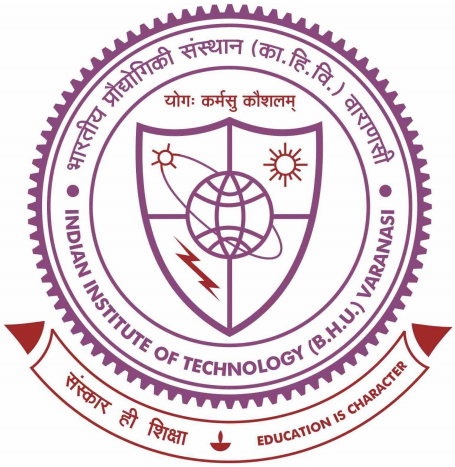
Asymptotic Behaviour of Certain Lambert Series Associated to Automorphic Forms and Moments of Divisor Function



Thesis submitted in partial fulfillment

for the Award of Degree

DOCTOR OF PHILOSOPHY

by

Babita

DEPARTMENT OF MATHEMATICAL SCIENCES

INDIAN INSTITUTE OF TECHNOLOGY

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It is certified that the work contained in this thesis titled "***Asymptotic behaviour of certain Lambert series associated to automorphic forms and moments of divisor function***" by ***Babita*** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of the Comprehensive Examination, Candidacy and SOTA for the award of a Ph.D. degree.

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I, ***Babita***, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of ***Dr. Abhash Kumar Jha*** from ***July 2019*** to ***July 2024*** at the ***Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi***. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports, dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

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Dedicated to my Mother

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Babita

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Symbols

$\mathbb{N}$	Set of natural numbers
$\mathbb{Z}$	Set of integers
$\mathbb{Q}$	Set of rational numbers
$\mathbb{R}$	Set of real numbers
$\mathbb{C}$	Set of complex numbers
$\Re(z)$	Real part of the complex number z
$\Im(z)$	Imaginary part of the complex number z
$\mathbb{H}$	Complex upper half-plane
$\mathbb{C}^*$	$\mathbb{C} \setminus \{0\}$
$\exp(z)$	e^z
$e(z)$	$\exp(2\pi i z)$
$a\|b$	a divides b
(a, b)	Greatest common divisor of a and b
$a \equiv b \pmod{c}$	$c\|a - b$
A^t	Transpose of a matrix A
0_n	$n \times n$ zero matrix
I_n	$n \times n$ identity matrix
$\det(A)$	Determinant of a square matrix A
$tr(A)$	Trace of a square matrix A
$\square$	End of the proof