence of thermal noise and present a method to produce analytic estimates of the bias and variance in the power spectrum. We model the residual gain errors as time-correlated Gaussian random variables with zero mean. To incorporate the time correlation of the gain errors, we model the two-point correlation function of the gain errors using a Gaussian function with correlation time given as the half-width of the Gaussian. We study the effect of time-correlated residual gain errors in presence of strong foregrounds and thermal noise analytically and provide the mathematical expressions for bias and variance in the power spectrum measurements. We used simulated observations using GMRT baseline configuration in the presence of point source foregrounds. We observe that the simulation results are in quite good agreement with our analytical estimates. Later we use this analytical framework to understand various effects of the correlated gain errors. As the standard deviation in the residual gain errors increases, the bias in the estimation supersedes the variance. It is observed that an optimal choice of the time over which the gain solutions are estimated minimizes the combination of bias and variance, collectively known as the

risk. We also find that the interferometers with higher baseline densities are preferred instruments for these studies.

As the complex antenna gains are both time and frequency-dependent, we incorporate the frequency dependence of the gains in our analytical framework as a part of our final project. Similar to the time correlation of the gain errors, we model the frequency correlation of the gains also using a Gaussian function. Our analytical estimates show that the bias is more at low-frequency separations and increases with an increase in the correlation frequency. On the other hand, the variance is dominated by the contribution of thermal noise, and it shoots up at higher frequency separations. This is because, at higher frequency separations, the number of independent estimates of the visibility correlation will be less. We compare the performance of GMRT and SKA - 1 Low baseline configurations and find that SKA -1 Low baseline configuration will require ~ 50 times less observation time than the GMRT. This is primarily because of the baseline configuration and the larger collecting area of the SKA- 1 Low.

Our work sheds light on the importance of assessing the effect of time and frequency-correlated gain errors in high dynamic range observations, such as EoR experiments. Although the mathematical estimates of the bias and variance depend upon the array configuration, the methodology we present in this work is rather general and can be used to estimate the bias and variance of the power spectrum estimates for any telescope, given the antenna gain properties are known. We believe this would add and aid the quest of detecting the redshifted 21- cm signal from the Epoch of reionization.