

CHAPTER – III

**DESIGN AND ANALYSIS  
OF EXISTING  
FRACTIONAL ORDER  
CONTROLLERS**

## INTRODUCTION

In control engineering, our motto is to design the controllers which will stabilize the given system according to users design specifications. In order to design the controller we have to compromise between the users demand and physical limits of the plant. The controller which is applied to plant is synthesized on the basis of a given parametric state of the system. Systems with the uncertain parameters are difficult to stabilize with the designed controller so the designed controller should have the capability of sensing the plant parameters variation in order to stabilize the plant.

Generally a control system is said to be robust when the open loop phase of the system is flat around the particular frequency interval this provides a constant overshoot of the time response to plant with gain like variations. On basis of this Oustaloup et al (1991) introduced fractional order algorithms for the control of the dynamic systems and demonstrated the superior performance of the CRONE controller (Commande Robuste d'Ordre Non Entier, Mean Non integer order Robust Control) technique over the traditional PID controller. He developed three generations of the crone controller namely First Generation Crone Control, Second Generation Crone Control and Third Generation Crone Control.

Some of the features of the CRONE controller are

- The design methods is purely based on frequency domain techniques using fractional order derivatives
- These controllers are applicable for continuous time and discrete time Single input single output systems (SISO) and Multi Input Multi Output Systems (MIMO).
- CRONE controllers proposed by Oustaloup et al (1991) assures the robustness in spite of plant parameters variations (Especially plant with gain like variations)
- Uses the basic unity feedback form
- Averting over-estimation of process disturbance, this provides non-conservative robust control systems and good performance as possible.

- It applicable for the plants of the type minimum phase or non minimum phase, unstable plants or plants with mechanical bending modes, time varying plants and non linear plants

Consider the plant with common unity feedback configuration as shown in Figure 3.1

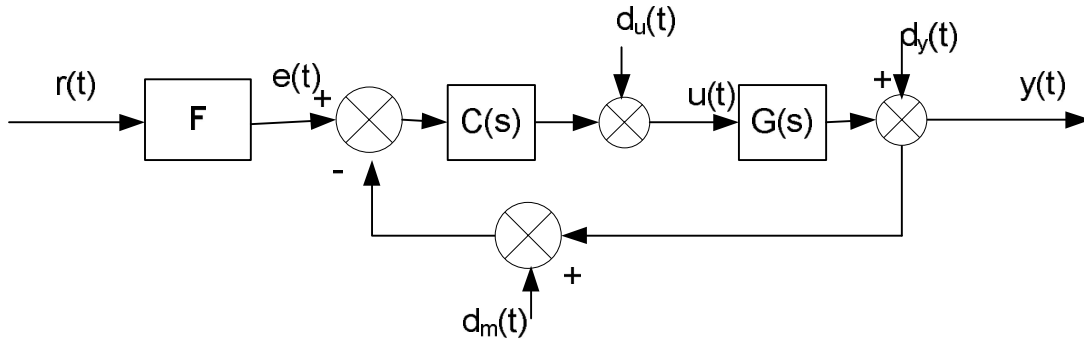


Figure 3.1- Block diagram of Control system

The other type of fractional controller called as fractional order Lead-Lag controller (FOLLC) proposed by Monje et al (2005) and Monje et al (2004). It is a generalization of the classical integer Lead-Lag controller. In frequency domain, FOLLC behaves similar to FOPID in some aspects. In order to design FOLLC simple relations between the parameters of the compensator are derived and specification like steady state error, phase margin ( $\varphi_m$ ) and gain cross over frequency ( $\omega_{cg}$ ) are achieved. Apart from above specifications robustness (flatness of phase curve) also fulfilled

The rest of the chapter is organized as follows: section 3.1 presents design methodology of first generation CRONE Controller and the same is applied to a system. The design technique of second-generation CRONE controller is presented in section 3.2 and shown the results with the help of Illustrative example. Section 3.2 provides the design of third generation CRONE controller and the results are shown with the help of Illustrative example. Next, an analytical design technique of fractional order Lead-Lag controller is presented in section 3.4, which also includes the simulation results. Finally, the conclusion is given in section 3.5.

### 3.1 FIRST GENERATION CRONE CONTROL

The first generation CRONE controller (Oustaloup et al (1991)) is applied for the plants with gain like uncertainties. The variation of open loop phase of the systems associated with the parametric variation of the plant. The purpose of the FIRST generation CRONE controller is to increase the phase margin of the plant by some suitable value. First generation CRONE controller is useful when the system to control is already having the constant phase around the gain cross over frequency. It provides the loop will be robust in face of plant gain variations, it assure the constant phase around the gain crossover frequency in spite of the plant gain variations.

The first generation CRONE controller is of the type

$$C(s) = C_0 s^\alpha \quad (3.1)$$

Where  $\alpha$  and  $C_0$  are the real numbers

Figure 3.2 represents the bode plot of the controller. This controller is valid in the frequency range  $(\omega_b, \omega_h)$  at the gain crossover frequency  $(\omega_{cg})$ . To implement this controller we can apply any approximation technique available in the literature whose frequency response fits the frequency response of the original fractional order controller in the frequency range  $(\omega_b, \omega_h)$ .

As it mentioned above it is applicable for the plants with constant open loop phase at gain crossover frequency. Since because of the control effort limitation is sometimes impossible to choose an open loop gain crossover frequency with in particular frequency range. Hence if the desired gain cross over frequency is out of the frequency interval, the crone controller cannot ensure the robustness of the closed loop system stability margins.

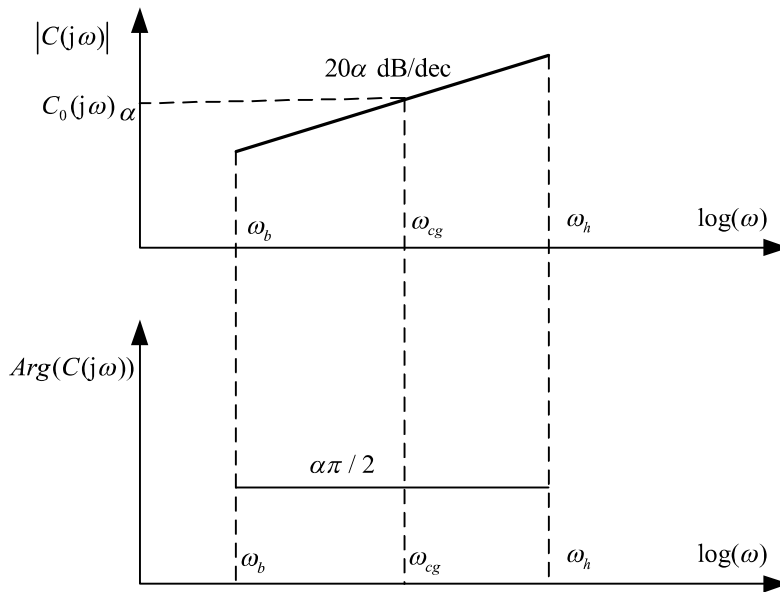


Figure 3.2- Bode plot of First Generation CRONE Controller  $C(s)$

### 3.1.1 ILLUSTRATIVE EXAMPLE

Consider the plant with perturbed gain

$$P(s) = \frac{k}{s(0.5s+1)} \quad \text{With } 3 \leq k \leq 7. \text{ The nominal plant is defined by } k=5$$

Give the designer specifications phase margin  $\varphi_m=50^\circ$  and gain crossover frequency  $\omega_{cg}=100\text{rad/sec}$

Using the methodology presented in Oustaloup et al (1991), the resultant First-generation CRONE controller as a rational function is

$$C(s) = \frac{2.465e07s^4 + 1.364e10s^3 + 7.173e11s^2 + 5.241e12s + 1e13}{s^5 + 5240s^4 + 7.172e06s^3 + 1.364e09s^2 + 2.465e10s}$$

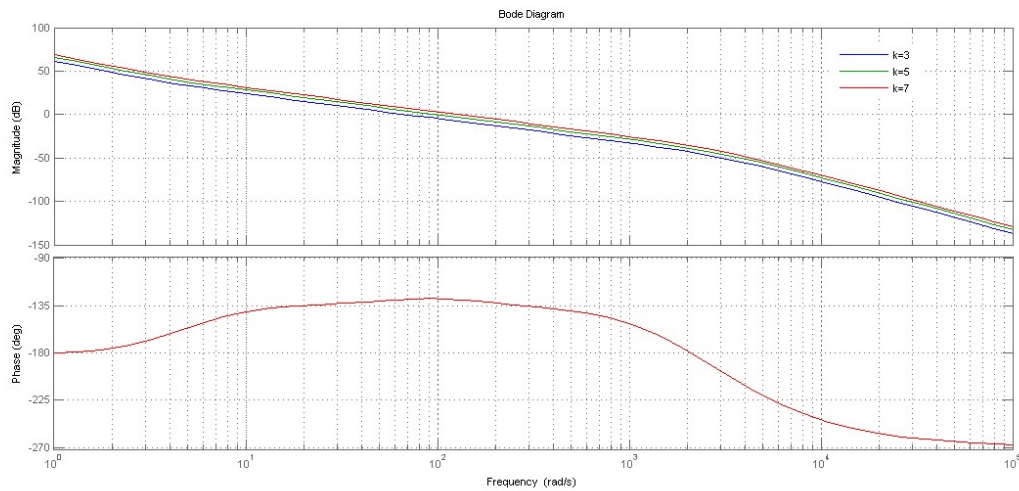


Figure 3.3- bode plot of the open loop system for  $kC(3,7)$ .

Figure 3.3 shows the frequency response of the open loop system with first generation CRONE controller. It is observed that the CRONE controller ensures to control the desired phase margin ( $\phi_m$ ) at the desired gain cross over frequency ( $\omega_{cg}$ ). Figure 3.3 shows the closed loop step response of the control system. The controller assures constant overshoot when the system is subjected to parametric variations. The controller shows the performance of robustness in front of frequency ( $\omega_0$ ) variations. It is clear that the robustness is much better by employing the first Generation CRONE controller.

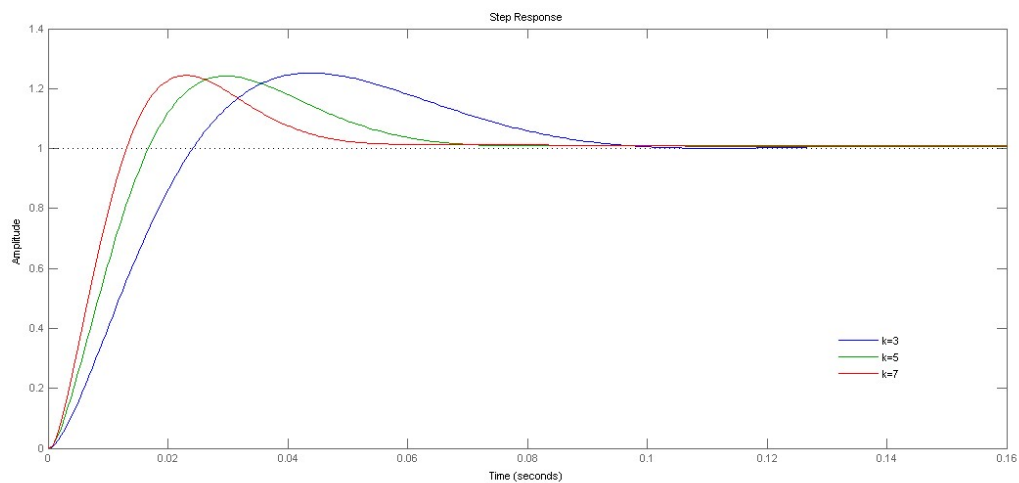


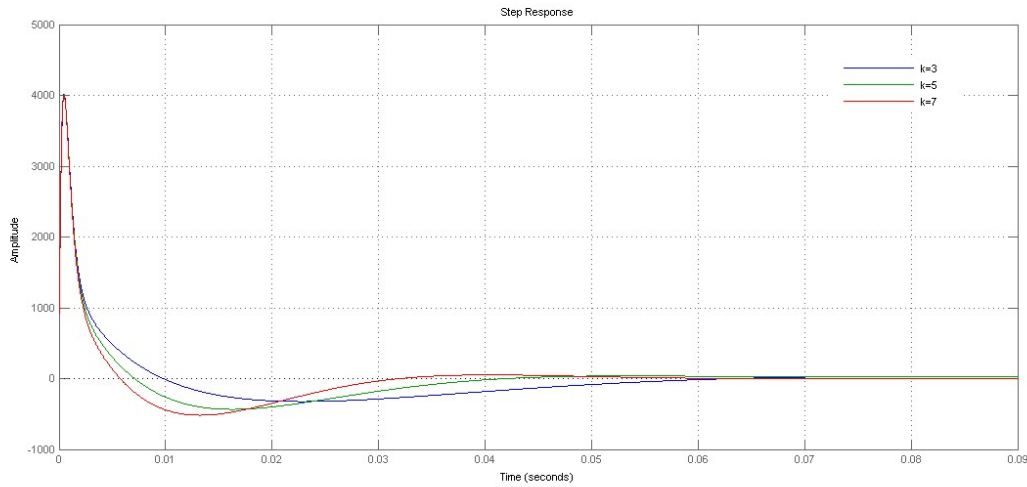
Figure 3.4- Step response of the plant with controller for  $kC(3,7)$ Figure 3.5-Step response of the controller output for  $kC(3,7)$ 

Figure 3.5 proves that first generation CRONE controller ensure the same sensitivity of the input of the plant in tracking and in regulation. Even if the robustness of the control system is very good by first generation CRONE controller it can be improved by using second generation CRONE controller by achieving a perfect phase locking of the open loop frequency response around the gain crossover frequency. It is possible to nullify the phase margin variations.

### 3.2 SECOND GENERATION CRONE CONTROLLER

The drawback of the first generation CRONE controller is that if plant does not have a constant phase with variation of gain this controller is invalid. However, it is possible to design a controller which assures a constant phase. When  $\omega_{cg}$  is within a frequency band where the plant uncertainties are gain like, the CRONE approach defines the open-loop transfer function, in the frequency band  $(\omega_b, \omega_h)$ , by that of a fractional order integrator

$$F(s) = C(s)P(s) = \left( \frac{\omega_{cg}}{s} \right)^\alpha \quad (3.2)$$

Where  $\alpha \in \mathbb{R}$  and  $\alpha \in [1, 2]$

Nichole chart of second generation CRONE control (Oustaloup et al (1991)) is showed in Figure 3.6. The vertical straight line, which is the desired shape of the open loop Nichols chart is called the template. In case of plant subjected to disturbances the vertical template assures the robustness of frequency domain specification and time domain specification such as phase margin, gain margin, resonant peak the damping ratio (which is related to closed loop poles).

Once the optimal Nichols chart is obtained, the fractional order controller is obtained from Equation (3.2) as

$$C(s) = \frac{F(s)}{P(s)} \quad (3.3)$$

By two ways we can approximate the fractional Order second generation Crone controller. One is by using Oustaloup recursive technique and another way to implement is to replace it by a rational order transfer function which has the same frequency response as  $\frac{F(s)}{P(s)}$  in frequency range of interest, using any frequency based identification technique.

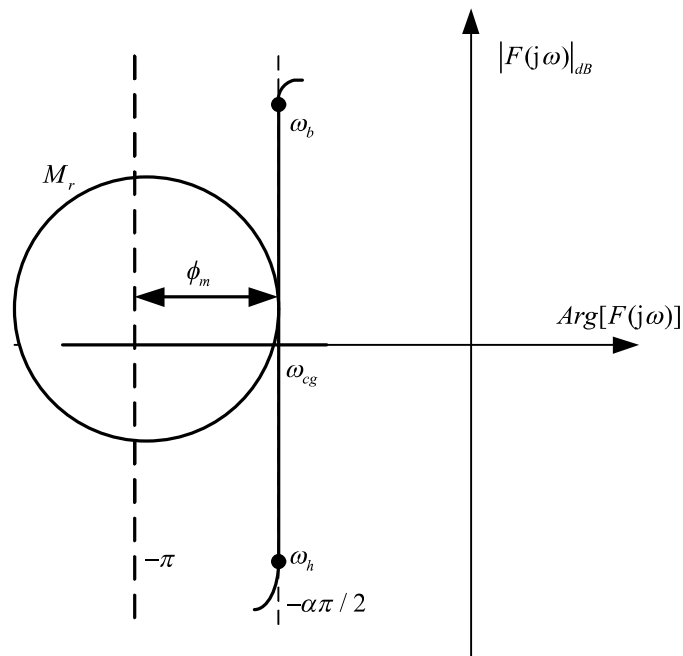


Figure 3.6- Nicholas chart of Second Generation CRONE Control

### 3.2.1 ILLUSTRATIVE EXAMPLE

Consider the plant with perturbed gain

$$P(s) = \frac{k}{s(0.5s + 1)} \quad \text{With } 3 \leq k \leq 7. \text{ The nominal plant is defined by } k=5.$$

Give the designer specifications phase margin  $\varphi_m=50^\circ$  and gain crossover frequency  $\omega_{cg}=100\text{rad/sec}$

Using the methodology presented in Oustaloup et al (1991), the resultant Second-generation CRONE controller as a rational function is

$$C(s) = \frac{7386s^6 + 1.725e07s^5 + 6.119e09s^4 + 3.875e11s^3 + 5.04e12s^2 + 1.858e13s + 2e13}{s^6 + 4289s^5 + 3.753e06s^4 + 6.084e08s^3 + 1.724e10s^2 + 7.385e10s}$$

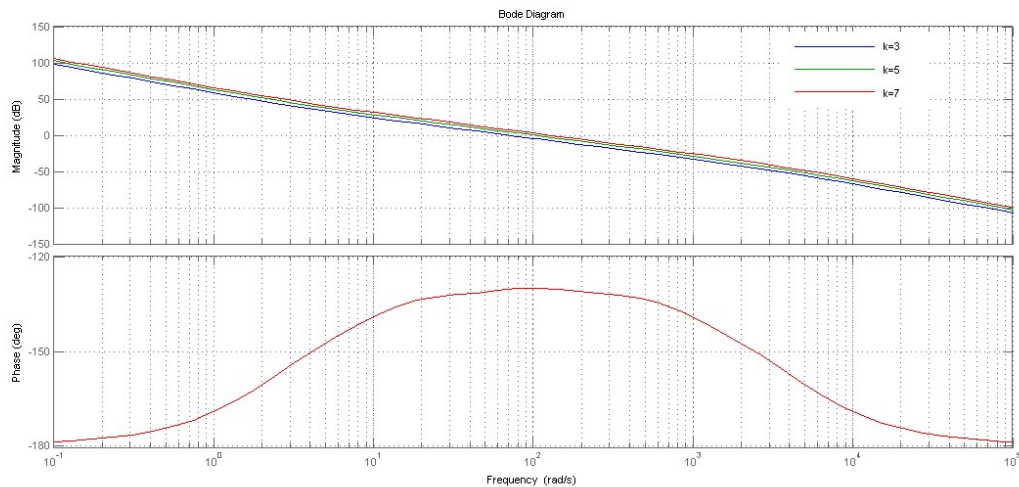
Figure 3.7- bode plot of the open loop system for  $kC(3,7)$ .

Figure 3.7 represents the open loop frequency response of the system with second generation CRONE controller. the results proves that CRONE controller successively assures the designer specification phase margin ( $\varphi_m$ ) at the desired gain cross over frequency ( $\omega_{cg}$ ), as the plant is under the influence of indifferent frequency behavior.

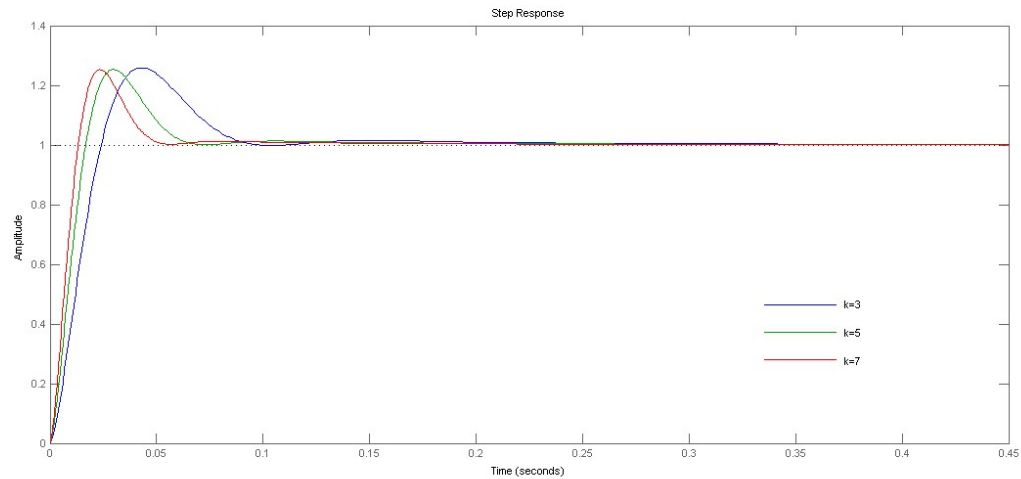


Figure 3.8-Step response of the plant with controller for  $k \in \{3, 7\}$

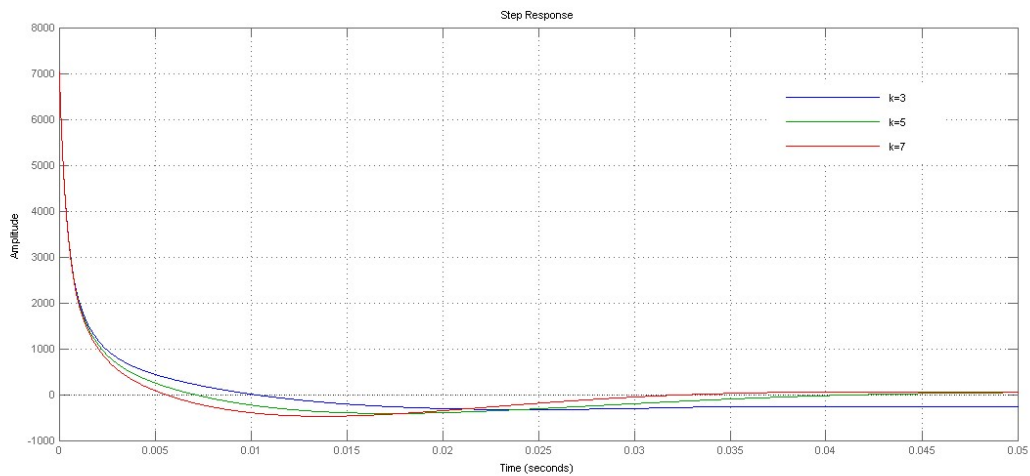


Figure 3.9-Step response of the controller output for  $k \in \{3, 7\}$

Figure 3.8 represents the closed loop step response of the control system. The controller assures the constant overshoot as system faces the parametric variations (gain like uncertainties).

Second generations CRONE controller offers some good robustness but at cost of performance. Here the rejection of input disturbance is slower and the sensitivity of the control effort to the measurement noise is greater. It is very important to manage well the tradeoff between robustness and performance while taking into account the main objectives of the specifications. A little loss of robustness can be necessary to ensure a good performance. The third generation CRONE controller over comes the same.

### 3.3 THIRD GENERATION CRONE CONTROL

Both first generation CRONE controller and second-generation CRONE controller are employed when the plant uncertainties are gain like, and they fail to assure robust for other types of uncertainties. The third generation CRONE controller (Oustaloup et al (1991)) employed when the plant uncertainties are of various types (including gain like uncertainties). The third generation CRONE controller tries to consider these cases.

Before designing the third generation CRONE controller, first we have to introduce the two important set of curves in the frequency response. These curves are of constant closed loop gain and curves of approximating constant damping ratio.

Let us consider the open loop systems

$$\Omega = |F(j\omega)| \tag{3.4}$$

$$\theta = \text{Arg}[F(j\omega)] \tag{3.5}$$

The closed loop will be given by

$$L|(j\omega)| = \frac{F(j\omega)}{1 + F(j\omega)} \tag{3.6}$$

And therefore, after the simple calculations

$$|L(j\omega)| = \left| \frac{\Omega e^{j\theta}}{1 + \Omega e^{j\theta}} \right| = \frac{\Omega}{\sqrt{1 + \Omega^2 + 2\Omega \cos \theta}} \quad (3.7)$$

Nichols curves of constant values of  $|L(j\omega)|$  are shown in Figure 3.10. These curves maintain the periodicity of  $2\pi$  rad in the phase axis and are symmetrical with respect to all vertical straight lines given by  $x = k\pi$ ,  $k \in Z$ . Figure 3.11 represents curves of constant values of the damping ratio in Nichols chart. These curves maintain the periodicity of  $2\pi$  rad in the phase axis and are symmetrical with respect to all vertical straight lines given by  $x = k\pi$ ,  $k \in Z$ .

Now we can proceed to design the third generation CRONE controller that assures the closed loop gain limit at certain value even the parameters of the system vary in certain interval and controller assures the closed loop-damping ratio will never get below a certain value, for a known range of variations of some plant parameters.

Once the desired frequency domain specifications are known the Third generation CRONE controller is designed.

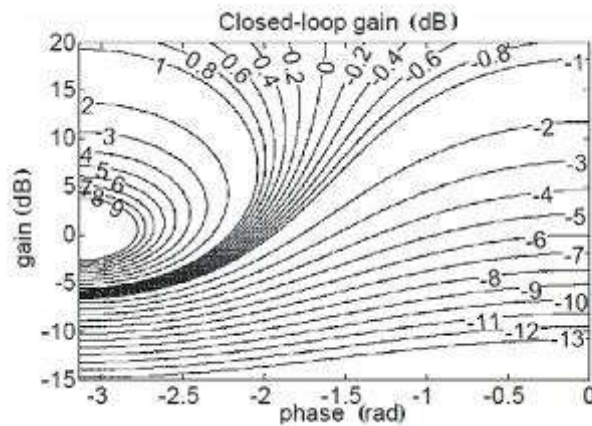


Figure 3.10 curves of constant values of closed loop gain in Nicholas chart

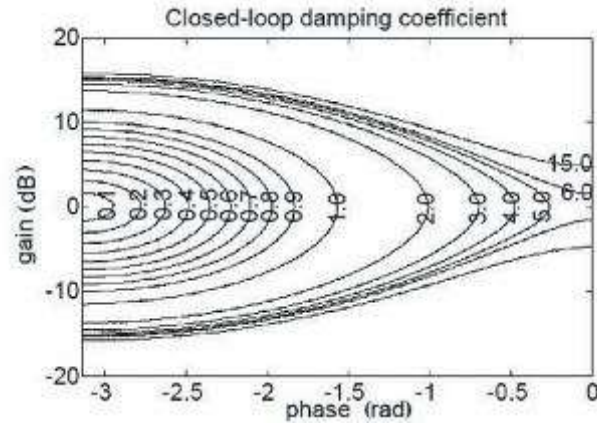


Figure 3.11 curves of constant values of closed loop damping ratio in Nicholas chart

### 3.3.1 ILLUSTRATIVE EXAMPLES

Consider the plant with perturbed gain

$$P(s) = \frac{k}{s(0.5s + 1)} \quad \text{With } 3 \leq k \leq 7. \text{ The nominal plant is defined by } k=5.$$

Give the designer specifications phase margin  $\phi_m=50^\circ$  and gain crossover frequency  $\omega_{cg}=10\text{krad/sec}$

Using the methodology presented in Oustaloup et al (1991), the resultant Third-generation CRONE controller as a rational function is

$$C(s) = \frac{2.263e13s^2 + 6.169e15s + 4.204e17}{s^3 + 5.674e05s^2 + 8.05e10s}$$

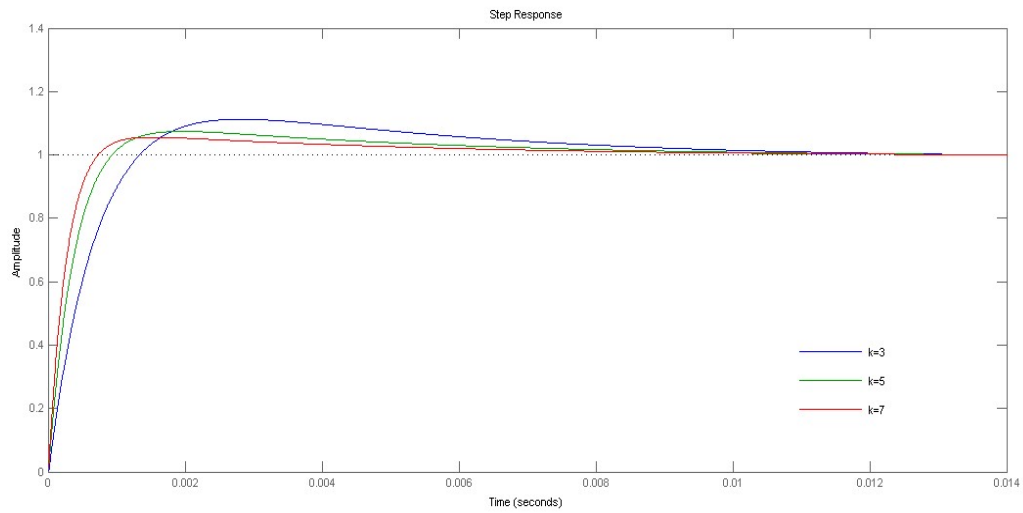


Figure 3.12-Step response of the plant with controller for  $k\mathcal{C}(3,7)$

From the Figure 3.12 it is observed that the controller responds to output disturbance are quick when compared to the CRONE second generation controller. The purpose of the controller is to completely reject the output disturbances in steady state with robustness.

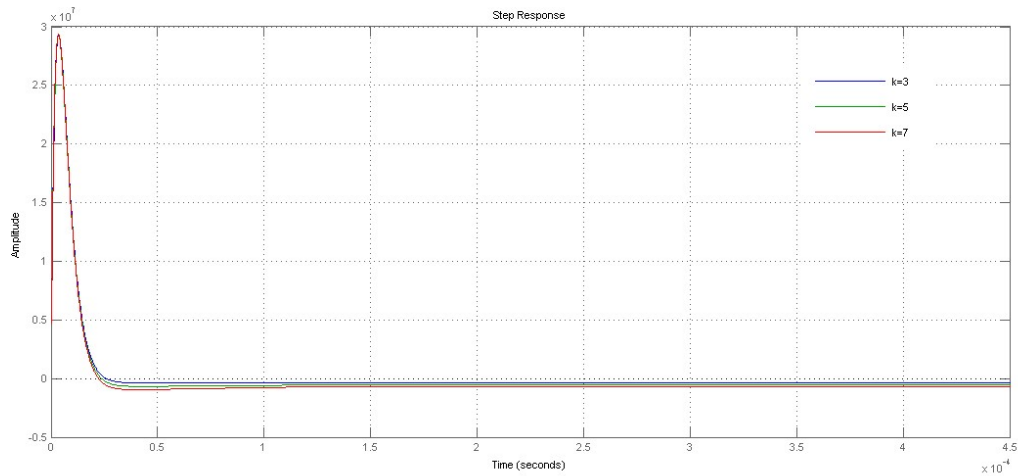


Figure 3.13-Step response of the controller output for  $k\mathcal{C}(3,7)$

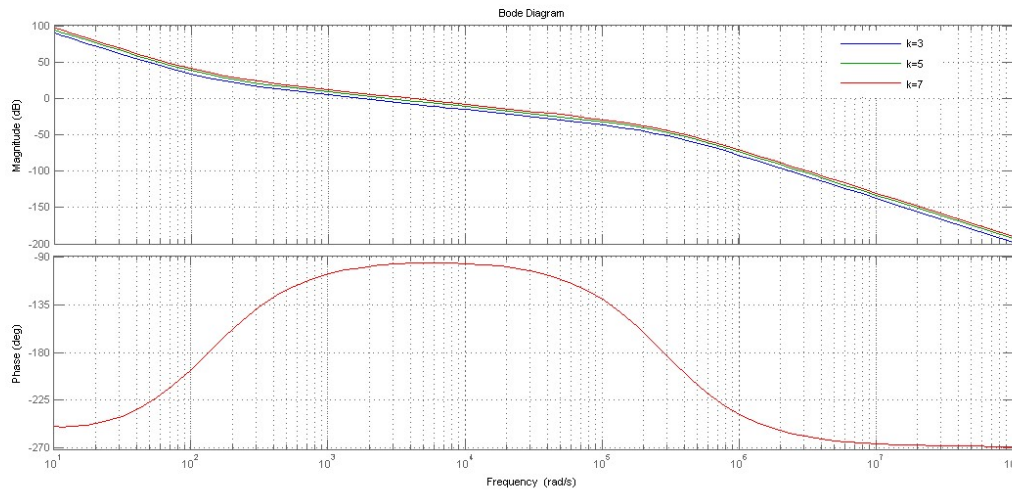


Figure 3.14- bode plot of the open loop system for  $k \in (3,7)$ .

Figure 3.14 represents the open loop frequency response of the system with third generation CRONE controller. The results prove that the CRONE controller successfully ensures the designer's specification phase margin ( $\varphi_m$ ) at the desired gain crossover frequency ( $\omega_{cg}$ ), as the plant is under the influence of indifferent frequency behavior. Figure 3.15 represents the closed loop step response of the control system. The controller ensures a constant overshoot as the system faces parametric variations (gain-like uncertainties).

### 3.4 FRACTIONAL ORDER LEAD-LAG CONTROLLER

The FOLLIC (Monje et al (2005)) transfer is of the type

$$C(s) = K_c \left( \frac{s + \frac{1}{\lambda}}{s + \frac{1}{x\lambda}} \right)^\alpha = K_c x^\alpha \left( \frac{\lambda s + 1}{x\lambda s + 1} \right)^\alpha, \quad 0 < x < 1 \quad (3.8)$$

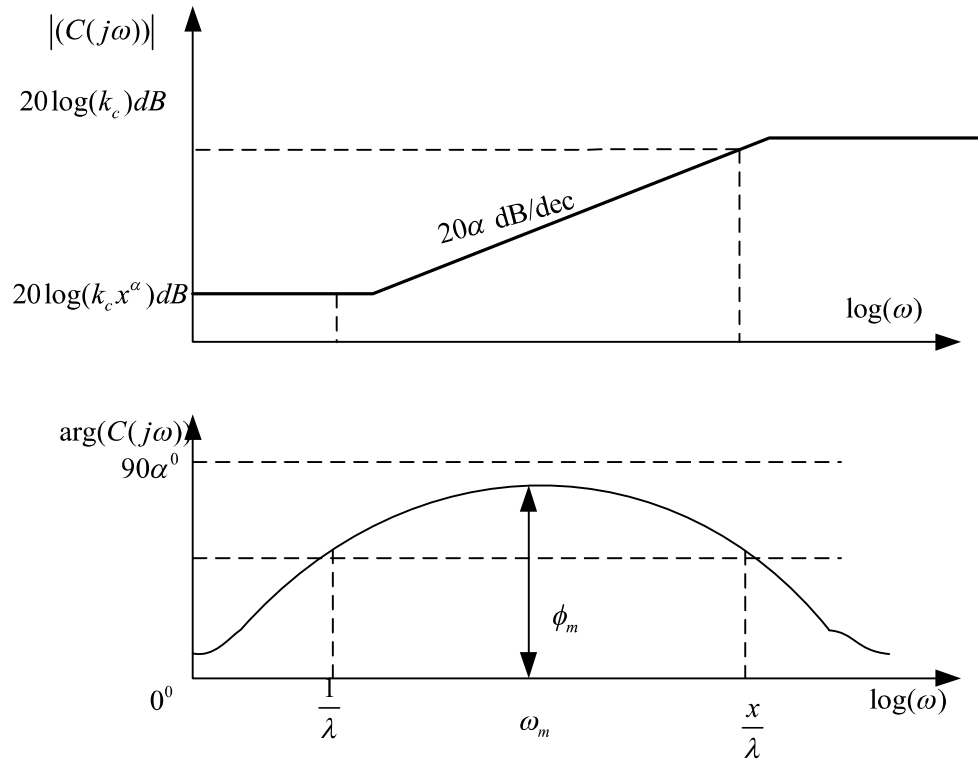
Where  $\alpha$  is the fractional order of the controller

$1/\lambda = \omega_{zero}$  is the zero frequency (when  $\alpha < 0$ ).

$1/(x\lambda) = \omega_{pole}$  is the pole frequency (when  $\alpha > 0$ ).

It is to be noted that the FOLLC corresponds to Lead controller when ( $\alpha > 0$ ) and  $0 < x < 1$  and FOLLC is corresponds to Lag corresponds when ( $\alpha < 0$ ) and  $0 < x < 1$ .  $x \in [0,1]$  in both cases. Fractional order lead controller behaves similar to a  $PD^\mu$  and Fractional order lag controller behaves similar to a  $PI^\lambda$ . The first step for a latter generalization of these structures to the fractional-order  $PI^\lambda D^\mu$  controller would be overcome.

The open loop frequency response of fractional order lead compensator when  $\alpha > 0$  is shown in Figure 3.15. For the lag controller when  $\alpha < 0$  the slope of the magnitude plot is negative and controller adds a phase lag. As can be seen in the Figure 3.15, the value of  $x$  sets the distance between the fractional zero  $\omega_{zero}$  and pole  $\omega_{pole}$ , the value of  $\lambda$  sets their position in the frequency axis, and the two values( $x, \lambda$ ) depend on  $\alpha$ . The higher value of  $\alpha$  provides the higher slope of the magnitude of FOLLC and the higher the maximum phase  $\phi_m$  that the FOLLC can give. In the case of integer order lead-lag controller the frequency  $\omega_m$  is the geometric mean of the corner frequency  $\omega_{zero}$  and  $\omega_{pole}$ , and its equation is given by  $\omega_m = \frac{\sqrt{x}}{\lambda}$ . The fractional parameter  $\alpha$  plays a crucial role in FOLLC. The lower the value of  $\alpha$ , the longer the distance between the zero and pole and vice versa, so that the contribution of phase at a certain frequency stands still. This concept provides FOLLC to be more flexible and assures robustness in the design

Figure 3.15-Bode plot of FOLLC( $\alpha > 0$ )

### 3.4.1 ILLUSTRATIVE EXAMPLE

The system to control is given by

$$P(s) = \frac{5}{s(0.1s + 1)}$$

The gain crossover frequency will be  $\omega_{cg} = 30 \text{ rad/sec}$ . At this frequency, the plant has a magnitude of  $-25.1188 \text{ dB}$  and a phase of  $-161.65^\circ$ . The velocity error constant will be set to  $K_v = 20$ , and the phase margin to  $\phi_m = 50^\circ$ . As can be observed, a lead compensator is needed in this case.

The FOLLC is designed by applying the method presented in Monje et al (2005) the

$$C(s) = 3 * \left( \frac{0.78s + 1}{0.0045s + 1} \right)^{0.51}$$

At the gain crossover frequency  $\omega_{cg} = 30$  rad/sec, the compensator has a magnitude of 26.1188 dB and a phase of 36.65. At that frequency the magnitude of the open-loop system  $F(s)$  is 0 dB, and the phase margin obtained is approximately  $50^\circ$ , as shown in Figure 3.16. Therefore, the specifications are correctly fulfilled. For the sake of implementation, this compensator has been approximated by using an inverse frequency identification method. This way, an integer-order transfer function is obtained that fits the frequency response of the FOLLC in a frequency range of interest.. The resulting step response of the closed-loop system using this controller approximation is shown in Figure 3.17.

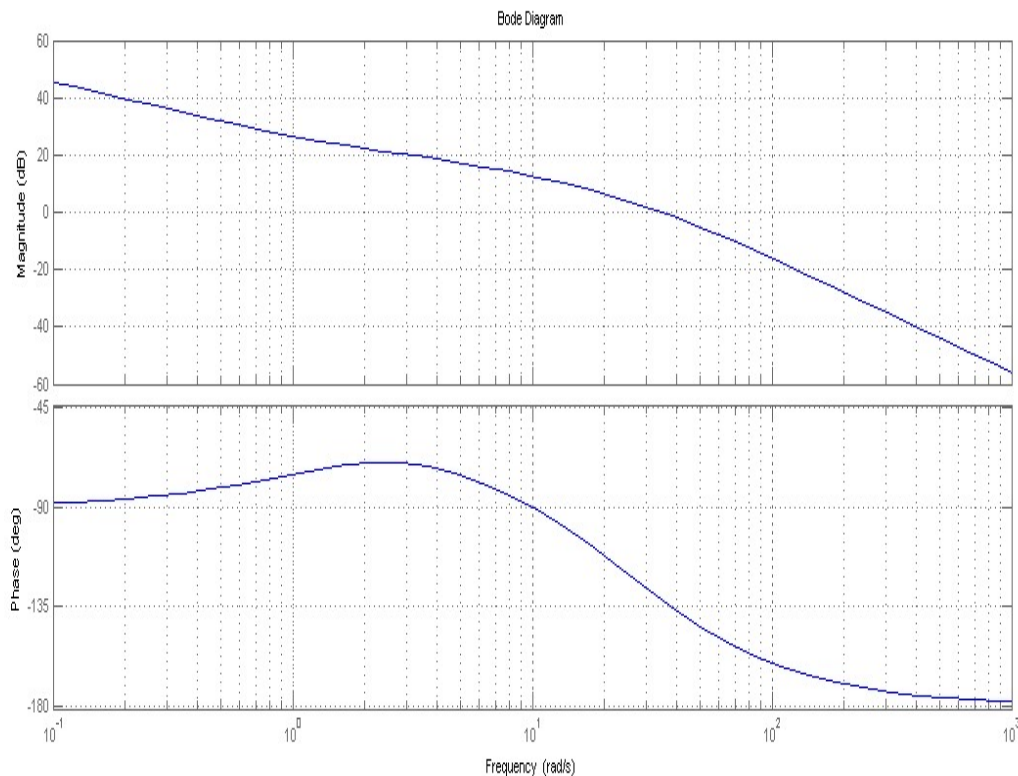


Figure 3.16- Frequency response of the open loop system with FOLLC

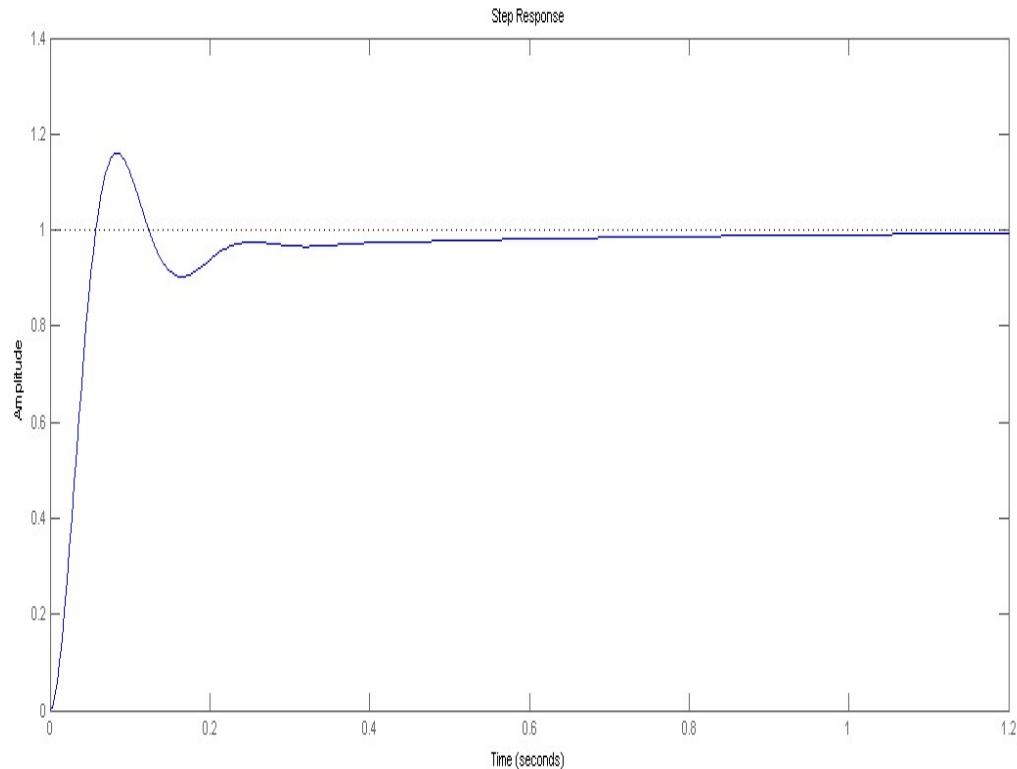


Figure 3.17- Step response of the system with FOLLC

### 3.5 CONCLUSION

This chapter gives a brief discussion on three generations of CRONE controller and presents their main design goals in frequency domain. The first generation CRONE controller ‘robustifies’ stability by reducing the phase variations in the open loop frequency response, It provides the constant phase around the gain crossover frequency. The second-generation CRONE controller improves the robustness by achieving a perfect phase locking of the open loop phase around the gain crossover frequency, thus avoiding the phase variations. The design methodology is based on an indifferent distribution of zeros and poles. This fills phase gap between desired phase and original phase. The time response keeps the constant overshoot under the gain variations of plant. The advantage of third generation CRONE controller lies in the fact

that it takes into account not only the genuine uncertainties of the plant, but also all types of the uncertainties.

Here the other fractional controller Fractional order Lead–Lag controller has been designed and analyzed with the help of example. The FOLLC fulfills the specifications like steady state error, phase margin, gain crossover frequency and robustness criteria (based on the flat phase around the gain crossover frequency).