

Chapter 7

Incorporating Heterogeneity Through Bayesian Hierarchical Models

7.1 Preface

This chapter presents the hierarchical modeling approach for incorporating site-based and vehicular heterogeneity in modeling crash risk. This fulfills objective 3 of this dissertation. Both pooled and hierarchical models were fitted, and the results were compared. This chapter utilized the univariate peak-over threshold approach described in section 4.1. Moreover, Bayesian inference (described in section 4.3) was used to estimate model parameters.

This chapter is organized as follows: Section 7.1 illustrates the review of relevant literature on approaches to modeling conflicts using Bayesian hierarchical extreme value. Section 7.2 presents the extraction of conflict indicators from the trajectory data, while Section 7.3 describes the Bayesian Hierarchical model framework for incorporating heterogeneity during crash risk assessment. Then, Section 7.4 presents the model estimates and a discussion of the results. Finally, Section 7.5 presents the summary of the significant findings.

7.2 Conflict-based Safety Assessment and Modeling Heterogeneity

Vehicular heterogeneity refers to the difference in microscopic traffic parameters (space and time headway, lateral gap) resulting from differences in vehicle characteristics. Conflict indicators derived using microscopic parameters depend on the type of L-F vehicles [38, 43, 160]. Although most of the conflict indicators are derived using space and time proximity between vehicles, vehicular heterogeneity has been mostly ignored in the reviewed studies. Most safety assessment studies [130, 131, 220, 221] have not incorporated the effect of vehicle type while safety assessment using microscopic conflict indicators. These studies have used a pooled model-based conflict technique for safety assessment, where a single threshold was used to segregate conflicts for all vehicles. In complete pooled models, all the subgroups in the population are lumped together, assuming that the population is homogeneous and a universal model is appropriate for all subgroups in the population. Since the crash risk of every leader and follower pair is modeled with the same parameters, a pooled model fails to account for the vehicle heterogeneity and can produce misleading conclusions in heterogeneous traffic conditions. To account for vehicular heterogeneity, few studies [160] have proposed separate models for each L-F pair in the traffic stream. However, a separate modeling approach assumes that the behavior of each L-F pair in the traffic stream is different and independent. Further, due to partition, each subgroup's sample size decreases, increasing the variance in model estimates.

Further, researchers have considered multiple sites for data collection to get a representative sample. For example, Paul et al. [119] utilized data from 14 three-legged and four-legged traffic intersections and fitted a negative binomial regression model by combining data from different sites. While researchers have utilized data from multiple sites, site specific heterogeneity has not been incorporated into the model framework.

The review of literature suggests that minimum spacing and microscopic conflict indicators depend upon the vehicle types. As traffic stream in heterogeneous traffic consists of multiple subgroups, vehicular heterogeneity should be incorporated in estimating crash risk. Further, while researchers have utilized data from multiple sites, site specific characteristics has not been incorporated into the model framework. Instead, a pooled modeling approach is adopted by most of the studies. To address this gap, this study aims to estimate the crash risk associated with rear-end crashes by incorporating site-based and vehicular heterogeneity into a hierarchical model framework.

7.3 Bayesian Extreme Value Modeling

In recent years, the Bayesian hierarchical extreme value models have been used by many researchers for conflict indicators-based safety assessment. With recent advancements in hierarchical EVT models, it is possible to account for non-stationarity and unobserved heterogeneity while estimating crash risk [222–224]. Since extremes are rare events, extreme value models are characterized by scarcity of data. To overcome the scarcity of data, the Bayesian hierarchical framework has been used by many researchers to combine conflict data from different sites for crash risk estimation [206, 225]. These studies show that conflict data from similar sites can be incorporated into a hierarchical model framework to address the paucity of conflict data. Literature suggests that hierarchical joint site EVT models may improve the accuracy of model estimate [56].

In this study, we quantify the risk of rear-end crashes in heterogeneous traffic based on extreme value theory using MTTC as a conflict indicator. We propose a Bayesian hierarchical framework to investigate the effect of vehicular heterogeneity on crash risk. The present study utilized conflict data from four locations in heterogeneous traffic for rear-end crash risk assessment. This study used site-based and L-F-based hierarchy levels for heterogeneity due to different sites and L-F pairs.

7.4 Traffic Conflict Data and Heterogeneity Tests

Defining traffic conflict in heterogeneous traffic is challenging since the vehicles do not follow a lane discipline. In this chapter, we focused on rear-end conflicts. To this end, vehicular interactions where the L-F vehicles move in line while maintaining a negative lateral gap were selected. To fit an extreme value-based crash risk model, traffic conflicts need to be quantified. Depending on the conflict type, various conflict indicators capture different crash mechanisms and have their own merits and demerits [20]. TTC, the most common indicator to quantify rear-end conflicts, is defined as the time until a crash occurs if both road users continue to maintain the same speed and travel direction [21]. The definition of TTC inherently assumes that vehicles would maintain the same velocity. However, vehicles take evasive action in the form of acceleration and deceleration, which need to be considered while defining conflict. MTTC is a modified form of TTC that incorporates vehicle acceleration [24]. Compared to TTC, MTTC estimates the risk for rear-end crashes more accurately [183]. Mathematically, MTTC is defined as

$$\text{MTTC} = \min \begin{cases} T_1 = \frac{X_{i-1}(t) - X_i(t) - L_{i-1}}{V_i(t) - V_{i-1}(t)} = \frac{d}{\Delta v} & \text{if } \Delta a = 0 \\ \begin{cases} T_2 = \frac{-\Delta v + \sqrt{\Delta v^2 + 2\Delta a d}}{\Delta a} \\ T_3 = \frac{-\Delta v - \sqrt{\Delta v^2 + 2\Delta a d}}{\Delta a} \end{cases} & \text{if } \Delta a \neq 0 \end{cases} \quad (7.1)$$

Where X_{i-1} and X_i represent the position of the leader and follower, respectively. L_{i-1} is the leader's length, d is the longitudinal spacing, Δv is the relative speed, and Δa is the relative acceleration between leader and follower vehicles. If Δa is zero, MTTC will be equal to T_1 . If Δa is not zero, two MTTC values are obtained. If both T_2 and T_3 are positive, the smaller value is taken as MTTC. However, if one is negative, the positive value is taken as MTTC. Fig.7.1 depicts the variation of MTTC among different sites and L-F pairs.

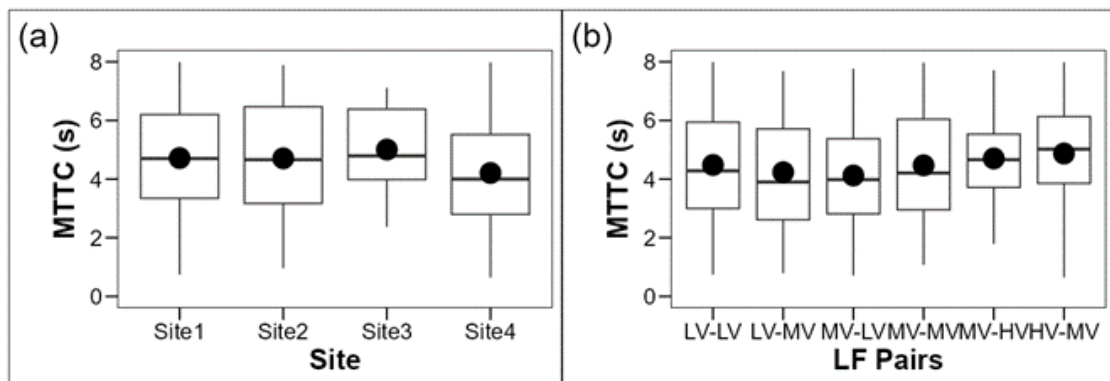


Fig. 7.1 Variation of MTTC among (a) sites (b) L-F pairs.

Since, this study modelled lower extreme values, MTTC values lower than 8 seconds were considered for further analysis. The average MTTC was found to be lowest for interactions that involved two and three-wheelers as either a leader or follower. These vehicles have relatively smaller speeds, maintaining a smaller longitudinal gap, resulting in smaller MTTC. Further, vehicle composition at the four study sites is similar, leading to comparable MTTC. The non-parametric Kruskal-Wallis's test [226] was used to check if MTTC was significantly different among the groups (L-F pairs and sites). For different L-F pairs, mean MTTC was found to be significantly different (p -value $< 8.17e-07$). Based on test results difference in mean MTTC across four sites was also significant (p -value $< 1.812e-15$). Therefore, hierarchical models incorporating vehicular and site-based heterogeneity were fitted as explained in Section 7.5.

7.5 Bayesian Hierarchical Extreme Value Models

This section describes extreme value modeling using peak over threshold approach for crash risk estimation using MTTC. Then it describes the Bayesian hierarchical structure for incorporating vehicular heterogeneity in modeling conflicts. Pickands [192] proposed the peak over threshold approach to model all the extreme observations exceeding a

specific high threshold. Suppose X_1, X_2, X_3, \dots is a sequence of independent and identically distributed random variables, and let $M_n = \max\{X_1, X_2, X_3, \dots, X_n\}$. Also, for large n ,

$$\Pr\{M_n \leq z\} \approx G(z) = \exp \left\{ - \left[1 + \xi \left(\frac{z - \mu}{\sigma} \right) \right]^{-1/\xi} \right\} \quad (7.2)$$

where $G(z)$ is the Generalized Extreme Value (GEV) distribution with μ , σ (>0), and ξ as location, scale, and shape parameters respectively. Then, for a large threshold u , the distribution of threshold excess $(X - u)$, conditional on $X > u$, is approximately given by

$$H(y) = 1 - \left(1 + \frac{\xi y}{\tilde{\sigma}} \right)^{-1/\xi} \quad (7.3)$$

where $\left(1 + \frac{\xi y}{\tilde{\sigma}} \right) > 0$ and $\tilde{\sigma} = \sigma + \xi(u - \mu)$. $H(y)$ represents the family of generalized Pareto distribution (GPD). In other words, threshold excess $(X - u)$, will follow a GPD with corresponding shape ξ and scale σ parameters. $H(y)$ models the probability of occurrence of maxima.

For modeling traffic conflict with GPD, a suitable threshold must be defined. An interaction would be a conflict if the conflict indicator MTTC exceeds its prespecified threshold value, while it would be a crash if it became less than or equal to zero. Since conflicts are defined based on minimum MTTC, negated MTTC or $-MTTC$ was fitted to $H(y)$, which models maxima. The probability of a crash (R), can be calculated based on estimated model parameters using Eqn.7.4.

$$R = \Pr(-MTTC \geq 0) = 1 - H(0) = \left(1 + \frac{\xi(0 - u)}{\sigma} \right)^{-1/\xi} \quad (7.4)$$

7.5.1 Threshold to segregate extreme values

Peak over threshold method allows efficient utilization of data while modeling extreme events. A critical step in fitting peak over threshold models is estimating the appropriate

threshold. Choosing a low threshold introduces non-extreme observations into the model, leading to bias. Also, selecting a very high threshold reduces the number of observations for model fitting that cause high variance. The mean residual life plot and threshold stability plot are two EVT-based techniques which are used for selecting the threshold of a univariate distribution. These techniques are described in Section 3.2.

7.5.2 Bayesian hierarchical structure

In the present study, the three-layer hierarchical structure is used to incorporate vehicular heterogeneity into the Bayesian hierarchical generalized Pareto distribution.

$$p(\theta|y) \propto \underbrace{p(y|\theta)}_{\text{Data}} \underbrace{p(\theta_1|\theta_2)}_{\text{Process}} \underbrace{p(\theta)}_{\text{Prior}} \quad (7.5)$$

The first level of the hierarchy is the data layer, where conflicts are modeled as GPD as shown in Eqn.7.3. The second level of the hierarchy is the process layer, which accounts for vehicular heterogeneity, which in the present case is discrete (vehicle type). The monolithic parameters (the scale (σ) and shape (ξ) parameters) were replaced by a set of k discrete parameters to incorporate the characteristics of individual L-F pairs. At the third level of the hierarchy, priors are defined for model parameters. The scale (σ) parameter is constrained to be positive for prior specification. Further, a wide prior was used for scale parameters as proposed in previous studies [223, 227]. Since GPD models are sensitive to their shape parameter (ξ), improperly defining prior would lead to a problem of model convergence. Therefore, information on shape parameter (ξ) from previous studies may be utilized. In recent conflict studies [183, 191, 228], the estimated shape parameters were found to be in the range of (-1, 1). Strongly informative priors would significantly affect posterior distributions. We want our posterior to be based more on data. Therefore, selecting a weakly informative prior is a better practice as it regularizes the estimates in the

case of small datasets and has no effect on the model parameters in the case of big datasets [194, 229]. Therefore, this study set a weakly informative prior for shape parameter (ξ) as Uniform $(-2, 2)$. The hierarchical model structure is depicted as follows:

Data layer:

$$G(y_k < z \mid \sigma_k, \xi_k) = 1 - \left[1 + \frac{\xi_k y_k}{\sigma_k} \right]^{-\frac{1}{\xi_k}} \quad (7.6)$$

Process layer:

$$\begin{cases} \sigma_k \sim N(\sigma_\mu, \sigma_\sigma) \\ \xi_k = \xi \end{cases} \quad (7.7)$$

Prior layer:

$$\begin{cases} \sigma_\mu \sim N(0, 100) \\ \sigma_\sigma \sim N(0, 100) \\ \xi \sim \text{Uniform}(-2, 2) \end{cases} \quad (7.8)$$

Where y_k is threshold exceedances from each sub-group k . In the above Bayesian hierarchical model structure, MTTC above the pre-specified threshold u_k are modeled as a generalized Pareto distribution. The process layer describes model parameters, which vary with vehicle groups or sites. The hyperparameters are the level-three parameters. With the determined thresholds for different subgroups, the hierarchical model can be specified by Eqns. (5) – (7). Using the Bayesian method for estimating model parameters, the posterior distribution for a given observation y can be written as

$$p(\theta \mid y) = \frac{p(y \mid \theta)p(\theta)}{\int p(y \mid \theta)p(\theta) d\theta} \quad (7.9)$$

Where the posterior $p(\theta \mid y)$ is the distribution of the model parameters, $p(y \mid \theta)$ is a likelihood, $p(\theta)$ is the prior distribution, and $\int p(y \mid \theta)p(\theta) d\theta$ is the normalizing term.

7.5.3 Analysis and results

This section describes how threshold excesses for conflict indicator MTTC, were modeled using the Bayesian hierarchical EVT approach. Analysis was completed using R software [230]. The package tidyverse [231] was used for data wrangling, while Rstan [232] and bayesplot [233] packages were used for fitting Bayesian models and visualizing the results, respectively.

Model estimation results

Before fitting the GPD model, selecting the appropriate threshold is essential. Threshold values for each model were selected using threshold stability plots by identifying regions where scale and shape parameters are stable. Threshold stability plot for pooled model is depicted in Fig.7.2, where red line segment represents the stable region for both the parameters.

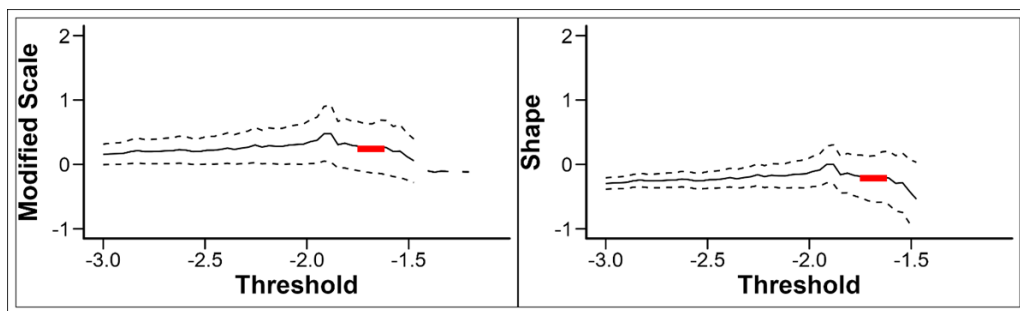


Fig. 7.2 Threshold stability plot for the pooled model.

Selected thresholds for different models are mentioned in Table. 7.1. The lower limit of the interval estimate was taken as the final threshold in all cases [218]. A smaller threshold ensures that more extreme observation will be selected while fitting the GPD model. The pooled model using Bayesian generalized Pareto distribution and L-F and site-based hierarchical models were fitted to the threshold excesses of negated MTTC.

All models were fitted using Stan [234], a probabilistic programming language which utilizes a No-U-Turn sampler (NUTS), and a variant of the Hamiltonian Markov chain Monte Carlo (MCMC) algorithm. NUTS has shown good performance in complex hierarchical models [235]. Posterior distribution was computed using sampling where four Markov chains were run for 6000 iterations for each model parameter. The first 2000 iterations from each chain were discarded as burn-in samples. From the remaining 4000 samples from each chain, posterior distribution was obtained.

Estimates of model parameters are presented in Table. 7.2. The posterior distributions obtained for hierarchical models are presented in Fig.7.3. Crash probability based on each model was computed using the estimated parameters using Eqn.7.4. Crash probability represents the rear-end crash risk based on conflict data. Further, using posterior distribution, 95% credible interval (CI) for crash probability were computed for each model.

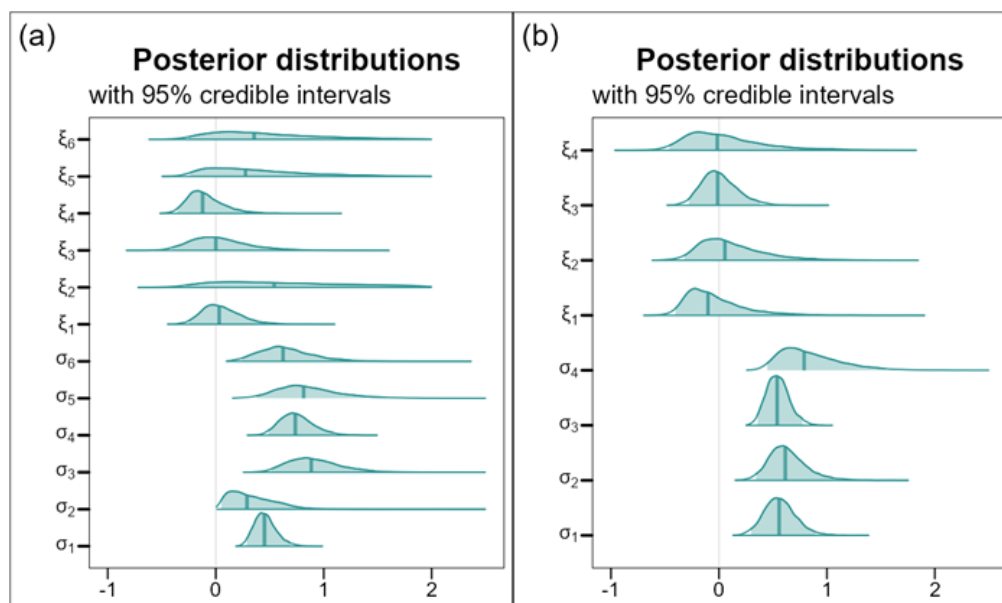


Fig. 7.3 Model parameters: posterior distribution (a) L-F-based models (b) site-based models.

The pooled model represents the overall crash risk for all vehicle groups at all the intersections. The hierarchical model leads to an L-F and site-based crash risk. The comparisons of crash risk obtained using pooled and hierarchical models are presented in Fig.7.4.

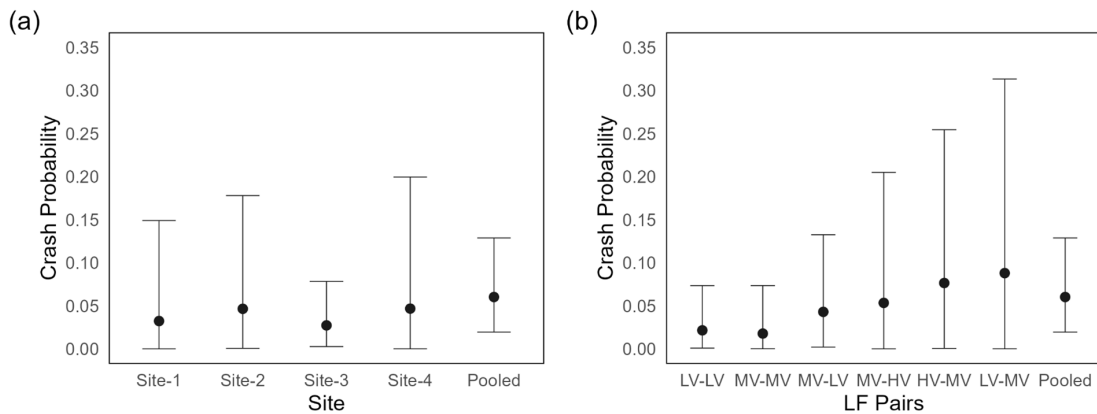


Fig. 7.4 Crash risk with a 95% CI based on hierarchical and pooled models (a) site-based models (b) L-F-based models.

Model diagnostics and posterior predictive check

The posterior distributions for each model parameter were estimated using MCMC algorithms. Samples generated from MCMC algorithms are biased as samples exhibit auto-correlation. If the algorithm is run for a large number of iterations, this correlation becomes negligible. Also, multiple chains are initiated to check for this bias. Each chain is initialized at a different location, and their convergence is monitored. One method of convergence diagnostic is to check \hat{R} (\hat{R} hat). If all the chains are converged, $\hat{R} = 1$ [236]. Trace plots are another method for visually inspecting the model convergence. A trace plot is a visual tool for checking the convergence of chains while sampling from the posterior distribution using MCMC sampler. As the chains start at different random locations, this plot shows the sampled posterior over each iteration for different chains. For an effective posterior sample, all the chains should have a similar distribution and be well mixed. The

present study used both \hat{R} and trace plots for model diagnostics. \hat{R} values for all the fitted models were equal to 1. Further, trace plots suggest that all chains converged and mixed well (Fig.7.5).

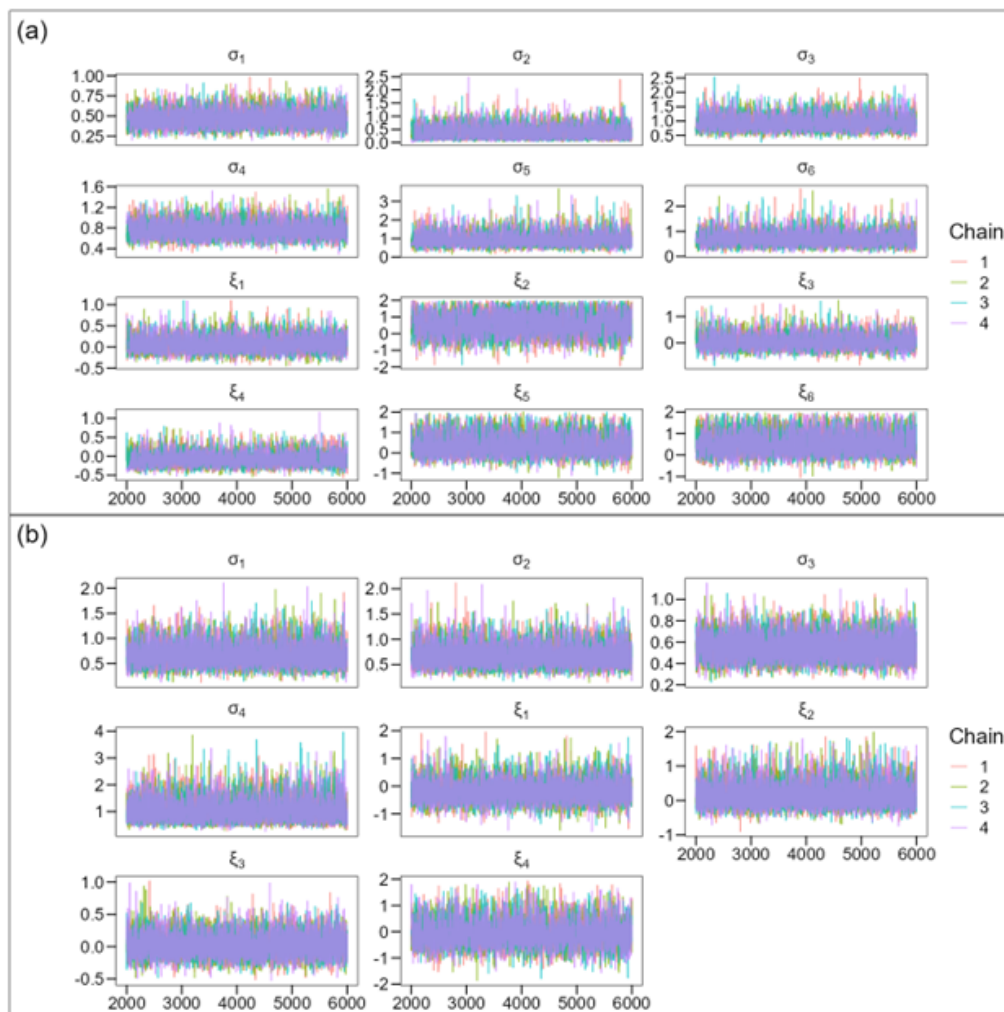


Fig. 7.5 Trace plot for hierarchical models (a) L-F- based models (b) site-based models.

After ensuring that sampling is properly done (with minimum to no autocorrelation) model fit is checked using posterior predictive checks where simulated data is compared with the observations [237]. To this end, samples were drawn from the posterior distribution and are used to generate data using the likelihood function. Predicted samples with 95% credible interval were plotted along with observed data. Fig.7.6 depicts the posterior

predictive check. The simulated data from the model were consistent with the observed data which implied that model fits the data well.

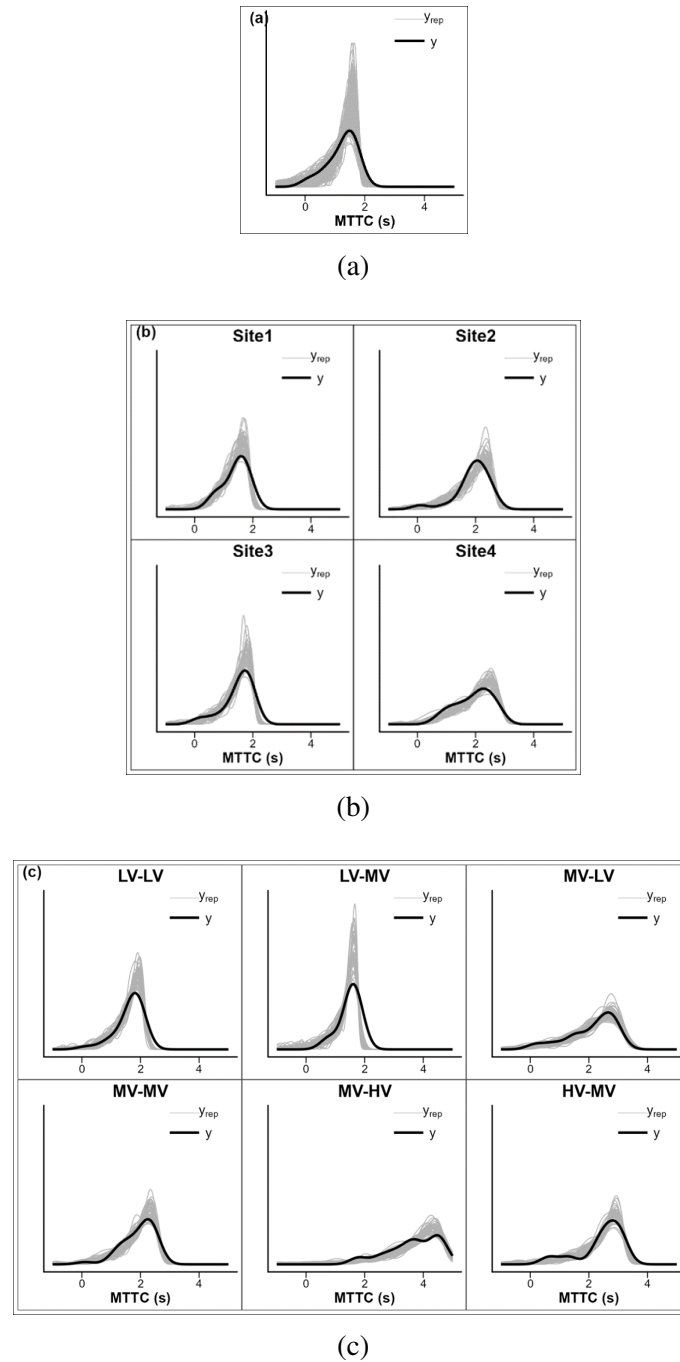


Fig. 7.6 Posterior predictive checks for (a) pooled model, (b) site-based hierarchical model, and (c) L-F-based hierarchical models.

Sensitivity analysis

Sensitivity of crash risk to threshold was performed. If the selected threshold is large enough, the exceedances will follow GPD with the same model parameters except for some sampling variability [172]. Since the model parameters remain almost constant, crash probability will also be comparable for different thresholds within the stable region [187, 218]. Threshold range presented in Table. 7.1 was used for crash risk comparison. Consider Threshold-1 and Threshold-2 as the upper and lower limits of the threshold range. The estimated crash probabilities with both the thresholds were compared as depicted in Fig.7.7. There is a considerable overlap between the 95% credible regions for crash risk estimated by the two thresholds. This implies that as long as the threshold is chosen within the range (stable region), crash risk will not be considerably different. Therefore, the lower limit of the threshold range was appropriate.

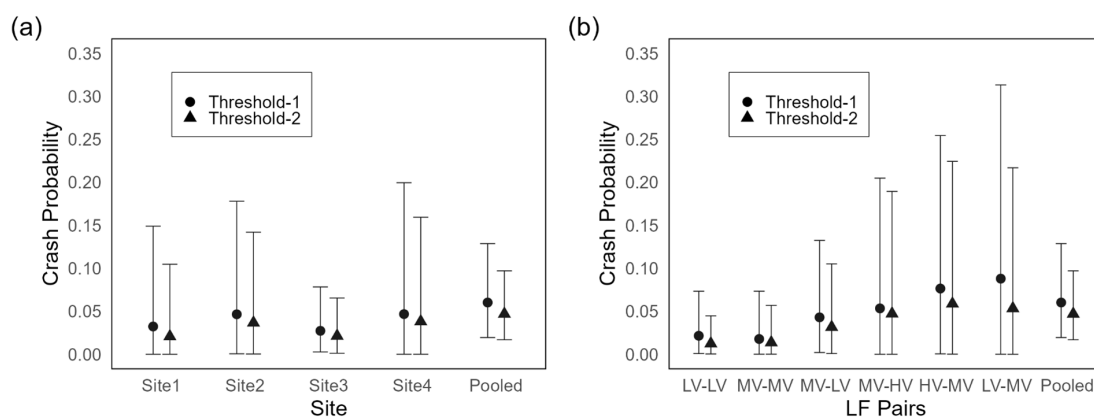


Fig. 7.7 Sensitivity analysis of crash risk (95% credible interval) with different thresholds (a) site-based models (b) L-F- based models.

7.6 Discussion

7.6.1 Conflict Identification in heterogeneous traffic

In heterogeneous traffic conditions, different vehicles have different behavior and maintain different gaps. MTTC varies for different L-F pairs, and interaction involving two and three-wheelers has lower MTTC than others (Fig.7.1). This can be attributed to the ability of two and three-wheelers to maintain smaller longitudinal gaps due to lower speeds and ease to manoeuvring. Global threshold cannot capture the same level of extreme event for all the L-F pairs. For some groups (interaction involving heavy vehicles), it hardly captures any extreme events; for others (light and medium vehicles), it classifies non-extreme events as conflict. Since two and three-wheelers maintain smaller gaps, normal interactions involving these vehicles will also be classified as conflict.

In a heterogeneous traffic, where traffic stream is composed of multiple vehicle types, this approach of conflict identification is not appropriate. Similar findings were reported by Kar, Venthuruthiyil, and Chunchu [177]. They used the GEV model for crash risk assessment on multilane rural highways in heterogeneous traffic. They pointed out problems with the block maxima approach, where the same block may not contain extreme events for all vehicle types. Therefore, in the present study, the threshold for each L-F pair was estimated separately.

7.6.2 Pooled versus hierarchical model

Vehicle composition and MTTC at the four sites were similar, indicating that the selected study sites are similar. When the population is homogeneous, the pooled model leads to more precise estimates (narrow CI) as in site-based models. The hierarchical model results in comparable model estimates but with a wider CI. However, when the population is heterogeneous, as in the L-F-based model, the pooled model leads to biased estimates

in both directions; it overestimates the crash risk for two and three-wheelers and underestimates for other vehicles. The hierarchical model leads to a more accurate estimate by incorporating the variability due to different vehicle types and sites (Fig.7.4). This finding is similar to Zheng et al. [228] where they showed that hierarchical models incorporating site-level heterogeneity performed better in estimating crash risk than separate models for each site. Based on hierarchical L-F based model results, crash risk varied across different leader and follower vehicles. Similar results were reported by Huang et al. [237], where they used crash-based models for studying the crash aggressiveness based on vehicle types using Bayesian hierarchical modeling approach.

7.6.3 Identification of critical vehicle groups

Crash risk assessment based on pooled model leads to a single crash probability of 0.05 for all vehicles assuming the crash risk to be homogeneous across all vehicle pairs. Identifying critical vehicle groups based on these models is impossible, as they predict the overall crash risk. Few studies which used pooled models have investigated the effect of vehicle composition on overall crash risk. These studies reported that with an increase in the composition of two and three-wheelers, crash risk increases, while with an increase in buses and heavy goods vehicles, crash risk decrease [89, 151]. Since the pooled model is usually fitted using a prespecified global threshold, increasing the proportion of two and three-wheelers would result in lower TTC or MTTC, increasing the number of conflicts and hence the crash risk. Similarly, increasing the proportion of heavy vehicles in the traffic stream would result in higher TTC or MTTC, reducing the number of conflicts and hence the crash risk. This may be the possible reason behind such findings from pooled model-based crash risk assessment which may not represent the actual crash risk.

Simultaneous analysis of conflict risk of multiple vehicle types in heterogeneous traffic using hierarchical models can help identify critical vehicle groups. This study showed a

higher crash risk when car and LCVs interact with other vehicles, as depicted in Fig.7.4. The possible reason may be that car and LCVs generally maintains a speed higher than other vehicles resulting in high-speed difference. Two and three-wheelers maintain the lowest speed in heterogeneous traffic, and interactions involving two and three-wheelers have the lowest crash risk. Crash data suggest a higher crash frequency for two and three-wheelers in low and middle-income countries. High crash frequencies of these vehicles may be attributed to their percentage composition in the traffic stream, which often attributed to their risky driving. In present study, composition of two and three-wheelers is around 55% at all sites. According to MORTH report in India, the share of motorized two-wheelers in Indian traffic is around 75%, and their crash involvement in total crashes is around 41%. The shares of car and LCVs in total traffic is around 14%, and their crash involvement is 19.7% [6]. Effectively car and LCVs are more likely to be involved in a crash than two and three-wheelers. The findings obtained from the proposed L-F-based hierarchical models corroborate these observations.

7.7 Chapter Summary

This chapter proposed a Bayesian hierarchical EVT model to account for vehicular and site-based heterogeneity in crash risk assessment. MTTC was used as a conflict indicator for quantifying rear-end conflicts. Data were collected at four uncontrolled T-intersections on four-lane divided highways in India. The extreme events were classified using threshold values obtained from threshold stability plots. Both multilevel and pooled models were fitted and compared. Three models were developed, namely, a Joint-site pooled model (Model-1), a model with site-level hierarchy (Model-2), and a Leader-Follower (L-F) based hierarchical model, which accounted for vehicular heterogeneity (Model-3). The posterior predictive check was performed to check the model's accuracy. The crash risk was computed for each vehicle type using the parameters (threshold, shape, and scale) of

the fitted model. Results from hierarchical model revealed that crash risk varied across different L-F pairs and sites. Due to similarity in sites (vehicle composition and road geometry), the variation in crash risk (0.025, 0.04) across sites was small. From the L-F-based hierarchical model, the crash risk was significantly different (0.02, 0.09) among L-F pairs, revealing the heterogeneity of vehicle types.