Chapter 4

Parametric and Design Optimization Investigation of Wavy Fin and Tube Air Heat Exchanger using Taguchi-Grey Technique

The focus of this chapter is to optimize the air-side performance of a wavy fin and tube heat exchanger at different design parameters on an individual target response using the Taguchi method. However, a statistical concept, Grey relational analysis, is also studied for combined optimization considering all target responses at one time. The study shows that with an increase in heat transfer, frictional pressure drop also increases but, the actual need for the heat exchanger is to have high heat transfer at a lower pressure drop. Thus, there must be a trade-off value and hence need optimization. Therefore, the present investigation aims at the effect of different design factors and levels of a wavy fin and tube configuration heat exchanger on the target response Colburn factor, TPF (Thermal performance factor), heat transfer coefficient, and the friction factor using the Taguchi technique. Also, the investigation focuses on the contribution ratio of the individual design factors to the defined target response. For the optimization of design parameters on the combined target response, the grey method has been used. The Grey relational method investigates the relational analysis between friction, TPF, and the Colburn factor. The overall target response is optimized quantitatively at various design factors using GRA (Grey relational analysis), and the optimal level of each design factor is explained as a novelty with the reproducibility of the optimal conditions.

4.1 Numerical Modelling

Taguchi methods help to determine the product and process quality based on statistical concepts and tools. To produce the best solution using a different combination of a factor at minimum cost and time (**Taguchi 1987; Kotciaglu et al., 2013**). Also, the Taguchi study is used to determine the design set of parameters for the wavy fin and tube heat exchanger, which will improve the heat transfer and pressure drop characteristic of a heat exchanger (**Wang et al., 1999; Chamoli et al., 2016; Celik et al., 2018; Nakhchi et al., 2019**). The flow diagram of Taguchi modeling is presented in Fig. 4.1. According to this method, primarily considers an orthogonal plan for the simulation of target response, solving the orthogonal plan in Engineering equation solver and then influencing the individual level of design factors on the target response (CF, FF, H, TPF) analyzed with the signal to noise ratio in the Minitab package. From the simulation plan and signal to noise ratio, investigated contribution ratio of the different design parameters on the target response. In the present study, seven factors of three-level have been considered, instead of considering full factorial analysis, which is equal to the 2187 different combinations for the solution, Taguchi orthogonal array $L_{27,}$ which consists of a total of 27 combinations. The orthogonal array simulation plan is prepared for the wavy fin and tube heat exchanger (Fig. 4.2.) with the dimensions as in Table 4.1, using the Minitab package and tabulated in Table 4.2.

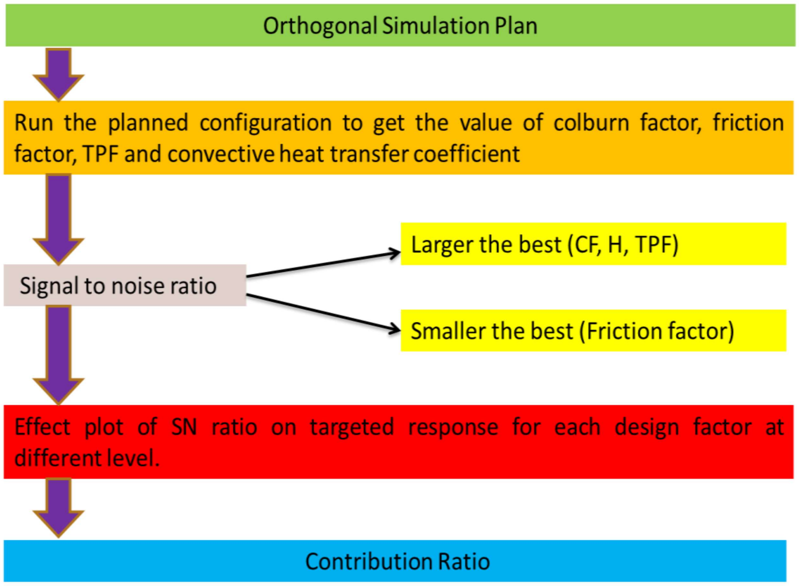

Fig. 4.1. Flow process of Taguchi analysis.

In this study, a total of seven different factors are considered, such as fin pitch (f_p), number of tube row (N), waffle height (P_d), fin thickness (δ_f), Air velocity, Longitudinal tube pitch (P_l) and Transverse tube pitch (P_t). This study aims to optimize the wavy fin heat exchanger by maximizing the Colburn factor, Thermal performance factor, and minimizing the friction factor. Furthermore, maximizing the heat transfer coefficient correlations needed for the wavy fin and tube heat exchanger depends on several considered parameters from an experimental investigation (**Wang et al. 1999**). The correlations which can predict the Colburn factor of 93.02 %, the friction factor of 91.8 % with mean deviations of 4.02 % and 4.9%, respectively, are used. These correlations apply to the selected level for the design factors. For the wavy fin heat exchanger, certain assumptions have been considered (**Wang et al. 2011**).

- Heat rejected by the coolant inside the tube is absorbed by the air flowing across the wavy fin.
- No heat generation is considered for the heat exchanger
- The heat exchanger operates under steady-state and steady flow conditions.
- Fluid properties remain constant, radiative and natural convection heat transfer neglected.
- Tube wall resistance is ignored.

The target response CF (Colburn factor) as a function of dependent geometrical parameters is given as,

$$CF = 1.79097\,\mathrm{Re}_{\mathrm{Dc}}^{-0.1707-1.374\left(\frac{P_l}{\delta_f}\right)^{-0.493}\left(\frac{F_p}{\mathrm{D_c}}\right)^{-0.886}(\mathrm{N})^{-0.143}\left(\frac{P_d}{\mathrm{X_f}}\right)^{-0.0296}} \left(\frac{P_l}{\delta_f}\right)^{-0.456} \mathrm{N}^{-0.27} \left(\frac{F_p}{\mathrm{D_c}}\right)^{-1.343} \left(\frac{P_d}{\mathrm{X}_f}\right)^{0.317} \tag{4.1}$$

The target response ff (friction factor) depending on geometrical parameters is given as,

$$ff = 0.007452\mathrm{Re}_{\mathrm{Dc}}^{f_1} \left(\frac{P_d}{X_f}\right)^{f_2} \left(\frac{F_p}{P_t}\right)^{f_3} \left(\frac{\mathrm{D}_h}{\mathrm{D_c}}\right)^{0.1325} \mathrm{N}^{0.02305} \tag{4.2}$$

Where, $\mathrm{Re}_{DC} = \frac{\rho V_{\max} D_c}{\mu}$, $D_c = D_0 + 2\delta_{fin}$, Dc is collar diameter, Re is Reynolds number

And the dependent non dimensional parameter f1, f2, f3, which depends upon the geometrical parameters is given as,

$f_1 = 0.1714 - 0.07372\left(\frac{F_p}{P_l}\right)^{0.25}\log_e(7.8)\left(\frac{P_d}{X_f}\right)^{-0.2}$, when F_p is fin pitch, P_d is waffle height and P_l is longitudinal pitch. (4.3)

$f_2 = 0.8733\left(\frac{F_p}{P_t}\right)^{0.3}$ where P_t is transverse tube pitch. (4.4)

$$f_3 = \frac{-10.2192}{\log_e Re_{Dc}} \qquad (4.5)$$

$D_h = \frac{4LA_c}{A_0}$, $\frac{A_c}{A_0} = 0.178$, D_h, is hydraulic diameter, A_c is minimum flow area, and A_0 is total area. (4.6)

$CF = \frac{h}{\rho V_{max} C_p}\mathrm{Pr}^{2/3}$, is used to evaluate heat transfer coefficient (4.7)

Non dimensional Prandtl number and the maximum velocity is given as

where $\mathrm{Pr} = \frac{\mu c_p}{k}$, $V_{\max} = V\frac{A_c}{A_{fr}}$

Where, CF is the Colburn factor, ff represents the friction factor, f_1, f_2, and f_3 are the friction factors dependent parameters which depends upon the geometrical parameters of wavy fin and tube heat exchanger, N is the number of tube row, D_c represents collar diameter, D_0 represents outer tube diameter considered as 10mm in the present study and h is the heat transfer coefficient. The Reynolds number range of the given parameter is valid $2000 < \mathrm{Re} < 10000$ (**Wang et al. 2011**.). Also, the minimum flow area (A_c) to the frontal area (A_{fr}) is kept constant to be 0.78.

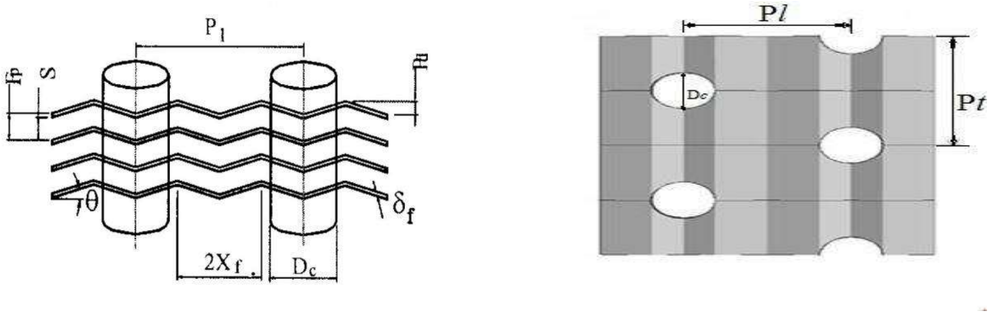

Fig. 4.2. The geometrical design of wavy fin and tube radiator of staggered arrangement.

Fin efficiency for wavy fin radiator (**Wang et al. 2011**)

Fin material is aluminum of thermal conductivity 177 W/mK

$$\eta_{fin} = \frac{tanh(mr\theta)}{mr\theta} \tag{4.8}$$

where, non-dimensional parameters are given as,

$$\theta = \left(\frac{R_{eq}}{r} - 1\right) * \left(1 + 0.35\left(ln\frac{R_{eq}}{r}\right)\right) \tag{4.9}$$

$$\frac{R_{eq}}{r} = 1.27 * \left(\frac{X_l}{X_m} - 0.3\right)^{0.5} \tag{4.10}$$

$$X_m = \mathrm{P}_l/2 \ ,m = \sqrt{\frac{2\,h}{k_{fin}\delta_{fin}}} \text{ and } \ X_l = \sqrt{\left(\frac{\mathrm{P}_t}{2}\right)^2 + \left(\frac{\mathrm{P}_l}{2}\right)^2} \tag{4.11}$$

TPF represents the thermal performance factor which indicates the performance of the wavy fin and tube heat exchanger (**Bhuiya et al. 2016**). The larger the value better the combined thermal and hydraulic performance enhancement of the wavy fin and tube heat exchanger. The TPF is defined as follows

$$TPF = \frac{CF}{(ff)^{1/3}} \tag{4.12}$$

Table. 4.1 Different factors and their level.

Factor		Level		
		1	2	3
A	fin pitch (f_p), mm	2.5	3	3.5
B	number of tube row (N)	2	3	4
C	waffle height (P_d), mm	1.4	1.8	2.2
D	fin thickness (δ_f), mm	0.15	0.18	0.24
E	Air velocity (v), m/s	2	3	5
F	Longitudinal tube pitch (P_l), mm	15.6	17.6	19.6
G	Transverse tube pitch (P_t), mm	24.8	31.75	38.1

The SN ratio is used in quality engineering. The SN ratio is needed to determine the best parameter of the control factor compared to other parameters for an individual target. Therefore, there is a trade-off between heat transfer and pressure drop; the SN ratio is determined for each run and is tabulated in Table 4.3.

Larger the best, for a target function, higher value is the requirement then SNR is given as,

$$SNR = 10\log_{10}\left[\frac{1}{\mathrm{n}}\sum_{i=1}^{\mathrm{n}}\frac{1}{\mathrm{Y}_i^{\,2}}\right] \tag{4.13}$$

Smaller the best, for a target function, lower value is the requirement then SNR is given as,

$$SNR = 10\log_{10}\left[\frac{1}{\mathrm{n}}\sum_{i=1}^{\mathrm{n}}\mathrm{Y}_i^{\,2}\right] \tag{4.14}$$

where,

Y_i represents the performance value of i^{th} run

N represents the total number of run

Furthermore, for the degree of agreement, the presumed SN represented as SNR_{pre} is determined using the SNR value at corresponding factors and levels from Table 4.3.

$$SNR_{pre} = SNR_m + \sum_{i=1}^{u}(\overline{SNR}_i - SNR_m) \tag{4.15}$$

Where SNR_m is the average total SN ratio, $\overline{SNR}_i$ is the value of the SN ratio at the optimum level condition, and *u* is the number of design parameters that influence the response factor significantly.

Table. 4.2 Simulation plan Taguchi method Orthogonal array of L_{27} (3^7).

Run	**Parameter and its level**							
	A	B	C	D	E	F	G	η_{fin}
1	1	1	1	1	1	1	1	0.69
2	1	1	1	1	2	2	2	0.43
3	1	1	1	1	3	3	3	0.29
4	1	2	2	2	1	1	1	0.80
5	1	2	2	2	2	2	2	0.56
6	1	2	2	2	3	3	3	0.38

7	1	3	3	3	1	1	1	0.81
8	1	3	3	3	2	2	2	0.66
9	1	3	3	3	3	3	3	0.47
10	2	1	2	3	1	2	3	0.63
11	2	1	2	3	2	3	1	0.60
12	2	1	2	3	3	1	2	0.75
13	2	2	3	1	1	2	3	0.37
14	2	2	3	1	2	3	1	0.35
15	2	2	3	1	3	1	2	0.42
16	2	3	1	2	1	2	3	0.52
17	2	3	1	2	2	3	1	0.50
18	2	3	1	2	3	1	2	0.61
19	3	1	3	2	1	3	2	0.44
20	3	1	3	2	2	1	3	0.52
21	3	1	3	2	3	2	1	0.44
22	3	2	1	3	1	3	2	0.56
23	3	2	1	3	2	1	3	0.65
24	3	2	1	3	3	2	1	0.57
25	3	3	2	1	1	3	2	0.38
26	3	3	2	1	2	1	3	0.42
27	3	2	2	1	3	2	1	0.36

4.2 Optimization Methodology

Grey relational tool is used for the design optimization of wavy fin and tube heat exchanger. The step-wise procedure of grey relational analysis for the selection of optimum design is shown in the process diagram of Fig. 4.3. Grey relational analysis helps determine the design parameter's influence using a grey system on multiple responses qualitatively. Taguchi SN ratio helps in determining the influence of each design parameter on the individual target response. But when there are multiple responses, which means combining all target responses for the optimization process, there is not a single design that satisfies all responses. Therefore, a systematic approach is needed, mainly grey relational analysis, which will result in selecting the optimum design parameter to influence each target response by using the weighting method. In the present investigation, equal weight is given to the target response, which is the Colburn factor, friction factor, TPF, and heat transfer coefficient. GRA's fundamental components are its level of variability and similarity of the control factor. This study aims to have a higher Colburn factor, air heat transfer coefficient, TPF, and lower friction factor simultaneously. Target response optimization of a wavy fin and tube follows steps as depicted in Fig. 4.3. After normalizing, the Grey relational coefficient (GRC) is evaluated using Eq. (4.14). After calculating the GRC value of the target response at the simulated planned chart, the Grey relational grade (GRG) is determined as per the weight assigned to the target response. Once GRG is calculated, Rank is assigned, preferring the highest value of GRG as better, for all the runs.

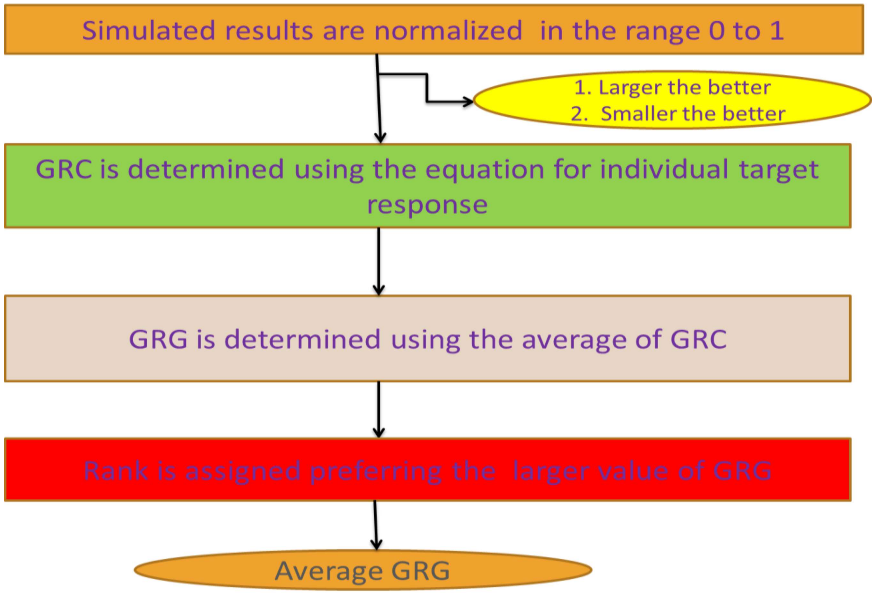

Fig. 4.3. Stepwise flow diagram of design optimization.

To produce an optimized design, simulation is done as per the planned orthogonal array to visualize the effect of the design parameter on the Colburn factor, friction factor, heat transfer coefficient, and TPF. This signal-to-noise ratio is used to estimate the performance analysis of a heat exchanger as tabulated in Table 4.3. Also, the orthogonal array of combinations for each run set was solved, and the Colburn factor, friction factor, TPF, and heat transfer coefficient codes are written in EES (Engineering Equation Solver). That value is then given as an input parameter in the Minitab package, and then SNR is determined for all runs with the entire response factor.

Table. 4.3 Simulation plan of the orthogonal array for the Colburn factor, friction factor, and air-side convective heat transfer coefficient with their SNR values.

Run	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**Colburn factor**	**SNR**	**Friction factor**	**SNR**	**Convective coefficient**	**SNR**
1	1	1	1	1	1	1	1	0.0085	-41.42	0.104	19.68	31.55	29.98
2	1	1	1	1	2	2	2	0.0095	-40.48	0.144	16.85	52.73	34.44
3	1	1	1	1	3	3	3	0.0086	-41.30	0.167	15.57	79.98	38.06
4	1	2	2	2	1	1	1	0.0067	-43.54	0.123	18.19	24.72	27.86
5	1	2	2	2	2	2	2	0.0077	-42.29	0.169	15.45	42.83	32.64
6	1	2	2	2	3	3	3	0.0071	-43.03	0.195	14.21	65.52	36.33
7	1	3	3	3	1	1	1	0.0054	-45.40	0.140	17.07	19.95	26.00
8	1	3	3	3	2	2	2	0.0064	-43.86	0.191	14.39	35.75	31.07
9	1	3	3	3	3	3	3	0.0059	-44.53	0.219	13.18	55.13	34.83
10	2	1	2	3	1	2	3	0.0091	-40.79	0.151	16.45	33.94	30.61
11	2	1	2	3	2	3	1	0.0087	-41.18	0.074	22.61	48.66	33.74
12	2	1	2	3	3	1	2	0.0036	-48.76	0.081	21.80	33.87	30.60
13	2	2	3	1	1	2	3	0.0167	-35.56	0.177	15.05	61.96	35.84
14	2	2	3	1	2	3	1	0.0153	-36.30	0.088	21.11	85.34	38.62
15	2	2	3	1	3	1	2	0.0094	-40.58	0.097	20.29	86.91	38.78
16	2	3	1	2	1	2	3	0.0114	-38.85	0.133	17.53	42.4	32.55
17	2	3	1	2	2	3	1	0.0106	-39.49	0.064	23.83	59.11	35.43
18	2	3	1	2	3	1	2	0.0058	-44.76	0.071	23.03	53.67	34.60
19	3	1	3	2	1	3	2	0.0165	-35.68	0.068	23.35	61.11	35.72
20	3	1	3	2	2	1	3	0.0111	-39.08	0.070	23.07	61.99	35.85
21	3	1	3	2	3	2	1	0.0108	-39.34	0.038	28.40	100.2	40.02
22	3	2	1	3	1	3	2	0.0122	-38.25	0.049	26.15	45.44	33.15
23	3	2	1	3	2	1	3	0.0080	-41.98	0.051	25.92	44.39	32.95
24	3	2	1	3	3	2	1	0.0078	-42.11	0.027	31.50	72.82	37.25
25	3	3	2	1	1	3	2	0.0151	-36.41	0.061	24.30	56.18	34.99

26	3	3	2	1	2	1	3	0.0121	-38.34	0.063	24.04	67.5	36.59
27	3	2	2	1	3	2	1	0.0110	-39.16	0.034	29.48	102.4	40.21

Generally, three different criteria are available for normalization: Larger the best, smaller the best, and nominal the best. Although Larger, the best is used for the Colburn factor, air heat transfer coefficient, and TPF while smaller the best is used for the friction factor. All the simulated planned results are normalized in the range of 0 to 1 using Equations (16) and (17), which are tabulated in Table 4.5.

$$N_i(k) = \frac{\max x_i(k) - x_i(k)}{max\ x_i(k) - \min x_i\ (k)} (\text{if a higher value is better }) \tag{4.16}$$

$$N_i(k) = \frac{x_i(k) - \max x_i\ (k)}{max\ x_i(k) - \min x_i\ (k)} (\text{if the lower value is better }) \tag{4.17}$$

where,$x_i(k)$ is the original sequence and $N_i(k)$ is the sequence for comparison, i = 1,2,3…….n and k = 1,2,3……,m. Where n is the total number of experiments, and m is the total number of responses.

$$GRC = \frac{D_{min} + \Psi . D_{max}}{D_{oi}(k) + \Psi . D_{max}} \tag{4.18}$$

where, D_{min} is the minimum value of the absolute difference of a comparing sequence and D_{max} is maximum the value of the absolute difference of all comparing sequences, and Ψ is a distinguished coefficient, or the identification coefficient lies between 0 to 1. This coefficient depends on the real system requirement. The standard value is chosen to be 0.5 to magnify the sequence [**Pandey et al., 2017; Naquiddin et al., 2018**]. D_{oi} is the deviation sequence of a reference sequence and the comparable sequence.

For the Grey relation grade, an equal weight of 0.25 is selected to determine the grey relational grade of multiple responses as in Eq. (4.19).

$$GRG = \frac{\sum_{K=1}^{m} GRC}{m} \tag{4.19}$$

$$GRG_{pre} = GRG_m + \sum_{i=1}^{u} (\overline{GRG}_i - GRG_m) \tag{4.20}$$

4.3 Validation, verification and optimization with Grey analysis

The optimal design parameter for the wavy fin and tube heat exchanger is A3B2C3D1E3F3G1 which represents fin pitch 3.5mm, number of tube row 3, waffle height 2.2mm, fin thickness 0.15mm, air velocity 5m/s longitudinal tube pitch 19.6mm and transverse tube pitch 24.8mm. The optimal condition of the present result of the wavy fin and tube heat exchanger is compared to the available simulation model for wavy fin and tube configuration (**Wang et al. 1999**) which consists of a fin pitch of 3.3mm, the number of tube row 6, waffle height 1.4mm, fin thickness 0.2mm, longitudinal tube pitch 27.5mm and transverse tube pitch 31.75mm at different Reynolds number. Results of the Colburn factor, friction factor, and Thermal performance factor are compared to the present optimized design factors of wavy fin and tube heat exchanger configuration as shown in the Fig. 4.4.

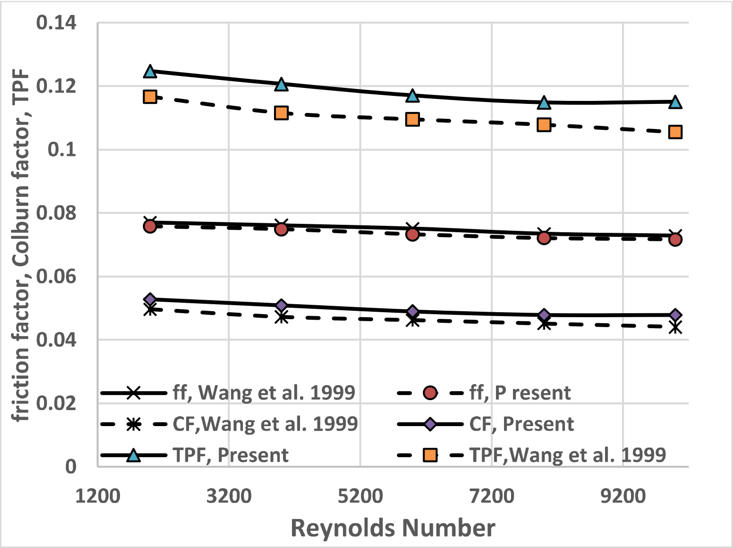

Fig. 4.4. TPF, ff and CF of optimized design with conventional model.

The present optimized dimension shows a similar variation of the CF, FF, and TPF to the wang model with the Reynolds number variation. The study found that trend variation is similar and the lower fin thickness with higher fin pitch results in a lower friction factor and higher Colburn factor and thus obtains higher TPF compared to the wang model investigation. Furthermore, With the use of the present design model and using the optimum dimension of wavy fin and tube heat exchanger resulted in 7.6% higher Colburn factor, 8.2% higher TPF, and 1.5% lower friction factor at 4000 Reynolds number compared to the design proposed by wang model.

4.4 Results and discussion

4.4.1 Effect of design parameter on the friction factor

Lower frictional pressure drops and higher heat transfer are the effective parameters for the heat transfer performance of a wavy fin and tube heat exchanger. Generally, the Colburn factor indicates the heat transfer performance, while the frictional factor represents the pressure drop. Therefore, to optimize the wavy fin and tube heat exchanger, the effect of design parameters on the friction factor is smaller, and the better is considered. The influence of each design parameter at three levels on the friction factor is shown in the SN ratio response Fig. 4.5. However, the steeper the SN ratio plot of a factor more significantly contributes to lowering the friction factor, but the highest value of each factor's SN ratio significantly impacts the objective. Therefore, larger value SN ratio of fin pitch at the 3rd level, waffle height at 1st level, air velocity at the 3rd level, longitudinal tube pitch at the 1st level, and transverse tube pitch at 1st level marked differently whose participation is more to achieve the number of tube row and fin thickness is not contributed much in the desired objective as all the SN ratio value at all levels remains almost the same.

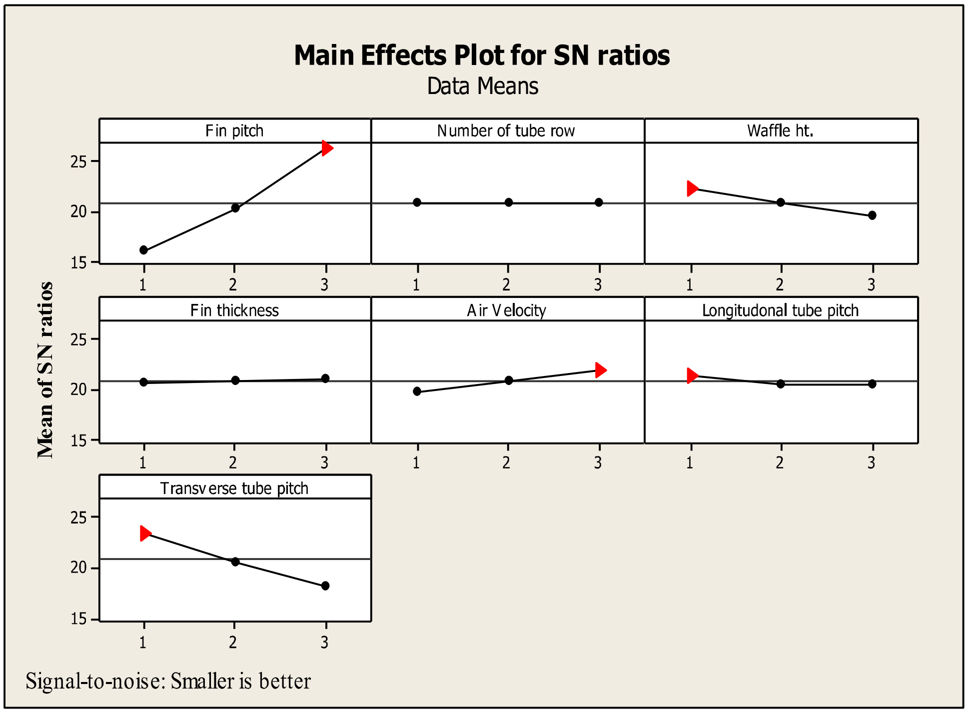

Fig. 4.5. Effect of design parameter on friction factor.

4.4.2 Effect of design parameter on Colburn factor

The response diagram of the Colburn factor is shown in Fig. 4.6. Larger is better considered for the Colburn factor as high heat transfer performance is the objective. The influence of each design factor at each level is represented in the SN ratio plot. Also, the higher the slope of the SN ratio represents a more significant impact on the objective defined as larger the better. Therefore, fin pitch at the 3rd level, number of tube rows at the 2nd level, waffle height at the 3rd level, fin thickness at 1st level, air velocity at 1st level, longitudinal tube pitch at the 3rd level, and Transverse tube pitch at 3rd level influences heat transfer significantly. Rest of the level of the design factors are least suitable for enhancing the Colburn factor performance of the wavy fin and tube heat exchanger. However, the CR of each factor is determined, which will help in determining the dominance in influencing each factor on the target.

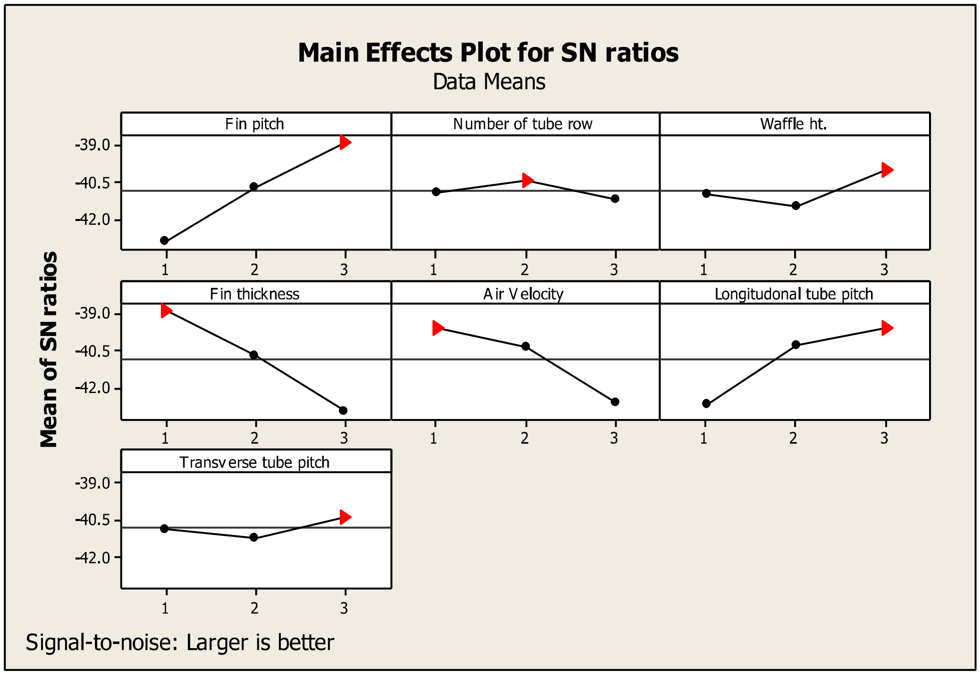

Fig. 4.6. Effect of design parameter on Colburn factor.

4.4.3 Effect of design parameter on the heat transfer coefficient

The heat transfer enhances by increasing the fin pitch and longitudinal tube pitch and reducing the fin thickness and air velocity. Also, the maximum value of the Colburn factor is obtained at 3.5mm of fin pitch, 19.6mm of longitudinal tube pitch, 2m/s of air velocity, and 0.15mm of fin thickness. Also, it is observed that the Colburn factor doesn't change significantly with transverse tube pitch and the number of tube rows. Therefore, the optimal parameter group for the heat transfer to be higher is A3B2C3D1E1F3G3. Also, the more the air velocity higher the heat transfer coefficient but also the cost of the penalty in friction factor. Thus, from Fig. 4.5 and Fig. 4.7, higher air velocity results in a higher friction factor as well as heat transfer coefficient. The heat transfer coefficient on the air side plays an important role in heat transfer performance.

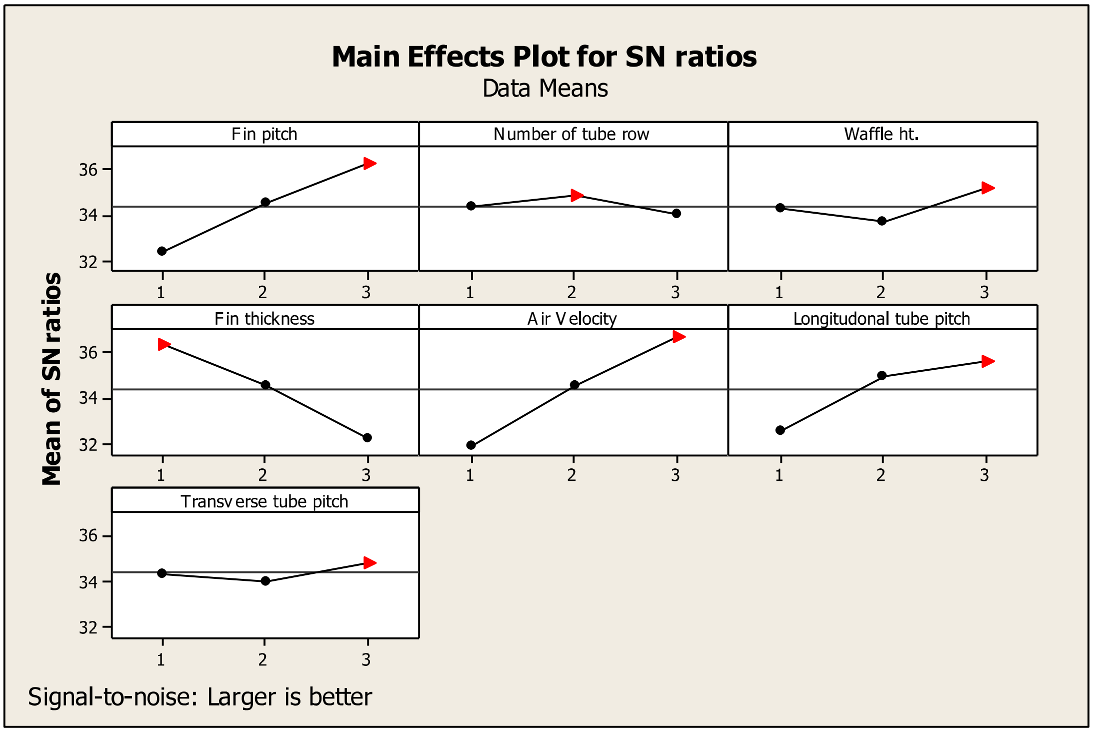

Fig. 4.7. Effect of design parameter on the heat transfer coefficient.

The effect of the design parameter at a different level on the heat transfer coefficient is depicted in the response Fig. 4.7. However, the higher the heat transfer coefficient is the objective, so the larger, the better is considered when estimating the signal-to-noise ratio. Therefore, design parameter fin pitch at 3rd level, number of tube rows at 2nd level, waffle height at 3rd level, fin thickness at 1st level, air velocity at 3rd level, longitudinal tube pitch at 3rd level, and transverse tube pitch at 3rd level influences heat transfer coefficient more than the same parameters at other levels. The higher the slope in the response diagram means the most influential design parameter on the target. It can be observed from the response diagram that increasing the air velocity, longitudinal tube pitch, and waffle height improve the heat transfer coefficient while it decreases with increasing the fin thickness. Also, from the plot, it is clear that the transverse tube pitch and the number of tube rows contribute little to the desired aim. Thus, the optimal parameter for the maximum heat transfer coefficient is A3B2C3D1E3F3G3.

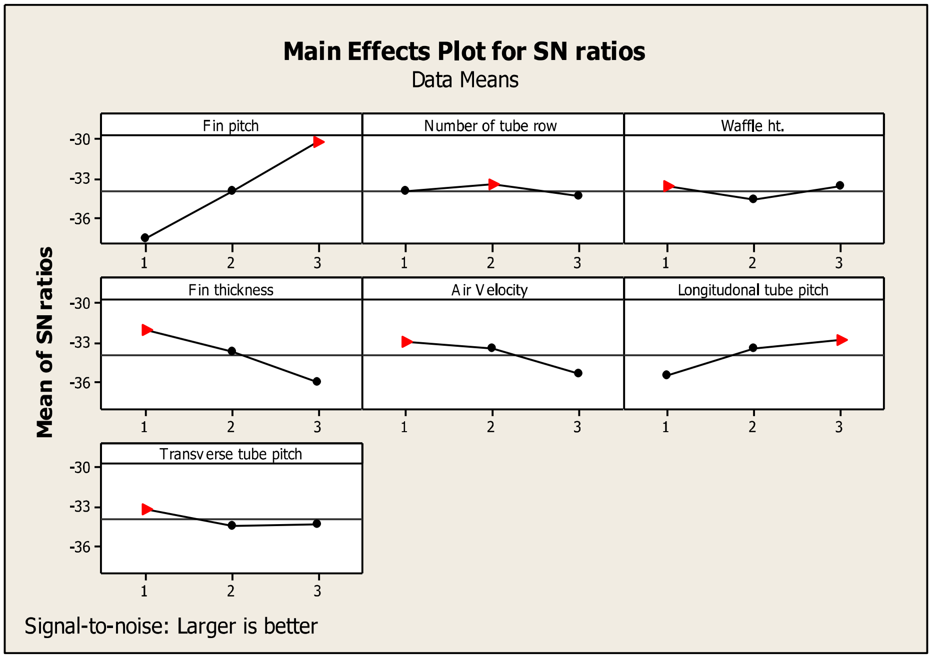

Fig. 4.8. Effect of design parameter on TPF.

TPF is a crucial parameter that suggests the thermo-hydraulic improvement of a wavy fin and tube heat exchanger. The effect of design factors at three different levels for higher TPF is presented in Fig. 4.8. The higher the TPF, the larger the SN ratio is preferred. Therefore, fin pitch and longitudinal tube pitch of 3rd level, fin thickness, transverse tube pitch, waffle height, and air velocity of 1st level improve the TPF more than the same design factors at other levels. From Fig. 4.8., the increment in fin pitch and the longitudinal tube pitch results in an improvement in TPF while TPF reduces with the increment in fin thickness, air velocity, and transverse tube pitch. The number of tube rows and waffle height is the least influential design factor which participates in the improvement of TPF due to their least slope variation. Response SN plot suggests the optimal design set of parameters for larger TPF is A3B2C1D1E1F3G1.

4.4.4 Reproducibility by the confirmation.

The optimal condition for each factor at a certain level is estimated with the help of Taguchi analysis. Now to reproduce the confirmation, the optimal condition of the heat transfer coefficient design factor is considered, and the set of combinations for optimal condition 1 is A3B2C3D1E3F3G3. However, comparing this optimal condition with the set A3B2C3D1E1F3G3 as another optimal condition 2, from the result of the Colburn factor. The presumed SN represented as $\mathrm{SNR}_{\mathrm{pre}}$ and the present analysis results

$$SNR_{pre} = SNR_{\mathrm{A3}} + SNR_{\mathrm{B2}} + SNR_{\mathrm{C3}} + SNR_{\mathrm{D1}} + SNR_{\mathrm{E3}} + SNR_{\mathrm{F3}} + SNR_{\mathrm{G3}} - 6 * \overline{SNR} = 40.17$$

Here, the numerical value of these subscript factor values is taken from Table 3.4. The SN ratio for optimal condition 2 is determined and was found to be 38.68. Therefore, there is reproducibility in this condition.

Table. 4.4 Factorial effect and its contribution ratio (A3B2C3D1E3F3G3).

Level	**(A) Fin pitch**	**(B) Number tube row**	**(C) Waffle Height**	**(D) Fin thickness**	**(E) Air Velocity**	**(F) Longitudinal tube pitch**	**(G) Transverse tube pitch**
1	32.36	34.34	34.27	36.39	31.86	32.58	34.35
2	34.53	34.82	33.73	34.55	34.59	34.96	34.00
3	36.30	34.03	35.19	32.24	36.74	35.65	34.84
Delta	3.95	0.80	1.46	4.15	4.88	3.08	0.85
Rank	3	7	5	2	1	4	6
CR (%)	20.6	4.17	7.62	21.63	25.46	16.07	4.43

4.4.5 Contribution Ratio

CR is defined as the ratio of difference of maxima and minima value of SN ratio of each factor to the sum of all R, and it shows the contribution of each factor to achieve the target. The contribution ratio of design factors for different objective functions is shown in Fig.4.8. For heat transfer from the air to be larger, the contribution of an individual factor on larger the better value of the Colburn factor is determined by the contribution ratio using SN ratio value, Fin thickness showed the largest contribution of 23.9% and hence ranked 1st followed by fin pitch (22.74 % CR) ranked 2nd, longitudinal tube pitch (17.73 % CR) ranked 3rd, air velocity (17.73% CR) ranked 4th, waffle height (8.4% CR) ranked 5th, transverse tube pitch (4.9 % CR) ranked 6th, and lowest CR is shown by the number of tube row (4.6% CR) and hence ranked 7th among the factors. The sum of CR is 100%. The most influential parameters were fin pitch to achieve the larger Colburn factor, while the least influential parameter which affected the desired objective was the number of tube rows.

The heat exchanger requirement must have a higher heat transfer capacity at a lower pressure drop, but generally, both aims are difficult to achieve simultaneously. The friction factor smaller is better; the contribution ratio explains the contribution of each parameter to influencing the desired aim. Fin pitch showed the largest value of contribution ratio of 47.06% and hence, ranked1stfollowed by transverse tube pitch (24.04% CR) ranked 2nd, waffle height (12.4% CR) ranked 3rd, air velocity (10.12% CR) ranked 4th, longitudinal tube pitch (4.50% CR) ranked 5th, fin thickness (1.4% CR) ranked 6th and minimum value of CR was found for Number of tube row(0.51% CR). Therefore, the fin pitch showed a maximum contribution to achieve the friction factor smaller is better while the minimum contribution was found when the number of tube row design factors was considered. Also, the slope of the SN ratio in the case of the number of tube rows doesn't seem to vary much and is the same as the case when the fin thickness design factor was considered as in Fig. 4.5. Hence those two parameters are not influencing the objective.

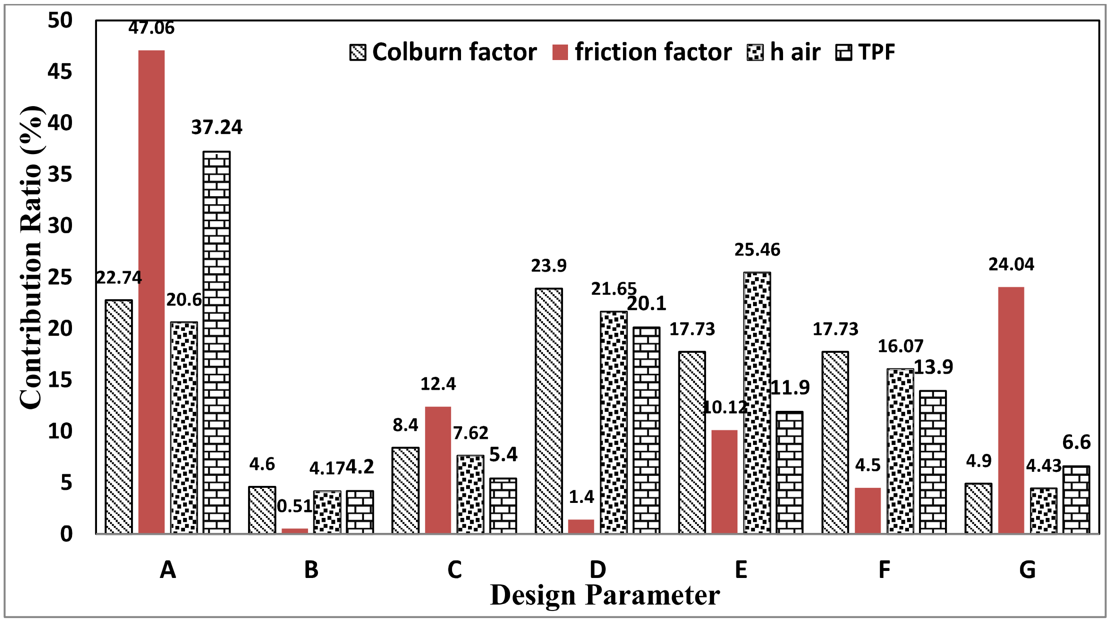

Fig. 4.9. Contribution ratio of each factor on target response.

CR is also determined at each design factor using the signal-to-noise ratio from the response diagram for the heat transfer coefficient and higher the CR recommended for achieving the target. CR for heat transfer coefficient under different design factors is shown in Fig. 4.9. Air velocity showed the largest CR value (25.46 %), and hence this parameter is ranked 1st in influencing the target followed by fin thickness (21.65%) ranked 2nd, fin pitch (20.6%) ranked 3rd, longitudinal tube pitch (16.07 %) ranked 4th, waffle height (7.62 %) ranked 5th, transverse tube pitch (4.43 %) ranked 6th and lowest CR value is estimated for the number of tube row (4.17 %) and hence ranked 7th among the design factors. Therefore, air velocity is the most influential parameter for the desired aim, and tube rows are the least influential design parameter. CR of seven different design factors for the higher TPF is also presented in Fig. 4.9. For larger TPF, the fin pitch design factor contributes a maximum of 37.24%, fin thickness 20.1% followed by longitudinal tube pitch 13.9%. TPF enhancement shows a negligible effect in the range of investigation for the number of tube rows and waffle height due to the least CR. Thus, CR suggested the effect of different design factors individually on the objective functions

Table. 4.5 Normalised response, grey relational grade, and the grey relational coefficient for the wavy fin tube type heat exchanger.

Runs	Normalized Response				GRC				GRG	Orders
	CF	FF	H	TPF	CF	FF	H	TPF		
1	0.372	0.599	0.141	0.303	0.443	0.555	0.368	0.418	0.446	19
2	0.446	0.392	0.398	0.303	0.474	0.451	0.454	0.418	0.449	18
3	0.381	0.274	0.728	0.227	0.447	0.408	0.648	0.393	0.474	17
4	0.231	0.499	0.058	0.156	0.394	0.499	0.347	0.372	0.403	23
5	0.310	0.262	0.278	0.173	0.420	0.404	0.409	0.377	0.402	24
6	0.261	0.128	0.553	0.118	0.404	0.364	0.528	0.362	0.414	22
7	0.132	0.411	0.000	0.061	0.366	0.459	0.333	0.347	0.376	26
8	0.212	0.148	0.192	0.086	0.388	0.370	0.382	0.354	0.373	27
9	0.176	0.000	0.427	0.045	0.378	0.333	0.466	0.344	0.380	25
10	0.421	0.357	0.170	0.275	0.463	0.437	0.376	0.408	0.421	20
11	0.390	0.754	0.348	0.388	0.450	0.670	0.434	0.450	0.501	14
12	0.000	0.716	0.169	0.001	0.333	0.638	0.376	0.333	0.420	21
13	1.000	0.221	0.510	0.668	1.000	0.391	0.505	0.601	0.624	8
14	0.895	0.682	0.793	0.816	0.826	0.611	0.707	0.731	0.719	5
15	0.438	0.636	0.812	0.376	0.471	0.579	0.727	0.445	0.555	12
16	0.596	0.448	0.272	0.438	0.553	0.475	0.407	0.471	0.477	15
17	0.534	0.804	0.475	0.567	0.518	0.719	0.488	0.536	0.565	11
18	0.164	0.772	0.409	0.175	0.374	0.687	0.458	0.377	0.474	16
19	0.982	0.785	0.499	1.000	0.966	0.700	0.500	1.000	0.791	2
20	0.573	0.774	0.510	0.582	0.540	0.689	0.505	0.545	0.569	10
21	0.548	0.941	0.973	0.743	0.525	0.894	0.949	0.660	0.757	3
22	0.659	0.883	0.309	0.783	0.594	0.810	0.420	0.697	0.630	7
23	0.331	0.876	0.296	0.412	0.428	0.801	0.415	0.460	0.526	13
24	0.322	1.000	0.641	0.560	0.424	1.000	0.582	0.532	0.635	6
25	0.880	0.822	0.439	0.941	0.807	0.737	0.471	0.895	0.728	4
26	0.649	0.812	0.577	0.692	0.588	0.727	0.542	0.619	0.619	9
27	0.566	0.964	1.000	0.808	0.535	0.933	1.000	0.722	0.798	1

Using the theoretical value, the target response is then normalized in the range of 0 to 1, as stated in the GRA steps. Onwards grey relational coefficient is determined analytically, described in the steps of GRA, and then Grey Relational Grade is estimated, which is tabulated in Table 4.5.

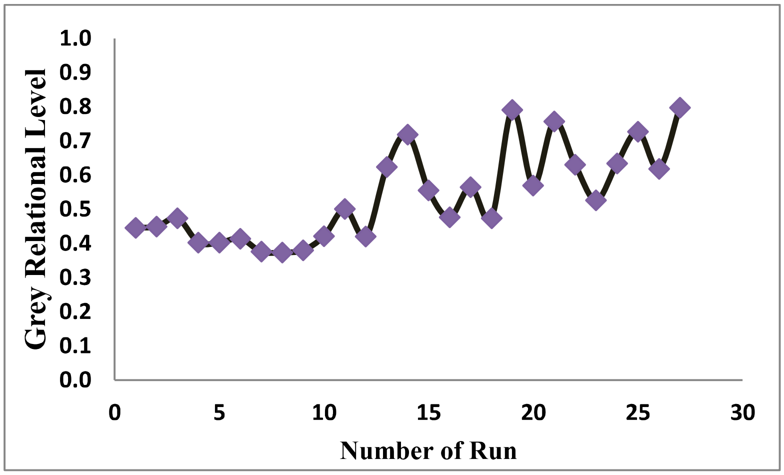

Fig. 4.10. Grey relational level at different run.

Grey Relational level varies in the range of 0 to 1, and how it varies at different run conditions is presented in Fig. 4.10. For a better planned orthogonal array, as the number of runs proceeds, the GRG value changes at a faster rate. Using grey relational grade higher, the grade is the priority and hence is arranged in the order. In the end, the response graph of the grey relational grade is represented in Fig. 4.11, which is plotted using the L_{27} orthogonal simulated plan and GRG on combining all responses.

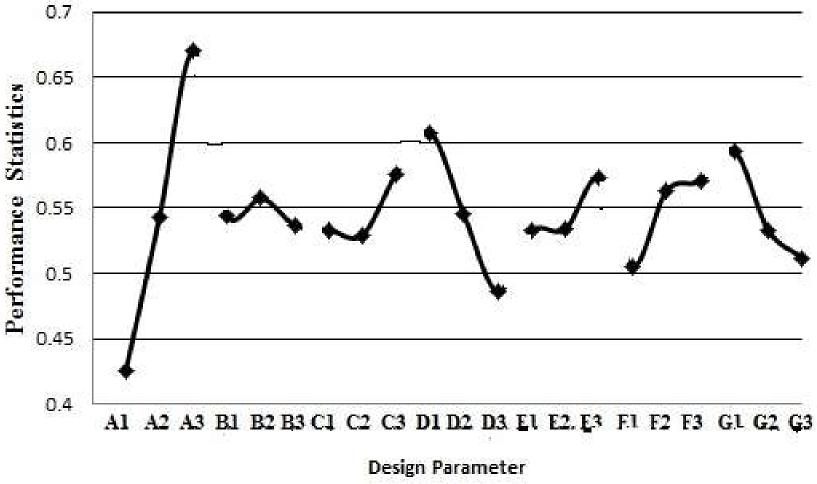

Fig. 4.11. Average GRG with a combination of all responses.

As depicted in Fig. 4.11, the best optimal condition considering all responses are fin pitch (A) at level 3 (3.5mm), number of tube row (B) at level 2 (3), waffle height (C) at level 3 (2.2mm) , fin thickness (D) at level 1 (0.15mm), air velocity (E) at level 3 (5m/s), longitudinal tube pitch (E) at level 3 (19.6mm) and transverse tube pitch (F) at level 1 (24.8mm). The optimal design parameter for wavy fin and tube heat exchanger is A3B2C3D1E3F3G1. On mapping the orthogonal array simulated plans with the optimal condition of Table 4.3, a confirmation test is required to conduct on the optimum design parameter to validate. And as per the grade order, the first rank is assigned to the optimum value of GRG, and others are arranged accordingly in decreasing order of GRG. The optimum GRG is predicted in a 27th run, which is ranked 1, reported in Table 4.5, and the corresponding design parameter and levels are A3B2C2D1E3F2G1.

The optimal condition set of design parameters using average GRG is found to be as A3B2C3D1E3F3G1, and the set of design parameters is determined analytically as A3B2C2D1E3F2G1. The GRG value for the second optimal condition shown in Table 4.5 is 0.823 and when the 1st case of optimal condition is considered as,

$$GRG_{pre} = GRG_{\mathrm{A3}} + GRG_{\mathrm{B2}} + GRG_{\mathrm{C3}} + GRG_{\mathrm{D1}} + GRG_{\mathrm{E3}} + GRG_{\mathrm{F3}} + GRG_{\mathrm{G1}} - 6 * GRG_{m} = 0.87$$

Therefore, the value determined from the analytical data is 0.82, and from the GRG analysis value is 0.87, and hence, there is reproducibility in the results as the difference is 0.047 only.

4.4 Highlights

The effect of different design parameters on target response is studied; the Colburn factor, heat transfer coefficient, friction factor, and TPF are considered as target response. The effect of those design parameters on each target response is studied using the Taguchi method, while the interaction between those target responses is evaluated by Grey relational analysis. Some conclusive remarks are highlighted as follows:

- A3C1E3F1G1 is the predicted set for optimum design parameter in influencing the smaller friction factor with fin pitch (47.06 %) and transverse tube pitch (24.04 %) as the most influencing parameter due to higher contribution ratio while fin thickness and number of tube row is fewer influences the friction factor.
- Taguchi method predicted the optimum influence parameter for the Colburn factor stated a set of control parameters as A3B2C3D1E1F3G3 with contribution ratio of fin thickness (23.9%) and fin pitch (22.74%) obtained as major influencing parameter comparatively while the least influential parameter is a number of tube row.
- Taguchi technique delivers a set of design parameters for higher TPF A3B2C1D1E1F3G1 with fin pitch (37.24%) and fin thickness (20.1%) as the most likely influential parameter and number of tube row and waffle height are the least influential design factor.
- Overall contribution ratio for each design factor on target response on taking the average is found higher for fin pitch then followed by air velocity, fin thickness, longitudinal tube pitch, transverse tube pitch, waffle height, and least average contribution is observed for the number of tube row design factor.

- Grey relational analysis produces the optimum condition on combining the target response. GRG gives an optimum set of design parameters A3B2C2D1E3F2G1 for wavy fin and tube of fin pitch of 3.5mm, number of tube row 3, waffle height 2.2 mm, fin thickness 0.15mm, air velocity 5m/s, longitudinal tube pitch 19.6mm, and transverse tube pitch of 24.8mm, at which TPF is maximum while the friction factor is minimum. Also, the average GRG gives another optimal condition as A3B2C3D1E3F3G1, which reproduces the above result.