

2 Literature review

2.1 General

This chapter includes a review of research work on various studies conducted in the direction of design, analysis and computational tools implemented to evaluate the behavior of tensile membrane structures (TMS). The main aim of this literature survey is to explore different techniques for analysis of fabric membranes, so that a gap in terms of parametric and reliability analysis can be bridged and can become basis for the present research work. Secondly, this literature study is carried out to know various methodologies implemented in the direction of exploring the structural behavior of membranes. The primary objective of designing a TMS is strength and stability. The failure of flexible fabric material is governed by inherent uncertainties associated with the environmental impacts (Dutta & Ghosh, 2019). Hence this study area should be directed towards the identification of parameters which may impart a major role in estimating the possible associated uncertainties with TMS. To this end an extensive literature survey is done focusing on parametric study on TMS and also various methods of reliability like, first order reliability method (FORM), or first order second moment reliability method (FOSM). Primarily a synopsis on computational design and simulation of membranes to carry out a parametric study is documented, this gives a strong base to apply reliability methods and develop basic design guidelines for fabric structures. Lastly, a literature study is conducted on the use of system reliability techniques to contribute to the strength and stability of TMS in general. The application of system reliability in present study was mainly conducted with a purpose to increase the accuracy of computationally obtained results where a TMS system behaves in a non-linear way, highly differently than other conventional structures.

2.2 Structural analysis of tensile membranes

Tensile membranes were first introduced by Frei Otto in 1950s as physical soap film models. These models were developed in a light weight structures lab in the University of Stuttgart; they were prepared to manifest the minimal surfaces of a real membrane structure. Although these models were a significant revolutionary work towards the development of membrane structures, but these models lack in precisely determining the

real time behavioral characteristics required for the construction of larger and complex membrane structural systems (Bradshaw et al., 2003) in short the soap film models lack the scope of scalability. Due to this drawback from the soap film models, the researchers started developing numerical models to obtain the mathematical relationships that describe the soap film model by Frei Otto. Horst Berger is considered as the first person to develop first mathematical model for soap film models.

The shape determination is the first step in the design process of membrane structures. There are number of researches based on computational form finding methods, the three most implied techniques are: (a) Transient stiffness matrix method, (b) Force density method, and (c) Dynamic relaxation method (Lewis, 2008). The transient stiffness method is all about obtaining the geometric and elastic stiffness of the membrane structure and unraveling the matrix equations. This method is the oldest technique and is implemented for form finding using the concepts of matrix method of structural analysis. This method is an efficient way to analyze cable nets and the final stable shape is obtained after step-wise solving of equilibrium equations (Argyris et al., 1974)(Levy & Spillers, 1998) . The stiffness matrix method is implemented for non-linear analysis of membrane structures through the general finite element large displacement formulation (Tabarrok & Qin, 1992).

The second method which is force density method accounts only geometric stiffness and also is independent of the material properties of tensile membrane structures, this method was originally described by (Schek Heidelberg, 1974). The technique involved in this method is, linearization of equations of form-finding for a given tension net. The characteristic of linearization facilitates the method to become material independent. In order to attain a unique state of equilibrium the force per unit length (force density) of the membrane network is considered for each element of the cable net or tensile membrane. This method was primarily used for analyzing cable-nets, further researches extended its applicability to tensile surfaces, through the concept of surface elements like constant strain triangular method, Maurin and Motro in 1998 termed this variation of method as surface stress density method. A major drawback to this method is the incompetence to control the final distribution of stresses. But, this can be compensated by updating force densities through iteration till the desired smoothness on stress distribution is accomplished. Based on this concept of force densities, number of researchers have devised similarly extended methods (Kai-Uwe Bletzinger, 1999)(Haber

& Abel, 1982). More recently, a multi-step force density iterative method, based on original concept is introduced in Spain, this process gives the desired stable forms of membranes along with smoother distribution of stresses (Sánchez et al., 2007). There is another technique developed in Japan called extended force density method to determine the initial shape of the membranes structures (Miki & Kawaguchi, 2010). The most recent assessments regarding force density methods are provided by Greco in Italy (Greco & Cuomo, 2012).

The most illustrious method for membranes form-finding is the dynamic relaxation (DR) method, this method was introduced by A.S.Day for the first time in the year 1962. Dynamic relaxation method is capable to provide solution to geometrically non-linear problem by altering it to a dynamic equilibrium (Barnes, 1999). Barnes implemented DR method for the purpose of form-finding for tension structures in his research. Basically, it is a finite difference technique applied to a discretized structural system where the equilibrium state is achieved after a while under the impact of damping. Due to its robustness towards form-finding, this method is used in most of the tensile membrane designing computer programs in the tensile membrane industry (Wakefield, 1999). The fundamental behind DR method is the Newton's second law, which states that the acceleration of a body is proportional to the net forces acting on it. Here for form-finding, the movement of all the nodes on the surface of membrane is calculated for a small time interval Δt , the final position of nodes are attained through an iterative process of residual force updating. This residual force is responsible for the movement of nodes, and this force is mainly caused due to the effect of internal force like pre-stress, and external loading like wind load or snow load. The acceleration imparted due to out of balance force provides the velocity and consequently the final position of nodes by stepping down through the time. The node position is adjusted with the motive to reduce the out-of-balance force, the equilibrium is said to be achieved when the residual force reaches a targeted value. A study based on computational framework have identified DR method with application of kinetic damping as an efficient and unwavering simulation technique for the complete analysis of a membrane structural system (Veenendaal & Block, 2012).

Besides above mentioned TMS analysis methods, there are several mutated methods are reviewed in the available literature on form finding analysis. A matrix method based on more standard non-linear analysis of structural system is used (de Borst et al., 2012).

Non-finite element techniques were also explored to get the stable surfaces using spline based methods (Brew & Lewis, 2003).

2.3 Current design problems of TMS

The material used for creating a membrane structure is known to exhibit non-linear orthotropic behavior whenever it undergoes the biaxial stress situation. Therefore the design techniques for membrane structure divide the complete process into two stages mainly: the form finding and the load analysis stage. The purpose of form finding procedure is to obtain an equilibrium shape of the membrane, equilibrium shape in the sense of minimized surface and the evenly distributed stresses mainly due to internally developed forces. The parameters in which the final state of membrane is dependent on are: predefined boundaries like, side cables, anchorages, supports and the targeted prestress of the membrane and cables as well. During the stage of form-finding the material properties does not influence the design process.

The soap film models were used in the initial design period of modern membrane structures. The soap films were considered to possess the similar characteristics. The surfaces generated by soap film naturally acquire a saddle shape in order to bear the tension and the effects of loads. It is understood that if a soap model shows highly divergent shapes, it is inevitable to face difficulties during scaling in its built form, for example there will be chances of failure of structure during due to unexpected deformations caused due to stress concentration imparted by the accumulation of rain water or snowfall. Undoubtedly the soap films are honest representation of a membrane structure in real world and also gives the impact of efficiency of the targeted TMS construction, the setup fabrication process for them is extremely demanding. Besides that they have the limitation of accounting self-weight and properties isotropically. (Brew & Lewis, 2003) have described the practical difficulties associated with transforming of soap film stable minimal surfaces into real TMS. Hence this method became obsolescent and the computational software like, ANSYS, Tensinet, Dlubal RFEM etc. replaced the orthodox technique of form finding.

The second stage of the TMS design process is load analysis. The material properties of the membrane are significant in this step. As mentioned earlier, the TMS materials exhibit high non-linearity. Moreover, membranes are incompetent to provide the

bending and compressive stiffness therefore, the effects of bending and compression due to wind and snow loads, are neglected during the calculation. Presently there are different approaches for TMS load analysis are in practice. Two main approaches are mentioned in section 1.2, a third method can be proposed with the combination of cable net and continuum surface topology. In discrete analysis, the membrane is approached by quantizing it into an orthotropic cable net, with an assumed constant young's moduli and linearly elastic comportment in both orthogonal directions. The Poisson's ratio and shear modulus are not considered. The continuum surface based analysis is a finite element centered procedure, it capitalizes the orthotropic membrane properties including the Poisson's ratio. Inclusion of constant elastic moduli in axial and shear direction is again an assumption in this method, and it can impart errors in the final calculation for TMS and consequently will lead to approximations of final results. The slight differences assumed to interpret results from final analysis process can culminate in imparting significant differences in the calculation of derived parameters e.g. the young's moduli (B. N. Bridgens & Gosling, 2004). For this reason the practice of using high safety factors are necessary to use for the complete tension membrane structural analysis. High safety factors are assumed to possibly compensate the load variable material behavior and the chances of creep and tear propagation.

Fabric material used for TMS is consisted of orthogonally woven yarns which provide strength, an impermeable coating is provided over the woven yarns for the purpose of waterproofing and to provide slight rigidness. The direction along the length is known as warp, and direction across the roll is called weft The most commonly used material are PVC (Poly Vinyl Chloride) coated polyester fabric yarns and PTFE (PolyTetraFloroEthylene) coated glass fibre yarns. A collective non-linear stress-strain response of yarn and coating along with the orthogonal yarns behavior is the main cause of complex behavior of fabric material (B. N. Bridgens & Gosling, 2004). The full visualization of the response of membrane material with respect to shear and biaxial in-plane loading has not been accomplished yet. Hence, assumed stiffness values are acknowledged for a given membrane material, however the real time stiffness values may deviate by a factor of 2-5 from the assumed values (Gosling, Bridgensben, et al., 2013). Considering the other parameters like elastic moduli, shear stiffness and Poisson's ratio, they are autonomously unconstrained by conventional limits and interactions for anisotropic materials.

The design of TMS is not codified precisely anywhere, besides few limited design guidelines which are recently compiled. There are international standards [ASCE/SEI. 55-10] to describe membrane design principles broadly, but it is the detailed design methodology for complete TMS analysis which is required. The available design codes broadly summarize the minimum stress factor value as 5 for permanent or semi-permanent structures, and this value is 7 at the constrained/ boundary points. Also the codes are designed for conventional, standard rigid building shapes and behavior. This makes the applicability of existing codes very difficult for TMS, since the geometrical orientation impacts not alone the magnitude of load, it also influences the loading effects distribution on the surface of the structure. Besides that, a precise estimation of the structural behavior of membranes is complicated when the variations in material properties is considerably huge. The variability in the form-fabric strength to that of the membrane applied in the field constructions after deformations and damages made during fabrication makes it almost impossible to predict TMS behavior.

The outcome of a round robin exercise (Gosling, Bridgens, et al., 2013) indicates the requirement for a world widely accepted design code and a standard design procedure for TMS. In this exercise, the results concerning initial shape (form), deformations, load bearing capacity and reaction forces were obtained using different computer software tools for four case studies. The lack of available benchmark problems for the purpose of validation of tensile membrane structural analysis imparts the high uncertainty in the results part (Lewis, 2008).

2.4 Simulation approaches of membranes

The computational simulation of TMS addresses the question of what is the comprehensive geometric alignment of a membrane surface covering the space between provided boundaries and at the same time coping up with the initial prestress and the effects of external loading. In general, the computation process starts with a pre-assumed form, embodied by mesh or nodes. From the past studies related to finite element analysis of structural systems, the whole structure is enforced to be discretized into small elements, and the collective response of these sub-domains is observed to obtain the response of the continuum. The interconnected node network gives a distinct surface of the model, this process of quantification of membrane surface into nodes and lines is known as mesh generation or surface discretization. It is a standard procedure in

the process of modeling of tensile membrane structures. A recent study on surface discretization (Brew & Lewis, 2003) have highlighted the applicability of cubic splines (a third order polynomials to define a space) for visualization of the membrane design along with suitably fitted smooth lines that enhances the accuracy of the surface discretization. The surface physiography is defined by the network of nodes and elements and in the context of the analysis method used in the present study; it is assumed that the membrane structure is discretized into predefined network of the finite elements. Based on the past literature, the simulation of membranes is accomplished by mainly two approaches; either the surface is discretized to cable net or continuum surface. Both the techniques have their advantages and disadvantages depending upon the nature of outcome and the rate of analysis (Lewis, 2008).

2.5 A probabilistic approach to the analysis of fabric structures

Presently, safety factor is widely accepted design parameter for TMS, although the design methodologies varies from country to country. The primary parameters to define safety factors are mainly based on the following aspects: a) material properties behavior and consequential environmental impacts, 2) Human error due to lack of skills, 3) incomplete knowledge of designer regarding physical behavior of all important variables. With so high unconventional behavior and with a large amount of uncertainties involved in the analysis of TMS, “With development in science and technology, the element of ignorance can be largely eliminated, while the uncertainties, being changed in the form and magnitude, can never be removed” – (ASCE-1945). Considering the present day requirement to fulfill the purpose, in the time when there is a huge revolution in terms of material developments and new forms of structural systems, the researchers have established an efficient tool to deal with the problem of uncertainties, through statistical exploration and stochastic investigation for robust and cost effective design. The researchers and practitioners from worldwide have adopted varieties of factors and coefficients contributing to the structural safety, obtained from different numerical and theoretical approaches. The most of the literature in the field contains procedures involving permissible stress method, unlike limit state design method which are implemented for conventional structures like, beam, column, steel truss etc. due to highly non-linear behavior of TMS, the limit state design method is not so pleasant experience to apply for analysis of membrane structures. There is a huge

scale of uncertainties exists in the membrane structure design and analysis, these uncertainties are compensated by a range of safety factors during design process.

Based on prevalent design practices and safety assessment methods, existing design guidelines are overviewed as follows:-

2.5.1 ASCE Standard

ASCE recommendations for TMS design is based on cyclic loading and different load cases and their combinations. In case of biaxial loading, it is anticipated that the material strength will reduce. Based on the design criteria, the permissible stress developed in both warp and weft directions is given by the following expression;

$$T_p = \beta_s \cdot L_t \cdot T_{sm} \quad 2.1$$

Where T_p is permissible stress, β_s is strength reduction factor depending upon the nature and combination of loading, a widely accepted and moistly used value of $\beta_s = 0.27$ is provided in codal provisions for almost all the loading combination cases for TMS, L_t is the life cycle factor, these are proficient in retaining at best 75% of the initial strength over the complete life span of TMS, the codes also mention $L_t = 0.75$ for fabric structures, and for the structures undergoing the repeated loading, the life cycle factor take the value of 0.6.

In case of biaxial loading the code proposes that stresses in both warp and weft direction collectively should not fall below $0.8 \beta \cdot (T_{sw} + T_{sf})$, where $(T_{sw} + T_{sf})$ is the indicated minimum breaking strengths in warp as well as weft (fill) direction. Due to huge amount of uncertainties involved in the behavior of TMS, factors including material characteristics, loading environment, working skill involvement etc. are considered in most of the design guidelines available to determine the safety coefficients. The use of high safety factors is adopted in order to compensate the effects of non-factored loads as they can alter the geometrical behavior by a large extent owing to high non-linearity associated with the TMS. Overall stress distribution is also sensitive to the ultimate limit states for membrane structure, hence it can lead to a diverge solution. Due to high ambiguity of fabric material and also due to unavailability of proper guidelines, the determination of safety factor might reflect an over safe (high reliability) and uneconomical TMS design. But, conversely the objective of achieving an optimized

design can be accomplished by predicting these safety factors through an appropriate reliability assessment.

2.5.2 IASS references

International Association for Shell and Spatial Structures (IASS) founded in 1959, Madrid, Spain, envisioned towards research and development in the domain of lightweight structural systems like shell structures, membrane structures, lattice and tension structures. Researching group from IASS have recommended an approach to estimate safety factors for tensile membranes. The parameter for safety factor estimation taken into consideration includes material physical characteristics, error in calculations, indeterminate loading and natural factors. The safety factor for complete structure is a collectively arranged safety system constituted of individual safety factor associated with each of the above mentioned parameters. The proposed expression for general safety factor is given as:

$$L = 2 \cdot l_1 \cdot l_2 \cdot l_3 \cdot l_4 \cdot l_5 \cdot l_6 \cdot l_7 \quad 2.2$$

Where l_1 represents the safety factor contributed by inconsistency of membrane surface, value of l_1 estimated in warp direction is 1.25 and for the weft direction it is fixed as 1.43, l_2 is associated with the safety factor imparted by accuracy of the calculations, if the accuracy is verified experimentally then l_2 is 1.25, and 1.3 in other scenarios, l_3 and l_4 are the safety factors engaged due to indeterminate nature of loading and load application, in both the cases it is equal to 1.0, l_5 is the representation of reliability aspect associated with authenticity of material and results obtained, in general l_5 takes the range of values between 1.1 to 1.3, for the other scenarios including undesirable characteristics it takes value 1.2. l_6 signifies the loading execution and is generally taken 1.0. l_7 is associated with the factors associated to environmental impacts such as cyclic loading, radiation damage, thermal impacts and moisture damage, the range of value it takes are 2.0 to 2.4.

Similar to other proposed code provisions the large safety factor is recommended by IASS the basis of which is mainly includes environmental effects instead of loading and to cope up with the material based impacts the unevenness of the fabric is emphasized. This brings up an ambiguity in the given approach as the immensity of environmental factors (e.g. creep, thermal damage..etc) is accountable to involve inaccuracy, also this approach of collective effects may be excruciating to implement.

2.5.3 EUROCODE 0

The European code design approach is derived from the provisions available in Euro code, based on the extent of damage from the failure of a general structure. Based on the amplitude of structural system failure, general structures are categorized into different classes as tabulated in the Table 1 below

Table 1: Target Reliabilities for different building categories given by Eurocode 0.

Class	Amplitude of failure	Duration of use (yrs.)	Target reliability index (β)
RC3	High consequence for loss of human life, or economic, social or environmental consequences very great	1	5.2
		50	4.3
RC2	Medium consequence for loss of human life, economic or environmental consequences considerable	1	4.7
		50	3.8
RC1	Low consequence for loss of human life, and economic, social or environmental consequences small or negligible	1	4.2
		50	3.3

.Based on the above mentioned classification, a TMS is foreseeable to be categorized on the basis of scope of applicability. For example the roofing system for a big parking area made of tensile membrane structure can be categorized into RC1 class with target reliability index 4.7 for 1 year of duration of use and 3.8 for 50 years of use with reference to the Euro-code 0. It is understandable that the reliability recommendations presented here are general provisions, however the actual safety index which is specific to a desired TMS may be obtained individually for each case. The fabric structures are known to possess high uncertainty in terms of material characteristics and their structural performances also the non-linearity in behavior makes it complicated to estimate the reliability of the structure.

Table 2 Target reliabilities of RC2 building type by limit states, reproduced from “Eurocode - Basis for Structural Design” Table C2 Target reliability index β for Class RC2 structural members [(Caner & Hsu, 1999)]

Limit State	Target reliability index, β for reference period of	
	1 year	50 years
Ultimate	4.7	3.8
Serviceability	2.9	1.5

The accurate reliability calculation for a TMS will need the sufficient information regarding uncertainty associated with materials, and a robust reliability approach to cope up with all the variables subjected to the structural response of a membrane structure.

2.5.4 French Design Code:

The French design guide mentions the formulation of the permissible stress T_p in the terms of material structural performance, the surface area of the TMS and environmental factors like radiation, pollution...etc. The expression for T_p is given as:

$$T_p = (K_q \cdot K_e \cdot T_{sm}) / S_f \quad 2.3$$

Where, K_q = factor associated with the material quality (1.0 for tested layers and 0.8 for others), K_e = scale factor (1.0 for TMS with surface area upto $50m^2$ and 0.8 for the surfaces between $50m^2$ and $1000m^2$), T_{sm} = minimum breaking strength mentioned for materials. S_f = factor related to environmental pollution/degradation (4.0 for low pollution, 4.5 for strong pollution region. The scale factor accounted in the French guide with an estimation that TMS occupying large area is supposed to have higher order of uncertainties involved whereas the small structures are assumed to possess low order of uncertainties.

2.5.5 Japan Design Guide :

The basis of safety factor estimation in Japanese design guide are mainly type of loading and type of structures. The impacts of environmental factors, and uncertainties associated with material are not accounted for the design proposal. The permissible stress is estimated with the help of specified minimum failure strength reduced by the safety factors associated according to the nature of load and type of structure. The safety

factors used are comparatively higher than the other available TMS design approaches, for a membrane cover of a framed or a primary spaced structure with some permanent nature of load will require a safety factor between 6.0 to 8.0, and similarly a temporarily loaded TMS supported with rigid frame will require the safety factor between 3.0 to 4.0. One of the main reason for using such a large value of safety factor may be is to compensate the effects of all the collective variables possessing uncertainties.

2.5.6 German design:

The German design code proposes the estimation of safety factor in terms of load factor along with the accountability of uncertainties involved with the membrane. This can be adopted as an efficient approach as the factor of safety intends to vary with the variation of loads. Also in German code there is a considerable amount of contribution of connection detailing is accounted in the design process.

The mathematical formulation for the allowable stresses f_d according to the code is given as:

$$f_d = \frac{f_{tk}}{\gamma_f \cdot \gamma_M \cdot A_i} = f_{tk} / A_{res} \quad 2.4$$

Where, f_{tk} is the tensile strength, γ_f is the load factor, γ_M is the material safety factor associated with material uncertainty and A_i is combination of reduction factors subjected different load cases.

2.5.7 Italian design:

The design approach proposed by Italian guide is similar to the German design for TMS. The only difference is the more emphasis on connections. The connections are categorized based on the approved connection designs from experts at different level. The detailed explanation about connection designs can be found in the Italian design code.

2.6 The reliability approach

In this section, the traditional uncertainties dealt for the general structural safety are summarized, and those particularly important for fabric structures are discussed. Then the general reliability methods to deal with these uncertainties are reviewed and discussed, and the reliability methods specific to the structural safety are highlighted and

their possible application on the fabric structures are also discussed. This section will demonstrate that FORM(at least initially) coupled with a good finite element method is the preferred option.

2.6.1 General uncertainties involved in a structural reliability analysis

Design and consequential results are normally combined on the basis of conventional mechanical theories and past field experience for traditional structural systems. However in case of non-conventional structural forms like tensile membrane structures, the past experience may prove to be insufficient and unreliable to characterize the existent statistical and structural behaviour. Also, new type of structural form will need advanced construction material, therefore efficient statistical approaches are required to incorporate the overall uncertainties involved. All the variables which can potentially mould the structural behaviour of these type of structures must be accounted as a general uncertainty in the design formulation for TMS. A categorized representation is done for this type of uncertainties as tabulated in Table 3, according to this study the general uncertainties related to structural analysis into 2 primary types:- the “strain” and the “resistance” along with this, the computational phase is also categorized as the intermediate source of uncertainties. Strain mainly accounts the uncertainties associated with all the loads, and resistance involves the uncertainties of the consequential effects of loads. The uncertainties are likely to have different importance regarding structural reliability of a given structural system, but the direct or indirect effect of them is concentrated towards safety of the system. The case specific safety factors may be convoluted by one or more uncertainties, depending upon their sensitivities towards structural safety.

Table 3: Classification of general uncertainties

Cause of Fluctuation in "Strain"	
Type A	I. Uncertainty and variability of loading conditions
	* Dead Load.
	* Live load (including dynamic effects)
	II. Uncertainty and variability of external conditions those are independent of the load.
	* Change of temperature.
	* Wind force.

	* Uncertainty of behavior of the subsoil
	Causes of uncertainty during "strain" computation.
Intermediate type	III. Variation of rigidity.
	IV. Imperfection of methods and shortcoming of assumptions.
	* Accuracy of method and tolerances of numerical computation.
	* Inadequacy of assumptions concerning initial and boundary conditions, stress concentration, and secondary strain.
	Causes of Fluctuation of Resistance.
Type B	V. Uncertainty and inaccuracy of the assumed mechanism of resistance.
	* Inaccuracy or inadequacy of conceived mechanism.
	* Variability of resistance limits of materials.
	VI. Variation of structural dimensions.

The uncertainties given in the table above embraces almost all the factors contributing to determine the reliability of TMS, hence should be very useful to closely observe behavior of the system. The important thing to keep in mind is to consider "resistance and "modeling" as more important aspects of the whole process. The reason for such consideration is non-other than the most important characteristic of membrane structures, and that is the intense geometrically induced non-linearity. This non-linearity is mainly due to the high variation in the material properties. The researchers have published a lot about the probabilities of "strain" influenced by various physical and environmental factors for almost all type of structural systems, while in the case of TMSs due to randomness in the material characteristics and variable young's moduli, extensive surveys and researches are required in the "resistance" aspect.

Similarly perceiving the fact that different approach in terms of reliability evaluation and data handling is required to tackle different types of uncertainty, another classification based on source of uncertainty is displayed in Table 4. The uncertainty is classified as cognitive and non-cognitive source of uncertainties.

Table 4: Classification based on the source of uncertainty

Non-cognitive (Quantitative)	1. Inherent uncertainty
	- Repeated measurements of the same physical quantity do not yield the same value due to numerous fluctuations in the environment, test procedure, instruments, observer, and so on non-cognitive sources
	2. Statistical uncertainty
	- No precise information about the variability of the physical quantity of interest due to limited data.
	3. Modeling uncertainty
	- System analysis models are only approximate representations of system behavior.
Cognitive (Qualitative)	1. The definitions of certain parameters, such as structural performance (failure or survival), quality, deterioration, skill and experience of construction workers and engineering, environmental impact of projects, and conditions of existing structures.
	2. Other human factors
	3. Definitions of the interrelationships among the parameters of the problem, especially for complex system.

The inherent randomness, modeling and statistical uncertainty are included in the quantitative or non-cognitive sources, whereas the ambiguity imparted in conglomeration of defining actual behavior through mathematical abstractions is counted in cognitive or qualitative source of uncertainty. First type of uncertainty is taken care through past experience and higher number of tests observations, the second type of uncertainty is addressed specifically through fuzzy set theory. This classification is of underuse until a conceptual approach is implemented to assess the uncertainty scenario of structure. In this research, the uncertainty analysis is concentrated in the basic parameters of TMS material and geometry, hence a statistical approach will be applied for the uncertainty assessment.

Another basis of uncertainty classification based on physical randomness and knowledge based uncertainty, these two sources are integrated using bayesian technique (Igusa et al., 2002). Based on cognitive source of uncertainties, the commendable researches have been performed (De Lima & Ebecken, 2000)(R. Zhang & Mahadevan, 2000), in order to

accomplish the reliability analysis for the structural systems. However, mathematical formulations to associate the qualitative uncertainties with design process is not very pleasing experience, this is due to the fact that authentication of the mathematical model implemented is typically demanding. In the other hand quantitative or non-cognitive or aleatory source based uncertainties which mainly includes loads and material properties, are controlled through statistical operations on test data and archives.

The knowledge from collective reviews of common uncertainties associated with engineering industry should be accounted for facilitating the reliability assessment of TMS by recognizing the randomness in structural parameters. The past studies on fabric structures indicates that reliability analysis offered with huge amount of uncertainties is a multifarious framework to deal with. In this scenario the relevant past recommendations and good guidance can ease the handling of uncertainties. The reliability analysis can be stratified on the basis of importance of uncertainty from structural response point of view, at first the limited number of uncertainties can be considered and can be further consideration of expanded uncertainty type.

2.6.2 A review of general reliability theories and methods

The structural reliability in general is viewed as the probability of the duration of time when structure is achieving the serviceability along with fulfillment of subjected limit states. To objectify the series of random variables like load, material characteristics etc. they are denoted through a vector X , represented through a joint probability density function $f_X(x)$. The safe zone (Ω_s) and zone of failure (Ω_f) for a given limit state function $G(x) = 0$, can be written as $\Omega_s = \{x | G(x) > 0\}$ and $\Omega_f = \{x | G(x) < 0\}$. Finally the probability of failure is denoted by:-

$$P_f = \int_{\Omega} f_X(x) dx \quad 2.5$$

This approach mentioned above cannot be directly implemented in the scenarios where there are large number of random variables along with number of limit states involved. To intensify the effectiveness of reliability analysis, variety of stochastic approaches has been explored. Some of the effective approaches includes Monte-Carlo method, Polynomial Chaos method, FORM/ FOSM etc. As a part of this literature contribution Monte-Carlo and FORM/FOSM is reviewed in further section.

First Order Reliability Method (FORM) is generally based on the model where a first order surface snug to design points substitutes the surface formed by the limit state parameters in the standard normal space. Applying statistical principles the mathematical representation for first order failure probability estimation is given as :-

$$P_f = \int_{\omega} \Phi(y) dy = \Phi(-\beta) \quad 2.6$$

In the above equation Φ_{ω} represents the standard normal density of the comprised independent statistical parameters to describe a structural system. β , which is identified as the safety index or reliability index, it is approximated as the closest possible distance of a limit state surface from the origin in a standard normal space. Reliability index is mathematically denoted as:-

$$\beta = \sqrt{t^* t} \quad 2.7$$

t^* and t represents the point vector coordinate of the limit state surface lying on standard normal space. A remarkable FORM algorithm developed by Rackwitz and Fiessler [242] which requires the accurate estimation of gradients associated with statistical variables but works efficiently for almost all structural systems.

All the non-normally distributed variables in FORM analysis are transformed as equivalent normally distributed variables on same cumulative density function at the design points. The random variables are denoted by vector X_s and represented as a set as:-

$$X_s = [X_1, X_2, X_3, \dots, X_d]^T \quad 2.8$$

These random variables are transformed to another independent and normally distributed standard random vector, U this random variable is a set of other equivalent random variables and is represented as :-

$$U = [U_1, U_2, U_3, \dots, U_d]^T \quad 2.9$$

These normal and non-normal variables are related using Rosenblatt transformation as:-

$$U = T(K) \quad 2.10$$

In the above relationship, the transformation is required to be repeated after every iteration in order to obtain most probable point lying over the safety criteria surface.

Along with the ease of transformation the Rosenblatt transformation has a drawback that the limit state function is liable to exhibit high nonlinearity as a function within the domain of independent standard normal variables. In its response an efficient normally distributed function is developed which work for most of the common distributions. This polynomial function is based on the use of fractile constraints in place of conventionally used statistical moment fitting (Hong, 1996).

The past researches have proved the utility of FORM as most effective approach for reliability assessment of a structural system (Zhao & Ono, 1999). For around past four decades FORM have been used to solve variety of reliability related problems in overall engineering domain, thus making it the basic and wide accepted reliability analysis method. The only drawback which has mentioned earlier is the nonlinear behaviour of limit state function will obtain less efficient reliability results.

To compensate this shortcoming of FORM, another higher order reliability method has been established, i.e. the Second Order Reliability Method (SORM). This new method is capable of improving the accuracy by implementing second order approximation to the limit state function. Although the acceptable accuracy requirement for FORM is hard to estimate, as till date the outcome of reliability analysis is authenticated by an event of reliability failure (Mitteau, 1999). The only way to supervise the accuracy of technique is the error control during each step of reliability estimation for a system.

A similar approach to SORM developed as a moment method based on the concept of implementing high order moments like, second order and third order moment method can be exploited to achieve the higher accuracy in terms of reliability by reducing error in approximating the limit state function (Zhao & Ono, 2001).

For the purpose of reducing the error in calculation and accuracy improvement, the limit state function can also be molded. A “bulge” method was developed on the basis of searching of accurate design points while implementing FORM/SORM (Der Kiureghian & Dakessian, 1998). It is moreover an iterative technique where the multiple design points of obtained deformed surface is estimated and updated after every iteration using a special function called “bulge” function.

The another research proposing the improvement in efficiency of FORM was based on an adaptive two-point nonlinear approximation, it is an algorithm where both first and

second order moments of a performance function is implemented. The design point searching and updating approach is used where the linear and non-linear approximation is alternatively used and the linearly derived design point is identified as the point obtained by previous iteration and the design point obtained through nonlinear approximation is considered as the design point for current iteration. In this way the approach claims the improvised reliability analysis for a system. Also this searching procedure for design point itself being an optimized process is helpful in reducing optimization and reliability analysis cost in terms of computational input.

As mentioned earlier in section 2.5, Eurocode 0 identifies the safety index as a design parameter categorized on the basis of limit state functions, on this basis the safety coefficient proposed currently by Eurocode 0 are bound to be deterministic. A diagrammatic representation of an overall reliability assessment by Eurocode 0 is displayed in Figure 1

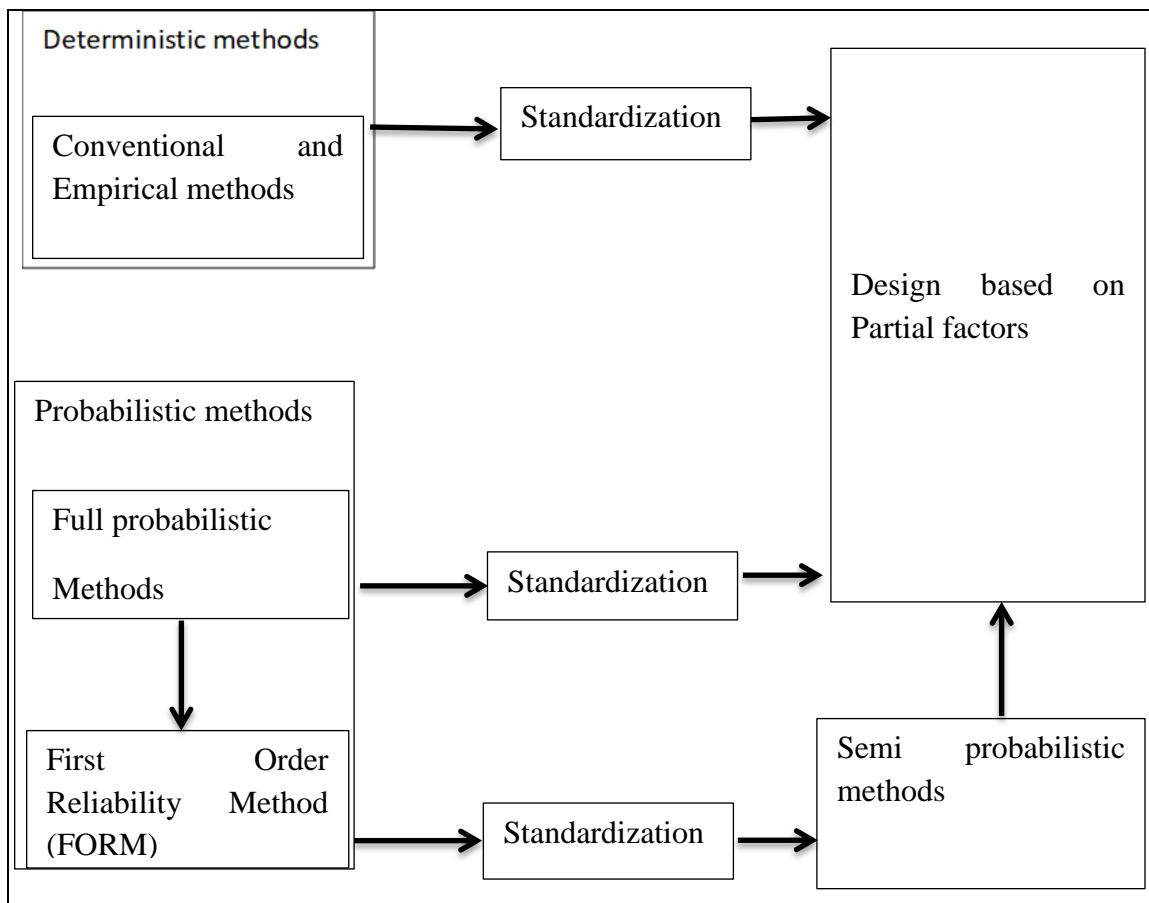


Figure 1 The schematic diagram of reliability

The safety factor approach in this code brings out the fact that an efficient probabilistic reliability can be universally accepted for reliability assessment, but due to insufficient knowledge and data it is rarely used for standardization of safety factor coefficient. The overall outcome of this particular section of literature review on persistent reliability approaches focusing FORM is that, this approach can be non-hesitantly implemented for advancement of design procedures for TMS, also it is certain that an appropriate approximation is necessary in order to improve the accuracy of the approach.

2.6.3 Monte-Carlo Method (MCM)

The Monte Carlo Method was developed in late 60s and is a proven persuasive statistical tool to accomplish random variable simulation. It has a large domain in terms of applicability in wide range of structures. It is a simple and instinctive in terms of applicability to simulate most of the random variables that fit in any type of existing distributions. One of the demanding factor for this method is the huge amount of calculation for obtaining relatively accurate outcome. This indicates towards a potential incapability of this approach for practical visualization of complex structures like TMS. To identify the other limitations in the technique several other researches have been listed for solving the sampling issue in Monte-Carlo (Rocco, 2003)(Ramirez-Marquez & Coit, 2005)(Naess et al., 2009)(Jin Guoliang et al., 1993)(Bjerager, 1990)(Bucher, 1988)(Mori & Ellingwood, 1993)(Tichý, 1994)(Bucher & Bourgund, 1990)(Zhao & Ono, 1999)(Zhao & Ono, 2001)(Mitteau, 1999)(Der Kiureghian & Dakessian, 1998)(Bjerager, 1991)(Ditlevsen et al., 1990).

A remarkable work combining finite element with MCM (Jin Guoliang et al., 1993), resulted in the development of an efficient Monte Carlo finite element method for reliability analysis, which is capable of handling complicated engineering structural systems. Similarly there are number of significant works which were directed towards the improvisation on the aspect of sampling methods to enhance the accuracy of this approach. However, in comparison to methods like FORM and its variants which uses moments of first order or higher order the MCM technique is still very much orthodox to analyse the system when the failure probability is low(Grooteman, 2008).

Other methods include directional simulation method (Ditlevsen et al., 1990)and importance directional simulation method (Melchers, 1994), which were developed to meet the purpose. The idea of directional simulation method is based on generation of

direction vectors which are uniformly distributed; along with this one-directional integration in every direction is also performed as a procedure for this technique. On the other hand importance directional simulation method is based on the importance sampling technique which is implemented to congregate the direction vectors towards the regions of interest. Both these methods are more or less efficient than MCM, but their accuracy decreases by great margin in the cases where the limit state function shows highly nonlinear behaviour. This concern can be handled but with a demanding requirement of huge number of sampling directions, whereas the reliability analysis requirement by a finite element technique is just opposite to it. For efficient reliability results the number of directions should be kept to minimum in order to limit the error in the general probability failure expression for a system.

Directional methods accuracy is dependent on the art of choosing the design points as in the case of MCM, and often are compelled to use several sampling approaches like importance sampling technology (Grooteman, 2008). But the researches have proved that directional methods accuracy is limited to the low dimensional regions only (Nie & Ellingwood, 2004).

2.6.4 Finite element probability approach

Estimation of response stresses is the primary importance for reliability analysis of a structural system. The simple physics laws are not sufficient to obtain the load responses in case of complex structures, hence the numerical methods become necessary to apply to analyse complicated structural systems. The finite element method has emerged as a robust numerical technique to analyze almost any type of structure.

The traditional form of finite element method was based on determinacy of structural system, the definite values of load and structural geometry were considered. The advancement in the field of statistics have proved parameters like, the material properties, loads, geometrical dimensions of system etc. as random variables. The overall structural reliability assessment requires knowledge of indeterminacy involved with each variable. Reliability estimation through computational technique implementing finite element method with some improvements in approach can be achieved effectively and accurately. If the structural responses can be obtained analytically in that case the reliability analysis can be accomplished without finite element. But the real is complex, most of the structural systems need FE approach to

obtain the structural response. FORM is a solid example of such a technique involving FE for reliability analysis of structures.

The early researches (Liu Pei-Ling, 1991)(Liu Pei-Ling, 1991) on finite element reliability estimation used first order or second order reliability methods with an objective to enhance the failure surface in standard normal space and also to record the gradients of structural responses like geometry variables, loads and material characteristic. This combination of finite element formulation with reliability approach has become very popular in modern-day reliability assessment, and is generally called “stochastic finite element reliability method”. The sensitivity analysis of basic statistical variables for finite element formulation at design points is done for pursuing safety index during iteration process.

Similar to FEM finite difference method has been developed to compute gradients of limit state functions with respect to basic statistical variables. This method is found to be time inefficient and is limited to use for problems involving few variables. The analytical method based on the differentiation chain rule(Y. Zhang & Der Kiureghian, 1993), is intended to behave similar to finite difference method with a goal to achieve better efficiency and accuracy while cost estimation of conventional structures like reinforced concrete.

The finite element formulation for FORM and SORM was first developed by Liu (Liu Pei-Ling, 1991) in order to estimate the reliability of structures who were non-linear geometrically and have plenty of uncertainties associated with it. The major finding of his research was the importance of computing vector gradients in examining the design points through finite element reliability method formulation, and also compared the computational wise efficiency. In order to prove the effectiveness of his approach, he combined analytical method and finite difference method with FORM and SORM, and the results proved that FORM approach gave better reliability estimation even for highly nonlinear structural systems containing non-normally distributed uncertainties. The computer efficiency for FORM was comparatively higher than SORM when the analytical method was coupled.

This finite element reliability approach was also applied to nonlinear concrete structures (Teigen et al., 1992). The nonlinearity was considered for geometry and material characteristics, and also the randomness in loading, geometry and material were

considered. The perturbation method is applied for obtaining the gradients based on structural response by exploiting the first and second order expressions from the limit state function expansion using Taylor series. The implicit limit state function and the reduced computational time were major highlights of his research. The numerical test results of this approach paves the way for further development of FE technique.

In order to prove the versatile nature of the FE formulated reliability approach, it was applied on geometrically nonlinear suspended truss system (Frangopol & Imai, 2000). The research highlighted the importance of material behavior for complete structural system without influencing the individual structural components. Other findings of the study were that, linearly modelled nonlinear geometrical system will provide conservative outcomes of tension and neoconservative compression results, also it was found that the correlation in load effects more obvious than correlation in resistances.

A variation of the finite element reliability approach was applied on RC frames with an objective of nonlinear analysis, it was based on the combination of FORM and the nonlinear FE structural modelling (Dimitri et al., 1997). A rational computational framework was developed to achieve the time-variant reliability assessment of RC structures. This approach can handle the computing cost and balances the precision of stochastic and mechanical models. It provides new material based models for time-invariant and fatigue response of the concrete. According to this literature the evaluation of long term life history including path dependence and sequence effects, is influenced by temporal uncertainties. And the overall reliability estimation was affected by spatial scatter of loads and material properties.

Suspension bridges were also studied using probabilistic FE geometrical nonlinear analysis procedure (Imai & Frangopol, 2000). The main source of uncertainty were different damage and loading circumstances. A general approach for reliability estimation of geometrically nonlinear structures exhibiting elastic behavior. The reliability analysis results decides an ideal maintenance of suspension bridges designed according to the allowable stress method.

Number of fabric structures have been designed and constructed since last several decades. And there has been the development in terms of new forms of material for constructing TMS. Even after so much researches conducted, a robust and effective reliability estimation method predicting the possibility of TMS failure is lacking. This is

because most of the attention are being paid to load effects and the demanding material strength instead of the uncertainties associated with them. The application of deterministic approaches will facilitate the identification of single failure point for a TMS containing fixed instantaneous values of material, loads and geometry, whereas when the structural response parallel to real time load scenarios and material case is concerned the safety index of the system is questionable.

To conclude all the efforts made with the objective to find an appropriate way of correctly visualizing a safety aspect of TMS, an effective reliability analysis is likely to contain number of uncertainties all depending on the situations and conditions to which a TMS is subjected to. Among all the uncertainties associated with material characteristic can be evaluated using numerical statistical approaches and to tackle other sources of uncertainties the methods like fuzzy set theory, particle swarm etc. can be implemented. An extensive review of literature regarding uncertainties in reliability evaluation of TMS provides a general understanding in considering uncertainties for present research and a general idea of modelling uncertainties related to material, load and geometry is developed. There are several existing methods like MCM, FORM, SORM and other probabilistic approaches, out of all the FORM has proven to be the most effective and robust reliability method to be efficiently applied to a wide range of structural systems including TMS. SORM implementation is intended to identify the second order failure boundaries for better simulation of closer to real cases, but their solution procedure is likely to be complex and less efficient, also the divergence of results is likely to take place. MCM is user friendly approach for estimation of reliability of almost all types of structures but the efficiency is good for only deterministic and simple structures. The full probabilistic methods for reliability analysis may result to an accurate reliability estimate, but the large number of uncertainty data is not available in most of the cases of TMS.

The FE reliability procedures have emerged as the combination of finite element formulation and reliability methods like FORM, and have successfully estimated the reliability of number of geometrically nonlinear materials. Although they have not been practiced much to be applied directly for TMS, they elaborate a basic idea for integration of FEM and reliability methods. It is expected that, if an efficient FE technique is developed exclusively for TMS, then FORM ensure an accurate reliability outcome based on the associated uncertainties within the control of finite element formulations.

2.7 Research Gap

Based on the intensive literature review about behavior and analysis of tensile membrane structures, the following findings in terms of research gap are proposed.

The current status of the standardization of design process of TMS highlights the importance for developing a design approach specified for membrane structures. So far reliability criteria have been estimated to be an efficient approach implemented by some of the developed countries in the calculation of safety factors to facilitate design process. The recommendations from Euro-code are solely based on the reliability requirements of a membrane structure. Hence the importance of developing reliability approach is projected through the present piece of work.

The Computer aided methods have been able to estimate the form finding stage with sufficient amount of assumptions related to material and other properties of TMS. The concept of minimal surface area is proven to be better approach to manifest the membrane surface. The finite element being an equilibrium solution based procedure can also be implemented to contain the membrane surface behavior. Therefore there is great need for the development of a finite element based methodology to analyze the non-linear geometries associated with the TMS.

The uncertainties associated with material properties are assessed using numerical approaches, and other qualitative uncertainties are estimated using advanced methods like fuzzy logic. The reliability analysis is prominently associated with including number of these uncertainties accounted according to available knowledge of limit state and the overall status of the structure. The literature about reliability involving uncertainties associated with TMS provides an urgent need to include more number of uncertainties for better assessment of the membrane structures.

With the availability of different structural reliability methods such as SORM, Monte-Carlo, FORM etc. The first order reliability method (FORM) is found to be the efficient most in analyzing large number of linear or nonlinear structures. Based on the outcome of the extensive literature study the overall handling of FORM in terms of ease of applicability in complex geometries and requirement of large amount of uncertainty information about membrane structures is more reliable and convenient method. This points out the essential demand for the development of the robust and efficient finite

element based reliability approaches which can specifically fulfill design guide requirement of TMS.

2.8 Scope of study

The scope of the study for present work covers the following aspects:-

- Checks the applicability of finite element based semi-probabilistic modeling approach for efficiently determining the form-finding and loading behavior of TMS
- To study the parameter change behavior for two most basic shapes, hyper paraboloid and the square base conic shaped TMS, by varying the overall height and the initial pre-stresses in warp and fill directions. The wind load, snow load and their combinations as per ASCE 7-16 (minimum design loads and associated criteria for buildings and other structures) are considered for including load behaviour of the TMS.
- A probabilistic analysis methodology for membrane related uncertainties based on the simulation data is proposed. An existing design criteria from Eurocode is adopted and implemented.
- Determination of structural responses in terms of in-plane stresses in both warp and fill directions, considering maximum deformations under different loading conditions for each prescribed TMS.
- The reliability indices calculation using a robust reliability tool, first order reliability method (FORM) for the COV range 0.1 to 0.25. The results of reliability are verified with the safety criteria mentioned in Euro-code.

2.9 Summary

The literature review performed for this study gives a strong basis for the research work which is organized in this thesis. To start with the assessment, an extensive review has been accomplished to evaluate the state-of-the-art analysis methodologies for membrane structures. From this study, it is concluded that non-linear finite element technique demonstrate a resilient numerical approach for form finding as well as the overall analysis of TMS. A complete stochastic analysis methodology using reliability based finite element investigations with the effect of variability of overall height of the TMS and the effect of change in pre-stress is proposed as one of the primary outcome of this

thesis. The accountability of uncertainties in geometry and loading conditions is the main focus of this study. The existing reliability methods are reviewed and the research gap are identified so that new and efficient methods can be developed in general. However, the present study is limited to the application of finite element based reliability methods in TMS with smaller number of uncertainties. Lastly the effectiveness of current reliability method in determining the pre-stress for form finding and the scope of its applicability in defining the limit states for a TMS is presented in this work.