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Appendix A

Out-of-time-order correlation and detection of phase structure in Floquet transverse Ising spin system

A-I Calculation of transverse magnetization OTOC

For the calculation of the transverse magnetization OTOC due to the local operators placed at different sites, we consider $\hat{V} = \hat{\sigma}_z^m$ and $\hat{W} = \hat{\sigma}_z^l$. Hence TMOTOC is defined as:

$$F_x^{l,m}(n) = \langle \phi_0 | \hat{\sigma}_z^l(n) \hat{\sigma}_z^m \hat{\sigma}_z^l(n) \hat{\sigma}_z^m | \phi_0 \rangle, \tag{A.1}$$

We transform the spin variables to fermionic creation $c^{l\dagger}$ and annihilation c^l operators at site l by using the Jordan-Wigner transformation [S249]

$$S_x^l = -\frac{1}{2} \prod_{j=1}^{l-1} (2c^{j\dagger} c^j - 1)(c^{l\dagger} + c^l) \quad \text{and} \quad S_z^l = c^{l\dagger} c^l - \frac{1}{2}. \tag{A.2}$$

The operators c^l and $c^{l\dagger}$ obey the the usual fermion anticommutation rules. The unitary operator for the closed chain is given as

$$\begin{aligned}\hat{U} &= \exp\Big[\frac{-it_1}{4}\Big(\sum_{l=1}^{N-1}(c^{l\dagger}-c^l)(c^{l+1\dagger}-c^{l+1})-(-1)^{N_F}(c_N^\dagger-c_N)(c^{l+1\dagger}-c_{N+1})\Big)\Big]\\ &\quad\times\exp\Big[-it_0\sum_{l=1}^{N}\big(c^{l\dagger}c^l-\frac{1}{2}\big)\Big], \qquad (A.3)\end{aligned}$$

where $N_F=\sum_{l=1}^{N}c^{l\dagger}c^l$ is the total number of fermions. We move in the momentum space using the Fourier transform of c^l which is defined as

$$c_q=\frac{exp(i\frac{\pi}{4})}{\sqrt{N}}\sum_{l=1}^{N}e^{-iql}c^l. \qquad (A.4)$$

Hence U can be written as [S54]

$$\hat{U}=e^{(-it_0\frac{N}{2})}\prod_{q>0}\mathscr{V}^q. \qquad (A.5)$$

The operator $\mathscr{V}_q$ in the above expression has the form

$$\begin{aligned}\mathscr{V}_q &= \exp\Big(-i\frac{t_1}{2}[\cos(q)(c^{q\dagger}c_q+c_{-q}^\dagger c_{-q})+\sin(q)(c_qc_{-q}c_{-q}^\dagger c_q^\dagger)]\Big)\\ &\quad\times\exp\Big(-2it_0(c^{q\dagger}c_q+c_{-q}^\dagger c_{-q})\Big). \qquad (A.6)\end{aligned}$$

For $\mathscr{V}_q$, the four basis state are $|0\rangle$, $|\pm q\rangle=c_{\pm q}^\dagger|0\rangle$, $|-qq\rangle=c_{-q}^\dagger c_q^\dagger|0\rangle$. The eigenstates of $\mathscr{V}_q$ are given by

$$\mathscr{V}_q|\pm q\rangle=e^{\left(-\frac{t_1}{2}cos(q)-it_0\right)}|\pm q\rangle, \quad and \quad \mathscr{V}_q|\pm\rangle=e^{\left(-\frac{t_1}{2}cos(q)-it_0\right))}e^{\pm i\gamma_q}|\pm\rangle,$$

where

$$|\pm\rangle=\alpha_{\pm q}|0\rangle+\beta_{\pm q}|-qq\rangle. \qquad (A.7)$$

In the above equation $\alpha_{\pm q}$ and $\beta_{\pm q}$ are given by eq. (11) and eq. (12) of the manuscript, respectively. The initial unentangled state is $|\psi_N(0)\rangle = |0\rangle^{\otimes N}$. In a Fock space, it is treated as vacuum. Time evolution operator of the fermionic annihilation operator in the momentum space is given as

$$c_q(n) = \mathscr{V}_q^{\dagger n} c_q \mathscr{V}_q^n = \Phi_q(n)^* c_q - \Psi_q(n) c_{-q}^\dagger. \tag{A.8}$$

The expansion coefficients $\Phi_q(n)$ and $\Psi_q(n)$ are defined in eq. (8) and eq. (9) of the manuscript, respectively, and phase angle (γ_q) is defined in eq. (10) of the manuscript. Let us apply the first spin operator on the initial state, we get

$$S_m^z(0)|0\rangle = \Big(c_m^\dagger c_m - \frac{1}{2}\Big)|0\rangle = -\frac{1}{2}|0\rangle. \tag{A.9}$$

In the above, $c_m^\dagger c_m$ is a number operator. The operation of the number operator on the vaccum gives zero eigenvalue. Time evolution of the spin operator at position l is

$$S_l^z(n) = \frac{1}{N}\sum_{a,b} e^{i(a-b)l} c_a^\dagger(n) c_b(n) - \frac{1}{2},$$

where a and b are indices in momentum space. By using eq. (A.8), we can write

$$\begin{aligned} S_l^z(n) &= \frac{1}{N}\sum_{a,b} e^{i(a-b)l}\Big[\Phi_a(n)c_a^\dagger - \Psi_a(n)^* c_{-a}\Big]\Big[\Phi_b(n)^* c_b - \Psi_b(n)c_{-b}^\dagger\Big] - \frac{1}{2}, \\ &= \frac{1}{N}\sum_{a,b} e^{i(a-b)l}\Big[\Phi_a(n)\Phi_b(n)^* c_a^\dagger c_b - \Phi_a(n)\Psi_b(n)c_a^\dagger c_{-b}^\dagger \\ &\quad -\Psi_a(n)\Phi_b(n)^* c_{-a}c_b + \Psi_a(n)^*\Psi_b(n)c_{-a}c_{-b}^\dagger\Big] - \frac{1}{2}. \end{aligned}$$

Application of time evolved spin operator on the vacuum gives

$$S_l^z(n)|0\rangle = \Big[-\frac{1}{N}\sum_{a,b} e^{i(a-b)l}\Phi_a(n)\Psi_b(n)c_a^\dagger c_{-b}^\dagger + \frac{1}{N}\sum_a |\Psi_a(n)|^2 - \frac{1}{2}\Big]|0\rangle, \tag{A.10}$$

and the Hermitian conjugate of the above equation is

$$\langle 0|S_l^z(n) = \langle 0|\Big[-\frac{1}{N}\sum_{p,r} e^{-i(p-r)l}\Phi_p(n)^*\Psi_r(n)^* c_{-r}c_p + \frac{1}{N}\sum_p |\Psi_p(n)|^2 - \frac{1}{2}\Big], \tag{A.11}$$

where p and r are indices in the momentum space. We can calculate $S_l^z(n)S_m^z(0)|0\rangle$ as

$$S_l^z(n)S_m^z(0)|0\rangle = -\frac{1}{2}\Big[-\frac{1}{N}\sum_{a,b} e^{i(a-b)l}\Phi_a(n)\Psi_b(n)c_a^\dagger c_{-b}^\dagger + \frac{1}{N}\sum_a |\Psi_a(n)|^2 - \frac{1}{2}\Big]|0\rangle.$$

Applying the third spin operator $S_m^z(0)$ on the state $S_l^z(n)S_m^z(0)|0\rangle$ we get

$$\begin{aligned} S_m^z(0)S_l^z(n)S_m^z(0)|0\rangle &= -\frac{1}{2}\Big[\frac{1}{N}\sum_{x,y} e^{i(x-y)m}c_x^\dagger c_y - \frac{1}{2}\Big] \\ &\times \Big[-\frac{1}{N}\sum_{a,b} e^{i(a-b)l}\Phi_a(n)\Psi_b(n)c_a^\dagger c_{-b}^\dagger + \frac{1}{N}\sum_a |\Psi_a(n)|^2 - \frac{1}{2}\Big]|0\rangle, \\ &= -\frac{1}{2}\Big[-\frac{1}{N^2}\sum_{x,y,a,b} e^{i(x-y)l}e^{i(a-b)l}\Phi_a(n)\Psi_b(n)\Big(c_x^\dagger c_{-b}^\dagger\delta(a,y) \\ &\quad -c_x^\dagger c_a^\dagger\delta(-b,y)\Big) + \frac{1}{2N}\sum_{a,b} e^{i(a-b)l}\Phi_a(n)\Psi_b(n)c_a^\dagger c_{-b}^\dagger \\ &\quad -\frac{1}{2}\Big(\frac{1}{N}\sum_a |\Psi_a(n)|^2 - \frac{1}{2}\Big)\Big]|0\rangle, \end{aligned} \tag{A.12}$$

where x and y are the indices in the momentum space. Now we take the scalar product of the states given by eq. (A.11) and eq. (A.12) and get TMOTOC as

$$\begin{aligned} F_x^{l,m}(n) &= 2^4\langle 0|S_l^z(n)S_m^z(0)S_l^z(n)S_m^z(0)|0\rangle, \\ &= -2^3\langle 0|\Big[-\frac{1}{N}\sum_{p,r} e^{-i(p-r)l}\Phi_p(n)^*\Psi_r(n)^* c_{-r}c_p + \frac{1}{N}\sum_p |\Psi_p(n)|^2 - \frac{1}{2}\Big] \\ &\quad \Big[-\frac{1}{N^2}\sum_{x,y,a,b} e^{i(x-y)m}e^{i(a-b)l}\Phi_a(n)\Psi_b(n)\Big(c_x^\dagger c_{-b}^\dagger\delta(a,y) - c_x^\dagger c_a^\dagger\delta(-b,y)\Big) \\ &\quad +\frac{1}{2N}\sum_{a,b} e^{i(a-b)l}\Phi_a(n)\Psi_b(n)c_a^\dagger c_{-b}^\dagger - \frac{1}{2}\Big(\frac{1}{N}\sum_a |\Psi_a(n)|^2 - \frac{1}{2}\Big)\Big]|0\rangle, \end{aligned}$$

$$\begin{aligned}F_x^{l,m}(n) &= -2^3\Big[\frac{1}{N^3}\Big(\sum_{p,a,r,x,y} e^{-i(p-a)l}e^{i(x-y)m}|\Psi_r(n)|^2\Phi_p^*(n)\Phi_a(n)\delta(p,x)\delta(a,y)\\ &\quad -e^{i(r+a)l}e^{i(x-y)m}\Psi_r(n)^*\Phi_p^*(n)\Phi_a(n)\Psi_{-p}(n)\delta(-r,x)\delta(a,y)\\ &\quad -e^{-i(p+b)l}e^{i(x-y)m}\Psi_b(n)\Psi_{-a}(n)^*\Phi_p(n)^*\Phi_a(n)\delta(p,x)\delta(-b,y)\\ &\quad +e^{i(r-b)l}e^{i(x-y)m}\Psi_b(n)\Psi_r(n)^*|\Phi_a(n)|^2\delta(p,a)\delta(-b,y)\Big)\\ &\quad -\frac{1}{2}\Big(\frac{1}{N}\sum_p|\Psi_p(n)|^2-\frac{1}{2}\Big)\Big(\frac{1}{N}\sum_a|\Psi_a(n)|^2-\frac{1}{2}\Big)\Big]\\ &\quad -\frac{1}{2N^2}\sum_{p,r}\Big(|\Psi_p(n)|^2|\Phi_r(n)|^2-\Psi_{-p}(n)\Psi_r(n)^*\Phi_p(n)^*\Phi_{-r}(n)\Big).\end{aligned} \tag{A.13}$$

Since the term

$$\begin{aligned}&\frac{1}{2N^2}\sum_{p,r}\Big(|\Psi_p(n)|^2|\Phi_r(n)|^2-\Psi_{-p}(n)\Psi_r(n)^*\Phi_p(n)^*\Phi_{-r}(n)\Big)\\ &+\frac{1}{2}\Big(\frac{1}{N}\sum_p|\Psi_p(n)|^2-\frac{1}{2}\Big)\Big(\frac{1}{N}\sum_a|\Psi_a(n)|^2-\frac{1}{2}\Big)\end{aligned}$$

is constant for all number of kicks (n) and system size (N) which comes out to be $\frac{1}{2^3}$. Since a and b are dummy indices, we replace them with q. Hence, the final formula of TMOTOC is

$$\begin{aligned}F_z^{l,m}(n) &= 1-\Big(\frac{2}{N}\Big)^3\sum_{p,q,r}\Big(e^{i(p-q)(m-l)}|\Psi_r(n)|^2\Phi_p^*(n)\Phi_q(n)\\ &\quad -e^{i(-r-q)(m-l)}\Psi_r(n)^*\Phi_p^*(n)\Phi_q(n)\Psi_{-p}(n)\\ &\quad -e^{i(p+q)(m-l)}\Psi_q(n)\Phi_{-r}(n)\Psi_r(n)^*\Phi_p(n)^*\\ &\quad +e^{i(q-r)(m-l)}\Psi_q(n)\Psi_r(n)^*|\Phi_p(n)|^2\Big).\end{aligned} \tag{A.14}$$

Now, we take a special case in which both the operators are the same local operator i.e. $W=\sigma_l^z$ and $V=\sigma_l^z$. Then the formula becomes

$$\begin{aligned}F_z^{l,l}(n) &= 1-\Big(\frac{2}{N}\Big)^3\sum_{p,q,r}\Big(|\Psi_r(n)|^2\Phi_p^*(n)\Phi_q(n)-\Psi_{-p}(n)\Psi_r(n)^*\Phi_p^*(n)\Phi_q(n)\\ &\quad -\Psi_q(n)\Psi_r(n)^*\Phi_p(n)^*\Phi_{-r}(n)+\Psi_q(n)\Psi_r(n)^*|\Phi_p(n)|^2\Big).\end{aligned} \tag{A.15}$$

A-II Calculation of Longitudinal Magnetization OTOC

Let us attempt to find the analytical expression of LMOTOC so that we calculate the phase structure of LMOTOC for higher system size. After moving some steps in analytical calculation of LMOTOC, we realize that the analytical expression of LMOTOC will take longer time than the numerical calculation. A few initial steps of our calculation of LMOTOCs are given below: S^x_l in the form of raising and lowering operator:

$$S^x_l = \frac{1}{2}[S^+_l + S^-_l] = \frac{1}{2}\exp\Big[-\pi i \sum_{j=1}^{l-1} c^\dagger_j c_j\Big](c^{l\dagger} + c^l) = -\frac{1}{2}\prod_{j=1}^{l-1}(2c^\dagger_j c_j - 1)(c^{l\dagger} + c^l)$$

The above equation is written by using the relation

$$S^+_l = c^{l\dagger}\exp[\pi i \sum_{j=1}^{l-1} c^{j\dagger} c^j], \quad S^-_l = \exp[-\pi i \sum_{j=1}^{l-1} c^\dagger_j c_j] c^l.$$

Now, we move in the momentum space by doing the Fourier transform of c^l and $c^{l\dagger}$. Hence S^x_l in momentum space can be written as:

$$\begin{aligned} S^x_l &= -\frac{1}{2}\prod_{j=1}^{l-1}\Big[2\sum_{q_j,p_j}\frac{1}{N}\exp[i(p_j - q_j)j]c^\dagger_{q_j}c_{p_j} - 1\Big]\Big[\sum_r\Big(\frac{1}{\sqrt{N}}\exp(\frac{-i\pi}{4})\exp(irl)c^\dagger_r \\ &+ \frac{1}{\sqrt{N}}\exp(\frac{i\pi}{4})\exp(-irl)c_r\Big)\Big] \end{aligned} \tag{A.16}$$

For the calculation of the time evolution of S^x_l *i.e.*, $S^x_l(n)$ we have to compute the time evolution of all the operators in the string of length N. Time evolution of such a large term will be too much complicated and unfruitful for our purpose because the calculation of LMOTOC involving product of four operators will be too much to handle.

Appendix B

Characteristic, dynamic, and near saturation regions of Out-of-time-order correlation in Floquet Ising models

B-I Calculation of TMOTOC in the non-integrable Floquet system using random state

If $\hat{V}$ and $\hat{W}$ are two Hermitian operators that are localized on different positions l and m, respectively, the OTOC [B19] is given as:

$$C^{l,m}(n) = -\frac{1}{2}\mathrm{Tr}\left([\hat{W}^{l}(n), \hat{V}^{m}]^{2}\right), \tag{B.1}$$

which is a measure of the noncommutativity of two operators $\hat{W}^{l}$ and $\hat{V}^{m}$. These are infinite temperature quantities and involve the entire spectrum of 2^N states. One can use the trick for evaluating OTOC by employing Haar random states of 2^N dimensions ($|\Psi_R\rangle$)

and calculate expectation value over $|\Psi_R\rangle$. OTOC will be

$$C^{l,m}(n) = -2^{N-1}\langle\Psi_R|[\hat{W}^l(n), \hat{V}^m]^2|\Psi_R\rangle, \tag{B.2}$$

Since, the behaviour of OTOC is similar in both the cases either taking random states or special initial states ($|\phi\rangle$ and $|\psi\rangle$ accordingly). So we consider special initial states and OTOC will be

$$C^{l,m}(n) = -2^{N-1}\langle\psi/\phi|[\hat{W}^l(n), \hat{V}^m]^2|\psi/\phi\rangle, \tag{B.3}$$

Fig. (B.1) is the behaviour of TMOTOC in the nonintegrable $\hat{\mathscr{U}}_x$ system using random initial state (ψ_R) drawn form the Harr measure. Characteristic time is independent of the Floqeut period [Fig. B.1(a)] and it depends on the separation between the observables. Number of kicks required to depart from unity is equal to separation between the observables [Fig. B.1(b)]. Dynamic region of TMOTOC for the nonintegrable is showing a power-law [Fig. B.1(c)] that is approximately similar to the [Fig. 3.4(c)]. The exponent of the power-law (b) depends on Δl [Fig. B.1(d)] and its behaviour is approximately similar as Fig. 3.4(d). Saturation of $\Re[F_z^{l,m}(n)]$ is following a linear decaying behaviour with a very small slope (0.004) for all Δl [Fig. B.1(e)]. There is very small oscillation in comparison of Fig. 3.4(e).

B-II Time evolution of TMOTOC

The Heisenberg evolution of an operator $\hat{W}(t)$ can be expanded using the Hausdorff-Baker-Campbel (HBC) formula as

$$\hat{W}(t) = \sum_{p=0}^{\infty} \frac{(it)^p}{p!}[\hat{H}, [\hat{H}, \cdots^{\text{p times}}, [\hat{H}, \hat{W}]]]. \tag{B.4}$$

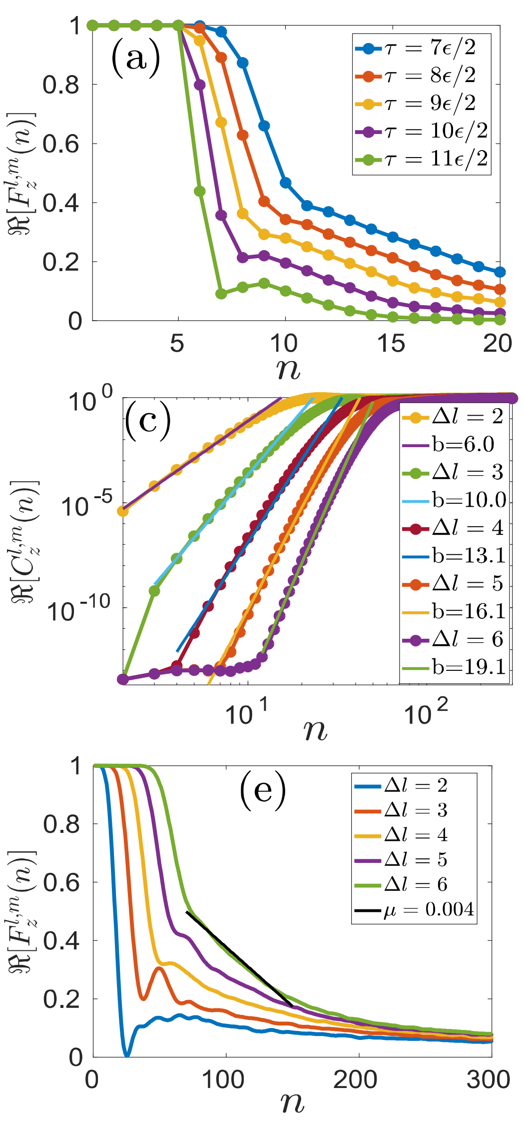

Fig. B.1 Nonintegrable closed chain transverse Ising Floquet system with $J_x = 1$, $h_z = 1$, and $h_x = 1$ of size $N = 18$. **(a)** Behaviour of $TMOTOC$ with number of kicks (n) by increasing Floquet period from $\frac{7\varepsilon}{2}$ to $\frac{11\varepsilon}{2}$ differing by $\frac{\varepsilon}{2}$ with fixed $\Delta l = 6$ ($\varepsilon = \frac{\pi}{28}$). **(b)** Initial region of $F_z^{l,m}$ with number of kicks and increasing distances between the spins (Δl) with fixed Floquet period $\tau = 6\varepsilon/2$. **(c)** $C_z^{l,m}$ with number of kicks ($\log - \log$) with increasing (Δl) at fixed $\tau = \frac{\varepsilon}{2}$

. **(d)** Changing of exponent of power-law with Δl. **(e)** Saturation of $F_z^{l,m}$ with number of kicks.

If $\hat{W} = \hat{\sigma}_l^{z/x}$, the HBC formula captures the spread of the operator over the spin sites and how it becomes more complex as time increases. Furthermore, direct replacement of

Eq. B.4 in Eq. 3.6 highlights the fact that the short-time growth is characterized by the smallest p on which

$$[\hat{H},[\hat{H},\cdots^{\text{p times}},[\hat{H},\hat{\sigma}_l^{x/z}]],\hat{\sigma}_m^{x/z}] \neq 0, \tag{B.5}$$

due to the time factor t^n that weights the terms in the expansion. We remark that this mechanism points out that the short-time growth is characterized by a general Hamiltonian structure of the system and not by the regular to chaotic regimes observed in the studied spin chains.

We consider Pauli operator in transverse direction of the coupling and $\hat{\mathscr{U}}_x = \hat{U}_{xx}\hat{U}_z$ where, $\hat{U}_{xx} = \exp\big[-i\tau(J_x\hat{H}_{xx} + h_x\hat{H}_x)\big]$ and $\hat{U}_z = \exp(-i\tau h_z\hat{H}_z)$. Using Eq. B.4, the Heisenberg evolution of the spin operator $\hat{\sigma}_z^l$ is obtained:

$$\hat{\sigma}_z^l(n) = (\hat{U}_z^\dagger\hat{U}_{xx}^\dagger)^n\hat{\sigma}_z^l(\hat{U}_{xx}\hat{U}_z)^n, \tag{B.6}$$

after applying first kick $\hat{\sigma}_z^l(1)$ is

$$\begin{aligned}
\hat{\sigma}_z^l(1) &= \hat{U}_z^\dagger\hat{U}_{xx}^\dagger\hat{\sigma}_z^l\hat{U}_{xx}\hat{U}_z, \\
&= \hat{U}_z^\dagger(\hat{\sigma}_z^l + i\tau[\hat{H}_{xx}+\hat{H}_x,\hat{\sigma}_z^l] + \frac{(i\tau)^2}{2!}[\hat{H}_{xx}+\hat{H}_x,[\hat{H}_{xx}+\hat{H}_x,\hat{\sigma}_z^l]+\cdots)\hat{U}_z, \\
&= \hat{\sigma}_z^l + i\tau\Big(\hat{U}_z^\dagger(-2i(\hat{\sigma}_x^{l-1}\hat{\sigma}_y^l + \hat{\sigma}_y^l\hat{\sigma}_x^{l+1} + \hat{\sigma}_y^l)\hat{U}_z\Big) + \cdots, \\
&= \hat{\sigma}_z^l + 2\tau\Big(\hat{U}_z^\dagger(\hat{\sigma}_x^{l-1}\hat{\sigma}_y^l + \hat{\sigma}_y^l\hat{\sigma}_x^{l+1} + \hat{\sigma}_y^l)\hat{U}_z\Big) + \cdots, \\
&= \hat{\sigma}_z^l + 2\tau\Big(\hat{}\sigma_x^{l-1}\hat{\sigma}_y^l + \hat{\sigma}_y^l\hat{\sigma}_x^{l+1} + \hat{\sigma}_y^l + i\tau(-2i[\hat{\sigma}_x^{l-1}\hat{\sigma}_x^l + \hat{\sigma}_y^{l-1}\hat{\sigma}_y^l + \hat{\sigma}_x^l\hat{\sigma}_x^{l+1} \\
&\quad + \hat{\sigma}_y^l\hat{\sigma}_y^{l+1} + \hat{\sigma}_x^l]) + \cdots\Big) + \cdots, \\
&= \hat{\sigma}_z^l + \Big(2\tau(\hat{\sigma}_x^{l-1}\hat{\sigma}_y^l + \hat{\sigma}_y^l\hat{\sigma}_x^{l+1} + \hat{\sigma}_y^l) + (2\tau)^2(\hat{\sigma}_x^{l-1}\hat{\sigma}_x^l + \hat{\sigma}_y^{l-1}\hat{\sigma}_y^l + \hat{\sigma}_x^l\hat{\sigma}_x^{l+1} \\
&\quad + \hat{\sigma}_y^l\hat{\sigma}_y^{l+1} + \hat{\sigma}_x^l) + \cdots\Big) + \cdots.
\end{aligned} \tag{B.7}$$

We apply second kick then $\hat{\sigma}_z^l(2)$ will be

$$\begin{aligned}\hat{\sigma}_z^l(2) &= \hat{U}_z^\dagger \hat{U}_{xx}^\dagger \Big(\hat{\sigma}_z^l + \Big(2\tau(\hat{\sigma}_x^{l-1}\hat{\sigma}_y^l + \hat{\sigma}_y^l\hat{\sigma}_x^{l+1} + \hat{\sigma}_y^l) + (2\tau)^2(\hat{\sigma}_x^{l-1}\hat{\sigma}_x^l + \hat{\sigma}_y^{l-1}\hat{\sigma}_y^l \\ &\quad + \hat{\sigma}_x^l\hat{\sigma}_x^{l+1} + \hat{\sigma}_y^l\hat{\sigma}_y^{l+1} + \hat{\sigma}_x^l) + \cdots \Big) + \cdots \Big) \hat{U}_{xx}\hat{U}_z, \\ &= \hat{\sigma}_z^l + \Big(4\tau(\hat{\sigma}_y^{l-1}\hat{\sigma}_x^l + \hat{\sigma}_y^l\hat{\sigma}_x^{l+1} + \hat{\sigma}_y^l) + (2\tau)^2(\hat{\sigma}_x^{l-1}\hat{\sigma}_x^l + \hat{\sigma}_y^{l-1}\hat{\sigma}_y^l + \hat{\sigma}_x^l\hat{\sigma}_x^{l+1} \\ &\quad + \hat{\sigma}_y^l\hat{\sigma}_y^{l+1} + \hat{\sigma}_x^l) + (2\tau)^2(\hat{U}_z^\dagger\hat{U}_{xx}^\dagger(\hat{\sigma}_y^{l-1}\hat{\sigma}_y^l + \hat{\sigma}_y^l\hat{\sigma}_y^{l+1})\hat{U}_{xx}^\dagger\hat{U}_z^\dagger) + \cdots \Big) + \cdots\end{aligned} \tag{B.8}$$

From the above equation, we extract the coefficient of τ^2 which contain $\hat{\sigma}_y^{l+2}$ term. This is given as

$$\begin{aligned}&(2\tau)^2(\hat{U}_z^\dagger\hat{U}_{xx}^\dagger(\hat{\sigma}_y^{l-1}\hat{\sigma}_y^l + \hat{\sigma}_y^l\hat{\sigma}_y^{l+1})\hat{U}_{xx}^\dagger\hat{U}_z^\dagger) \\ &\qquad = (2\tau)^2\hat{U}_z^\dagger\Big(\hat{\sigma}_y^l\hat{\sigma}_y^{l+1} + i\tau[\hat{H}_{xx} + \hat{H}_x, \hat{\sigma}_y^l\hat{\sigma}_y^{l+1}] + \cdots\Big)\hat{U}_z, \\ &\qquad = (2\tau)^2\Big(\cdots - 2\tau\hat{\sigma}_y^{l-1}\hat{\sigma}_z^l\hat{\sigma}_y^{l+1} - 2\tau\hat{\sigma}_y^l\hat{\sigma}_z^{l+1}\hat{\sigma}_y^{l+2} + \cdots\Big).\end{aligned} \tag{B.9}$$

For $m = l+2$, $C_z^{l,m}(2) = 64\tau^6$ We apply the third kick then $\hat{\sigma}_z^l(3)$ will be

$$\begin{aligned}\hat{\sigma}_z^l(3) &= \hat{U}_z^\dagger\hat{U}_{xx}^\dagger\Big(\hat{\sigma}_z^l + \Big(4\tau(\hat{\sigma}_y^{l-1}\hat{\sigma}_x^l + \hat{\sigma}_y^l\hat{\sigma}_x^{l+1} + \hat{\sigma}_y^l) + (2\tau)^2(\hat{\sigma}_x^{l-1}\hat{\sigma}_x^l + \hat{\sigma}_y^{l-1}\hat{\sigma}_y^l \\ &\quad + \hat{\sigma}_x^l\hat{\sigma}_x^{l+1} + \hat{\sigma}_y^l\hat{\sigma}_y^{l+1} + \hat{\sigma}_x^l) + \cdots\Big) + \cdots\Big)\hat{U}_{xx}\hat{U}_z, \\ &= \Big[\hat{\sigma}_z^l + \Big(6\tau(\hat{\sigma}_y^{l-1}\hat{\sigma}_x^l + \hat{\sigma}_y^l\hat{\sigma}_x^{l+1} + \hat{\sigma}_y^l) + (2\tau)^2(\hat{\sigma}_x^{l-1}\hat{\sigma}_x^l + \hat{\sigma}_y^{l-1}\hat{\sigma}_y^l + \hat{\sigma}_x^l\hat{\sigma}_x^{l+1} \\ &\quad + \hat{\sigma}_y^l\hat{\sigma}_y^{l+1} + \hat{\sigma}_x^l) + \cdots\Big) + \cdots\Big].\end{aligned} \tag{B.10}$$

For $\Delta l = 1$, $m = l+1$ dominating exponent of the power-law of the OTOC will be

$$\begin{aligned}C_z^{l,l+1}(1) &= 4\tau^2\langle\phi_0|[(\hat{\sigma}_y^l\hat{\sigma}_x^{l+1} + \hat{\sigma}_y^l), \hat{\sigma}_z^m]^2|\phi_0\rangle, \\ &= 4\langle\phi_0|(-i\hat{\sigma}_y^l\hat{\sigma}_y^{l+1})^2|\phi_0\rangle = 4\tau^2.\end{aligned} \tag{B.11}$$

Similarly, $C_z^{l,l+1}(2) = 16\tau^2$ and $C_z^{l,l+1}(3) = 36\tau^2$.

For $\Delta l = 2$, $m = l+2$, dominating exponent of the power-law of the OTOC will be $C_z^{l,l+2}(1) = 0$, $C_z^{l,l+2}(2) = 64\tau^6$. For $\Delta l = 2$, $m = l+2$. This power-law growth approximately matches the dynamic region of the Eq. (3.18).

B-III Time evolution of LMOTOC

We consider Pauli operator in longitudinal direction of the coupling. Using Eq. B.4, the Heisenberg evolution of the spin operator $\hat{\sigma}_x^l$ is obtained:

$$\hat{\sigma}_x^l(n) \quad = \quad (\hat{U}_z^\dagger \hat{U}_{xx}^\dagger)^n \hat{\sigma}_x^l (\hat{U}_{xx}\hat{U}_z)^n,$$

after applying first kick $\hat{\sigma}_x^l(1)$ is

$$\hat{\sigma}_x^l(1) = \hat{U}_z^\dagger \hat{U}_{xx}^\dagger \hat{\sigma}_x^l \hat{U}_{xx}\hat{U}_z = \hat{U}_z^\dagger(\hat{\sigma}_x^l + i\tau[\hat{H}_{xx}+\hat{H}_x, \hat{\sigma}_x^l] + \frac{(i\tau)^2}{2!}[\hat{H}_{xx}+\hat{H}_x, [\hat{H}_{xx}+\hat{H}_x, \hat{\sigma}_x^l] + \cdots)\hat{U}_z.$$

Since, $[\hat{H}_{xx}+\hat{H}_x, \hat{\sigma}_x^l] = 0$, then

$$\hat{\sigma}_x^l(1) = \hat{U}_z^\dagger \hat{\sigma}_x^l \hat{U}_z = \hat{\sigma}_x^l + i\tau[\hat{H}_z, \hat{\sigma}_x^l] + \frac{(i\tau)^2}{2!}[\hat{H}_z, [\hat{H}_z, \hat{\sigma}_x^l] + \cdots = \hat{\sigma}_x^l(1) = \hat{\sigma}_x^l - 2\tau\hat{\sigma}_y^l + \cdots.$$

We apply second kick then $\hat{\sigma}_x^l(2)$ will be

$$\begin{aligned}
\hat{\sigma}_x^l(2) &= \hat{U}_z^\dagger \hat{U}_{xx}^\dagger \hat{\sigma}_x^l(1)\hat{U}_{xx}\hat{U}_z = U_z^\dagger \hat{U}_{xx}^\dagger(\hat{\sigma}_x^l - 2\tau\hat{\sigma}_y^l + \cdots)\hat{U}_{xx}\hat{U}_z, \\
&= \Big(U_z^\dagger \hat{U}_{xx}^\dagger \hat{\sigma}_x^l \hat{U}_{xx}\hat{U}_z - 2\tau U_z^\dagger \hat{U}_{xx}^\dagger \hat{\sigma}_y^l \hat{U}_{xx}\hat{U}_z + \cdots\Big), \\
&= \Big(\hat{\sigma}_x^l - 2\tau\hat{\sigma}_y^l - 2\tau U_z^\dagger(\hat{\sigma}_y^l - 2\tau(\hat{\sigma}_y^{l-1}\hat{\sigma}_z^l + \hat{\sigma}_z^l\hat{\sigma}_y^{l+1} + \hat{\sigma}_z^l) + \cdots)\hat{U}_z + \cdots\Big),
\end{aligned}$$

$$\begin{aligned}\hat{\sigma}_x^l(2) &= \Big(\hat{\sigma}_x^l - 2\tau\hat{\sigma}_y^l - 2\tau[\hat{\sigma}_y^l - 2\tau\hat{\sigma}_y^{l-1}\hat{\sigma}_z^l - 2\tau\hat{\sigma}_z^l\hat{\sigma}_y^{l+1} - 2\tau\hat{\sigma}_z^l + i\tau(-2i\hat{\sigma}_x^l \\ &\quad - 2\tau(-2i)\hat{\sigma}_z^l\hat{\sigma}_y^{l+1}) + \cdots] + \cdots\Big), \\ &= \Big(\hat{\sigma}_x^l - 4\tau\hat{\sigma}_y^l + (2\tau)^2(\hat{\sigma}_z^l\hat{\sigma}_y^{l+1} + \hat{\sigma}_x^l + \hat{\sigma}_z^l) + (2\tau)^3\hat{\sigma}_z^l\hat{\sigma}_y^{l+1}) + \cdots\Big).\end{aligned} \tag{B.12}$$

We apply third kick then $\hat{\sigma}_x^l(3)$ will be

$$\begin{aligned}\hat{\sigma}_x^l(3) &= \hat{U}_z^\dagger\hat{U}_{xx}^\dagger\Big(\hat{\sigma}_x^l - 4\tau\hat{\sigma}_y^l + (2\tau)^2(\hat{\sigma}_z^l\hat{\sigma}_y^{l+1} + \hat{\sigma}_x^l + \hat{\sigma}_z^l) + (2\tau)^3\hat{\sigma}_z^l\hat{\sigma}_y^{l+1}) + \cdots\Big)\hat{U}_{xx}\hat{U}_z, \\ &= \Big(\hat{\sigma}_x^l - 6\tau\hat{\sigma}_y^l + 2(2\tau)^2(\hat{\sigma}_z^l\hat{\sigma}_y^{l+1} + \hat{\sigma}_x^l + \hat{\sigma}_z^l) + 2(2\tau)^3\hat{\sigma}_z^l\hat{\sigma}_y^{l+1}) + \cdots\Big) + \cdots\end{aligned} \tag{B.13}$$

Consider $\Delta l = 1$, $m = l + 1$ $C_x^{l,l+1}(1) = 0$, $C_x^{l,l+1}(2) = 64\tau^6$, and $C_x^{l,l+1}(3) = 256\tau^6$. Exponent of the power-law approximately matches the Eq. (3.20).

Appendix C

Out-of-time-order correlators of nonlocal block-spin and random observables in integrable and nonintegrable spin chains

C-I Calculation of post-scrambling OTOC using random unitary operator

We calculate long-time saturation values of OTOC for spin-block operators $\hat{V}$ and $\hat{W}$ are calculated by replacing the unitary operator $\hat{U}$ with random CUE of size 2^N and averaging over it. Two- and four-point correlation functions $C_2(n)$ and $C_4(n)$ are calculated as below:

C-I.1 Calculation of two-point correlation

Two point correlation ($C_2(n)$) averaged over random U drawn from CUE of size 2^N is given by

$$\overline{C_2(n)}^U = \frac{1}{d_A d_B}\overline{\mathrm{Tr}(\hat{W}(n)^2\hat{V}^2)}^U. \tag{C.1}$$

Since time evolution of $\hat{W}$ is given by Heisenberg time evolution as $\hat{W}(n) = \hat{U}(n)^\dagger \hat{W}\hat{U}(n)$. Hence,

$$\begin{aligned}\overline{C_2(n)}^U &= \frac{1}{d_A d_B}\overline{\mathrm{Tr}(\hat{U}^\dagger\hat{W}^2\hat{U}\hat{V}^2)}^U = \frac{1}{d_A d_B}\sum_{j=1}^{d}\overline{\langle j|\hat{U}^\dagger\hat{W}^2\hat{U}\hat{V}^2)|j\rangle}^U, \\ &= \frac{1}{d_A d_B}\sum_{j,k,l,m}\overline{\langle j|\hat{U}^\dagger|k\rangle\langle k|\hat{W}^2|l\rangle\langle l|\hat{U}|m\rangle\langle m|\hat{V}^2|j\rangle}^U = \frac{1}{d_A d_B}\sum_{j,k,l,m}\overline{\hat{U}^*_{kj}\hat{U}_{lm}}^U\hat{W}^2_{kl}\hat{V}^2_{mj}.\end{aligned}$$

Since, $\overline{\hat{U}^*_{kj}\hat{U}_{lm}}^U = \sum_{j,k,l,m}\delta_{kl}\delta_{jm}|\hat{U}_{kj}|^2$ and $|\hat{U}_{kj}|^2 = \frac{1}{d}$

$$\overline{C_2(n)}^U = \frac{1}{d_A d_B}\frac{1}{d}\sum_{j,k,l,m}\delta_{kl}\delta_{jm}\hat{W}^2_{kl}\hat{V}^2_{mj} = \frac{1}{d_A d_B}\frac{1}{d}\sum_{k,j}\hat{W}^2_{kk}\hat{V}^2_{jj} = \frac{1}{d_A d_B}\frac{1}{d}\mathrm{Tr}(\hat{W}^2)\mathrm{Tr}(\hat{V}^2).$$

Since, $d_A d_B = 2^N$. Hence $C_2(n)$ will be $C_2(n) = \frac{1}{2^{2N}}\hat{\mathrm{T}}\mathrm{r}(\hat{W}^2)\hat{\mathrm{T}}\mathrm{r}(\hat{V}^2)$.

Since, block observables are localized spin block observables defined by Eq. (4.5). Then calculate $\mathrm{Tr}(\hat{W}^2)$ will be

$$\mathrm{Tr}(\hat{W}^2) = \frac{4}{N^2}\mathrm{Tr}\left(\sum_{l=1}^{\frac{N}{2}}(\hat{\sigma}^x_l)^2 + \sum_{l\neq l'}\hat{\sigma}^x_l\hat{\sigma}^x_{l'}\right). \tag{C.2}$$

By using the properties of Pauli operator, square of Pauli operators are equal to identity matrix. Hence first term of Eq. C.2 will be equal to $\frac{2}{N}2^N$. And second term, $\sum_{l\neq l'}\hat{\sigma}^x_l\hat{\sigma}^x_{l'}$ is equal to zero because Pauli observable follow the anti-commutation relation. Hence, $C_2(n)$ for the spin block observables is

$$\overline{C_2(n)}^U = \frac{1}{2^{2N}}\frac{4}{N^2}2^{2N} = \frac{4}{N^2}. \tag{C.3}$$

C-I.2 Calculation of four point correlator

Four-point correlator $[C_4(n)]$ averaged over random U drawn from CUE of size 2^N is given by

$$\begin{aligned}
\overline{C_4(n)}^U &= \frac{1}{d_A d_B}\overline{\mathrm{Tr}(\hat{W}(n)\hat{V}\hat{W}(n)\hat{V})}^U = \frac{1}{d_A d_B}\overline{\mathrm{Tr}(\hat{U}^\dagger\hat{W}\hat{U}\hat{V}\hat{U}^\dagger\hat{W}\hat{U}\hat{V})}^U, \\
&= \frac{1}{d_A d_B}\sum_{i_1,i_2,\cdot,i_8}\overline{\langle i_1|\hat{U}^\dagger|i_2\rangle\langle i_2|\hat{W}|i_3\rangle\langle i_3|\hat{U}|i_4\rangle\langle i_4|\hat{V}|i_5\rangle\langle i_5|\hat{U}^\dagger|i_6\rangle\langle i_6|\hat{W}|i_7\rangle\langle i_7|\hat{U}|i_8\rangle\langle i_8|\hat{V}|i_1\rangle}^U, \\
&= \frac{1}{d_A d_B}\sum_{i_1,i_2\cdot i_8}\overline{\hat{U}^*_{i_1,i_2}\hat{U}_{i_3,i_4}\hat{U}^*_{i_6,i_5}\hat{U}_{i_7,i_8}}^U\hat{W}_{i_2,i_3}\hat{V}_{i_4,i_5}\hat{W}_{i_6,i_7}\hat{V}_{i_8,i_1}, \\
&= \frac{1}{d_A d_B}\sum_{i_1,i_2\cdot i_8}\Bigg(\delta_{i_2,i_3}\delta_{i_1,i_4}\delta_{i_6,i_7}\delta_{i_5,i_8}|\hat{U}_{i_2,i_1}|^2|\hat{U}_{i_6,i_5}|^2\hat{W}_{i_2,i_3}\hat{V}_{i_4,i_5}\hat{W}_{i_6,i_7}\hat{V}_{i_8,i_1} \\
&+\delta_{i_2,i_7}\delta_{i_1,i_8}\delta_{i_3,i_6}\delta_{i_4,i_5}|U_{i_2,i_1}|^2|\hat{U}_{i_3,i_4}|^2\hat{W}_{i_2,i_3}\hat{V}_{i_4,i_5}\hat{W}_{i_6,i_7}\hat{V}_{i_8,i_1}\Bigg) \\
&-\frac{1}{d_A d_B}\sum_{i_1,i_2\cdot i_8}\Bigg(\delta_{i_2,i_3}\delta_{i_1,i_4}\delta_{i_6,i_7}\delta_{i_5,i_8}\hat{U}^*_{i_2,i_1}\hat{U}_{i_2,i_4}\hat{U}^*_{i_6,i_5}\hat{U}_{i_6,i_8}\hat{W}_{i_2,i_3}\hat{V}_{i_4,i_5}\hat{W}_{i_6,i_7}\hat{V}_{i_8,i_1} \\
&+\delta_{i_2,i_7}\delta_{i_1,i_8}\delta_{i_3,i_6}\delta_{i_4,i_5}U^*_{i_2,i_1}\hat{U}_{i_6,i_5}\hat{U}^*_{i_2,i_1}\hat{U}_{i_6,i_5}\hat{W}_{i_2,i_3}\hat{V}_{i_4,i_5}\hat{W}_{i_6,i_7}\hat{V}_{i_8,i_1}\Bigg), \\
&= \frac{1}{d_A d_B}\frac{1}{d^2-1}\sum_{i_1,i_2\cdot i_8}\Bigg(\delta_{i_2,i_3}\delta_{i_1,i_4}\delta_{i_6,i_7}\delta_{i_5,i_8}\hat{W}_{i_2,i_3}\hat{V}_{i_4,i_5}\hat{W}_{i_6,i_7}\hat{V}_{i_8,i_1} \\
&+\delta_{i_2,i_7}\delta_{i_1,i_8}\delta_{i_3,i_6}\delta_{i_4,i_5}\hat{W}_{i_2,i_3}\hat{V}_{i_4,i_5}\hat{W}_{i_6,i_7}\hat{V}_{i_8,i_1}\Bigg) \\
&-\frac{1}{d_A d_B}\frac{1}{d(d^2-1)}\sum_{i_1,i_2\cdot i_8}\Bigg(\delta_{i_2,i_3}\delta_{i_1,i_4}\delta_{i_6,i_7}\delta_{i_5,i_8}\hat{W}_{i_2,i_3}\hat{V}_{i_4,i_5}\hat{W}_{i_6,i_7}\hat{V}_{i_8,i_1} \\
&+\delta_{i_2,i_7}\delta_{i_1,i_8}\delta_{i_3,i_6}\delta_{i_4,i_5}\hat{W}_{i_2,i_3}\hat{V}_{i_4,i_5}\hat{W}_{i_6,i_7}\hat{V}_{i_8,i_1}\Bigg), \\
&= \frac{1}{d_A d_B}\frac{1}{d^2-1}\sum_{i_1,i_2\cdot i_8}\Bigg(\hat{W}_{i_1,i_2}\hat{V}_{i_1,i_5}\hat{W}_{i_6,i_6}\hat{V}_{i_5,i_1}+\hat{W}_{i_2,i_3}\hat{V}_{i_4,i_4}\hat{W}_{i_3,i_2}\hat{V}_{i_1,i_1}\Bigg) \\
&-\frac{1}{d_A d_B}\frac{1}{d(d^2-1)}\sum_{i_1,i_2\cdot i_8}\Bigg(\hat{W}_{i_2,i_2}\hat{V}_{i_4,i_4}\hat{W}_{i_6,i_6}\hat{V}_{i_8,i_8}+\hat{W}_{i_2,i_3}\hat{V}_{i_4,i_5}\hat{W}_{i_6,i_7}\hat{V}_{i_8,i_1}\Bigg),
\end{aligned}$$

$$\overline{C_4(n)}^U = \frac{1}{d_A d_B}\frac{1}{d^2-1}\left((\mathrm{Tr}\hat{W})^2(\mathrm{Tr}\hat{V})^2 + (\mathrm{Tr}\hat{W}^2)(\mathrm{Tr}\hat{V})^2\right)$$
$$-\frac{1}{d_A d_B}\frac{1}{d(d^2-1)}\left(\mathrm{Tr}(\hat{W}^2)\mathrm{Tr}(\hat{V}^2) + (\mathrm{Tr}\hat{W})^2(\mathrm{Tr}\hat{V})^2\right) + O\left(\frac{1}{d(d^2-1)}\right).$$

Considering traceless observables such that $\mathrm{Tr}(\hat{W}) = 0$ and $\mathrm{Tr}(\hat{V}) = 0$, and $d_A d_B = d$ we get

$$\overline{C_4(n)}^U = -\frac{1}{d}\frac{1}{d(d^2-1)}(\mathrm{Tr}\hat{W}^2)(\mathrm{Tr}\hat{V}^2) = -\frac{1}{d^2(d^2-1)}(\mathrm{Tr}\hat{W}^2)(\mathrm{Tr}\hat{V}^2). \tag{C.4}$$

For traceless observables $C_2(n)$ will be

$$\overline{C_2(n)}^U = \frac{1}{d^2}\mathrm{Tr}(\hat{W}^2)\mathrm{Tr}(\hat{V}^2). \tag{C.5}$$

Hence, OTOC for the traceless observables will be

$$\begin{aligned}\overline{C(n)}^U &= \overline{C_2(n)}^U - \overline{C_4(n)}^U = \frac{1}{d^2}(\mathrm{Tr}\hat{W}^2)(\mathrm{Tr}\hat{V}^2)\left(1+\frac{1}{d^2-1}\right),\\ &= \frac{1}{d^2}(\mathrm{Tr}\hat{W})^2(\mathrm{Tr}\hat{V})^2\frac{d^2}{d^2-1} = \frac{1}{d^2-1}(\mathrm{Tr}\hat{W})^2(\mathrm{Tr}\hat{V})^2 = \frac{1}{2^{2N}-1} \approx \frac{1}{2^{2N}}.\end{aligned} \tag{C.6}$$

Appendix D

Quantum information diode based on a magnonic crystal

D-I Diagonalization of Hamiltonian of 2D square lattice

2D square-lattice spin system with nearest-neighbor J_1 and the next nearest-neighbor J_2 coupling constants (taking $\hbar = 1$):

$$
\begin{aligned}
\hat{H} &= J_1 \sum_{\langle n,m \rangle} \hat{\sigma}_n \hat{\sigma}_m + J_2 \sum_{\langle\langle n,m \rangle\rangle} \hat{\sigma}_n \hat{\sigma}_m - \mathbf{P} \cdot \mathbf{E}, \\
&= J_1 \sum_{\langle n,m \rangle} \hat{\sigma}_n \hat{\sigma}_m + J_2 \sum_{\langle\langle n,m \rangle\rangle} \hat{\sigma}_n \hat{\sigma}_m - D \sum_n (\hat{\sigma}_n \times \hat{\sigma}_{n+1})_z, \\
&= \frac{1}{4} \Big[J_1 \sum_{\langle n,m \rangle} \hat{S}_n \hat{S}_m + J_2 \sum_{\langle\langle n,m \rangle\rangle} \hat{S}_n \hat{S}_m + \frac{D}{i} \sum_n (\hat{S}_n^+ \hat{S}_{n+1}^- - \hat{S}_n^- \hat{S}_{n+1}^+) \Big], \\
&= \frac{1}{4} \Big[J_1 \sum_{\langle n,m \rangle} \frac{1}{2} \Big\{ \Big(\hat{S}_n^- \hat{S}_m^+ + \hat{S}_n^+ \hat{S}_m^- \Big) + \hat{S}_n^z \hat{S}_m^z \Big\} + J_2 \sum_{\langle\langle n,m \rangle\rangle} \frac{1}{2} \Big\{ \hat{S}_n^- \hat{S}_m^+ + \hat{S}_n^+ \hat{S}_m^- \Big) + \hat{S}_n^z \hat{S}_m^z \Big\} \\
&+ \frac{D}{i} \sum_n (\hat{S}_n^+ \hat{S}_{n+1}^- - \hat{S}_n^- \hat{S}_{n+1}^+) \Big]. \qquad \text{(D.1)}
\end{aligned}
$$

Spin-half systems have two permitted states on each site, *i.e.*, $|\uparrow\rangle$ and $|\downarrow\rangle$. Operation of spin operators on these state are given as

$$\begin{aligned}
&\hat{S}^{+}|\downarrow\rangle = |\uparrow\rangle,\ \hat{S}^{+}|\uparrow\rangle = 0,\\
&\hat{S}^{-}|\uparrow\rangle = |\downarrow\rangle,\ \hat{S}^{-}|\downarrow\rangle = 0,\\
&\hat{S}^{z}|\uparrow\rangle = \frac{1}{2}|\uparrow\rangle,\ \hat{S}^{z}|\downarrow\rangle = -\frac{1}{2}|\downarrow\rangle,
\end{aligned} \tag{D.2}$$

Transformation of the spin operators in hard-core bosonic creation and annihilation operators are given as

$$\begin{aligned}
\hat{S}^{+}_{m,n} &= \hat{a}_{m,n},\\
\hat{S}^{-}_{m,n} &= \hat{a}^{\dagger}_{m,n},\\
\hat{S}^{z}_{m,n} &= 1/2 - \hat{a}^{\dagger}_{m,n}\hat{a}_{m,n}
\end{aligned} \tag{D.3}$$

Hamiltonian in the bosonic representation is given as

$$\begin{aligned}
\hat{H} &= \frac{1}{4}\Big[J_1 \sum_{\langle n,m\rangle} \Big(\hat{a}^{\dagger}_{n}\hat{a}_{m} + \hat{a}_{n}\hat{a}^{\dagger}_{m} - \hat{a}^{\dagger}_{n}\hat{a}_{n} - \hat{a}^{\dagger}_{m}\hat{a}_{m}\Big)\\
&+ J_2 \sum_{\langle\langle n,m\rangle\rangle} \Big(\hat{a}^{\dagger}_{n}\hat{a}_{m} + \hat{a}_{n}\hat{a}^{\dagger}_{m} - \hat{a}^{\dagger}_{n}\hat{a}_{n} - \hat{a}^{\dagger}_{m}\hat{a}_{m}\Big) + \frac{D}{i}\sum_{n}\Big(\hat{a}_{n}\hat{a}^{\dagger}_{n+1} - \hat{a}^{\dagger}_{n}\hat{a}_{n+1}\Big)\Big].
\end{aligned} \tag{D.4}$$

Fourier transform of $\hat{a}^{\dagger}_{n}(\hat{a}_{n})$ is $\hat{a}^{\dagger}_{\vec{k}}(\hat{a}_{\vec{k}})$.

$$\hat{a}^{\dagger}_{\vec{k}} = \frac{1}{\sqrt{N}}\sum_{n} e^{i\vec{k}\vec{r}_{n}} a^{\dagger}_{n}, \qquad \hat{a}_{\vec{k}} = \frac{1}{\sqrt{N}}\sum_{n} e^{i\vec{k}\vec{r}_{n}} a_{n}. \tag{D.5}$$

Inverse Fourier transform is given as

$$\hat{a}^{\dagger}_{n} = \frac{1}{\sqrt{N}}\sum_{n} e^{i\vec{k}\vec{r}_{n}} a^{\dagger}_{\vec{k}}, \qquad \hat{a}_{n} = \frac{1}{\sqrt{N}}\sum_{n} e^{i\vec{k}\vec{r}_{n}} a_{\vec{k}}. \tag{D.6}$$

After summing over n we get Hamiltonian (Eq. D.1) in $\vec{k}$ space as

$$\hat{H} = \sum_{\vec{k}} \omega_{\vec{k}} \hat{a}^{\dagger}_{\vec{k}} \hat{a}_{\vec{k}} - D \sum_{\vec{k}} \sin(\vec{k}a) \hat{a}^{\dagger}_{\vec{k}} \hat{a}_{\vec{k}} = \sum_{\vec{k}} \omega(\pm D, \mathbf{k}) \hat{a}^{\dagger}_{\vec{k}} \hat{a}_{\vec{k}} \tag{D.7}$$

where,

$$\begin{aligned} &\omega(\pm D, \mathbf{k}) = \big(\omega(\vec{k}) \pm \omega_{DM}(\vec{k})\big), \;\; \omega_{DM}(\vec{k}) = D\sin(k_x a), \\ &\omega_k = 2J_1(1-\gamma_{1,\mathbf{k}}) + 2J_2(1-\gamma_{2,\mathbf{k}}), \; \gamma_{1,\mathbf{k}} = 1/2(\cos k_x + \cos k_y), \\ &\gamma_{2,\mathbf{k}} = 1/2(\cos(k_x+k_y) + \cos(k_x-k_y)). \end{aligned} \tag{D.8}$$

D-II Calculation of left and right out-of-time ordered correlation functions

We will calculate OTOC exactly for one magnon excitation state given in Eq. (7) as

$$\begin{aligned} C(t) = \frac{1}{2}\Big\{ &\langle \hat{\eta}_n \hat{\eta}_m(t) \hat{\eta}_m(t) \hat{\eta}_n \rangle + \langle \hat{\eta}_m(t) \hat{\eta}_n \hat{\eta}_n \hat{\eta}_m(t) \rangle \\ &- \langle \hat{\eta}_m(t) \hat{\eta}_n \hat{\eta}_m(t) \hat{\eta}_n \rangle - \langle \hat{\eta}_n \hat{\eta}_m(t) \hat{\eta}_n \hat{\eta}_m(t) \rangle \Big\}. \end{aligned} \tag{D.9}$$

Here, $\hat{\eta}_{m/n} = \hat{\sigma}^z_{m/n}$ is Hermitian and unitary, therefore, Eq. (D.9) transforms in the form given as

$$C(t) = 1 - \langle \hat{\eta}_m(t) \hat{\eta}_n \hat{\eta}_m(t) \hat{\eta}_n \rangle = 1 - F(t), \tag{D.10}$$

where $F(t)$ is given as

$$F(t) = \langle \phi | \hat{a}_n \hat{\eta}_m(t) \hat{\eta}_n \hat{\eta}_m(t) \hat{\eta}_n a^{\dagger}_n | \phi \rangle. \tag{D.11}$$

In the above equation, the expectation value is taken over one magnon excitation state $\hat{a}_n^\dagger|\phi\rangle$, where $|\phi\rangle$ is the vacuum state, equivalent to a polarized state. First of all we calculate the product four obsevables in $F(t)$ (Eq. (D.11))in bosonic representation as

$$\begin{aligned}
\hat{\eta}_m(t)\hat{\eta}_n\hat{\eta}_m(t)\hat{\eta}_n &= [1-2\hat{a}_m^\dagger\hat{a}_m(t)][1-2\hat{a}_n^\dagger\hat{a}_n][1-2\hat{a}_m^\dagger\hat{a}_m(t)][1-2\hat{a}_n^\dagger\hat{a}_n],\\
&= \left[1-2\hat{a}_m^\dagger\hat{a}_m(t)-2\hat{a}_n^\dagger\hat{a}_n+4\hat{a}_m^\dagger\hat{a}_m(t)\hat{a}_n^\dagger\hat{a}_n\right]\\
&\times \left[1-2\hat{a}_m^\dagger\hat{a}_m(t)-2\hat{a}_n^\dagger\hat{a}_n+4\hat{a}_m^\dagger\hat{a}_m(t)\hat{a}_n^\dagger\hat{a}_n\right],\\
&= 1-4\hat{a}_m^\dagger\hat{a}_m(t)-4\hat{a}_n^\dagger\hat{a}_n+4\hat{a}_m^\dagger\hat{a}_m(t)\hat{a}_n^\dagger\hat{a}_n+4\hat{a}_n^\dagger\hat{a}_n\hat{a}_m^\dagger\hat{a}_m(t)\\
&+ 4\hat{a}_m^\dagger\hat{a}_m\hat{a}_m^\dagger\hat{a}_m(t)+4\hat{a}_n^\dagger\hat{a}_n\hat{a}_n^\dagger\hat{a}_n+4\hat{a}_m^\dagger\hat{a}_m\hat{a}_n^\dagger\hat{a}_n+4\hat{a}_n^\dagger\hat{a}_n\hat{a}_m^\dagger\hat{a}_m\\
&- 8\hat{a}_m^\dagger\hat{a}_m\hat{a}_m^\dagger\hat{a}_m(t)\hat{a}_n^\dagger\hat{a}_n-8\hat{a}_n^\dagger\hat{a}_n\hat{a}_m^\dagger\hat{a}_m(t)\hat{a}_n^\dagger\hat{a}_n\\
&- 8\hat{a}_m^\dagger\hat{a}_m(t)\hat{a}_n^\dagger\hat{a}_n\hat{a}_m^\dagger\hat{a}_m(t)-8\hat{a}_m^\dagger\hat{a}_m(t)\hat{a}_n^\dagger\hat{a}_n\hat{a}_n^\dagger\hat{a}_n\\
&+ 16\hat{a}_m^\dagger\hat{a}_m(t)\hat{a}_n^\dagger\hat{a}_n\hat{a}_m^\dagger\hat{a}_m(t)\hat{a}_n^\dagger\hat{a}_n. \qquad \text{(D.12)}
\end{aligned}$$

Further, we calculate the expectation value of the last term of Eq. (D.12) over one magnon excitation state *i. e.*, $\langle\phi|\hat{a}_n\hat{a}_m^\dagger\hat{a}_m(t)\hat{a}_n^\dagger\hat{a}_n\hat{a}_n\hat{a}_m^\dagger\hat{a}_m(t)\hat{a}_n^\dagger\hat{a}_n\hat{a}_n^\dagger|\phi\rangle$, using the properties of bosonic operators $[\hat{a}_i,\hat{a}_j^\dagger]=\delta_{ij}$, $(\hat{a}_i)^2=0$, and $(\hat{a}_i^\dagger)^2=0$. We get

$$\begin{aligned}
\langle\phi|\hat{a}_n\hat{a}_m^\dagger\hat{a}_m(t)\hat{a}_n^\dagger\hat{a}_n\hat{a}_n\hat{a}_m^\dagger\hat{a}_m(t)\hat{a}_n^\dagger\hat{a}_n\hat{a}_n^\dagger|\phi\rangle &= \langle\phi|\hat{a}_n e^{i\hat{H}t}\hat{a}_m^\dagger\hat{a}_m e^{-i\hat{H}t}\hat{a}_n^\dagger\hat{a}_n e^{i\hat{H}t}\hat{a}_m^\dagger\hat{a}_m e^{-i\hat{H}t}\hat{a}_n^\dagger|\phi\rangle,\\
&= \langle\Psi(t)|\Psi(t)\rangle,
\end{aligned} \qquad \text{(D.13)}$$

where $|\Psi(t)\rangle = \hat{a}_n e^{i\hat{H}t}\hat{a}_m^\dagger\hat{a}_m e^{-i\hat{H}t}\hat{a}_n^\dagger|\phi\rangle$. Fourier transformation of the $|\Psi(t)\rangle$ and diagonalized Hamiltonian will provide

$$\begin{aligned}
|\Psi(t)\rangle &= \frac{1}{N}\sum_k e^{i(-k(m-n)+\omega_k t/\hbar)}\frac{1}{N}\sum_{k'} e^{i(k'(m-n)-\omega_{k'}t/\hbar)}|\phi\rangle\\
&= \frac{1}{N^2}\Omega_1\Omega_2|\phi\rangle.
\end{aligned}$$

Hence,

$$\langle\Psi(t)|\Psi(t)\rangle = \frac{1}{N^4}\Omega_1\Omega_2\Omega_1\Omega_2. \tag{D.14}$$

Similarly,

$$\langle\phi|\hat{a}_n\hat{a}_m\hat{a}_m(t)\hat{a}_n^\dagger|\phi\rangle = \frac{1}{N^2}\Omega_1\Omega_2 \tag{D.15}$$

After doing some simple bosonic algebra, time dependent terms of Eq. (D.12) are converted either in the form of Eq. (D.13) or Eq. (D.15). By using Eq. (D.14) and Eq. (D.15), we calculate $F(t)$ as

$$\begin{aligned} F(t) &= 1-\frac{4}{N^2}\Omega_1\Omega_2-4+\frac{4}{N^2}\Omega_1\Omega_2+\frac{4}{N^2}\Omega_1\Omega_2+\frac{4}{N^2}\Omega_1\Omega_2+\frac{4}{N^2}\Omega_1\Omega_2 \\ &\quad +\frac{4}{N^2}\Omega_1\Omega_2+4-\frac{8}{N^2}\Omega_1\Omega_2-\frac{8}{N^2}\Omega_1\Omega_2-\frac{8}{N^4}\Omega_1\Omega_2\Omega_1\Omega_2-\frac{8}{N^2}\Omega_1\Omega_2 \\ &\quad +\frac{16}{N^4}\Omega_1\Omega_2\Omega_1\Omega_2, \\ &= 1-\frac{8}{N^2}\Omega_1\Omega_2+\frac{8}{N^4}\Omega_1\Omega_2\Omega_1\Omega_2. \end{aligned} \tag{D.16}$$

Then, we get left and right OTOCs' analytical expression as

$$\begin{aligned} C_L(t) &= \frac{8}{N^2}\Omega_1^L\Omega_2^L-\frac{8}{N^4}\Omega_1^L\Omega_2^L\Omega_1^L\Omega_2^L, \\ C_R(t) &= \zeta^4(D)\Big(\frac{8}{N^2}\Omega_1^R\Omega_2^R-\frac{8}{N^4}\Omega_1^R\Omega_2^R\Omega_1^R\Omega_2^R\Big), \end{aligned} \tag{D.17}$$

where frequencies $\Omega_{1/2}^{L/R}$ are given as

$$\begin{aligned} \Omega_1^R &= \Omega_2^{R*}=\sum_{m_0}\exp\left(-\frac{im_0\pi r_{1,2}}{a_0}\right)\exp\left(\frac{i\omega_{m_0}t}{\hbar}\right), \text{ and} \\ \Omega_1^L &= \Omega_2^{L*}=\sum_{m_0}\exp(-ik_s^- r_{1,2})\exp\left(\frac{i\omega_{m_0}t}{\hbar}\right). \end{aligned} \tag{D.18}$$

List of Publications

1. “Out-of-time-order correlation and detection of phase structure in Floquet transverse Ising spin system", **Rohit Kumar Shukla**, Gautam Kamalakar Naik, and Sunil Kumar Mishra, EPL **132**, 47003 (2021).
2. “Characteristic, dynamic, and saturation regions of Out-of-time-order correlation in Floquet Ising models", **Rohit Kumar Shukla** and Sunil Kumar Mishra, Phys. Rev. A **132**, 47003 (2022).
3. “Out-of-time-order correlation of nonlocal block-spin and random observables in integrable and nonintegrable spin chains", **Rohit Kumar Shukla**, Arul Lakshminarayan, and Sunil Kumar Mishra, Phys. Rev. B **105**, 224307 (2022)
4. “Quantum information diode based on the magnonic crystal", **Rohit Kumar Shukla**, L. Chotorlishvili, Vipin Vijayan, Harshit Verma, A. Ernst, S. S. P. Parkin, and Sunil K. Mishra, under review in npj Quantum Information.