

Certificate

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Declaration

I, **Neelam Singh**, certify that the work embodied in this thesis is my own bona-fide work and carried out by me under the supervision of **Dr. Gauhar Abbas** from **July 2019 to January 2025** at the **Department of Physics**, Indian Institute of Technology (BHU), Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers whenever and wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any others' work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, theses, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

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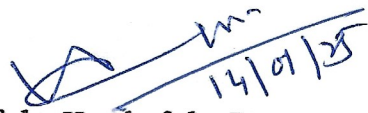
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"It is good to have an end to journey toward; but it is the journey that matters, in the end." – Ursula K. Le Guin

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Neelam Singh

Abstract

In this thesis, we discuss the investigation of solutions to the flavour problem of the standard model based on newly proposed ideas. These ideas include the Froggatt-Nielsen mechanism based on the $\mathcal{L}_N \times \mathcal{L}_M$ flavour symmetry, and an atypical solution based on the VEVs hierarchy. The latter solution is called the Standard Hierarchical VEVs Model, where the different fermionic mass scales are addressed in terms of different order parameters originating from the hierarchical VEVs. We first present a comprehensive and thorough review of the flavour problem, tracing back to the time when the standard model was just shaping into a robust framework. Following this, we extensively discuss various standard mechanisms and paradigms which are capable of providing a solution to the flavour problem. After this, we discuss the solution of the flavour problem based on the $\mathcal{L}_N \times \mathcal{L}_M$ flavour symmetry. This symmetry allows us to implement the Froggatt-Nielsen mechanism without introducing a gauged continuous $U(1)$ symmetry, thus, resulting in different phenomenological signatures. The phenomenology of the models based on the $\mathcal{L}_N \times \mathcal{L}_M$ flavour symmetry is investigated by deriving the bounds from quark as well as lepton flavour physics data. Moreover, collider signatures of different $\mathcal{L}_N \times \mathcal{L}_M$ flavour symmetries are presented at three different hadron colliders, the high-luminosity Large hadron Collider, the high-energy Large hadron Collider, and a 100 TeV future collider. The flavour bounds on the parameter space of the Standard Hierarchical VEVs model are derived, and the phenomenology of the ALPs of the Standard Hierarchical VEVs model is discussed. Furthermore, we have investigated collider signatures of the Standard Hierarchical VEVs model, and have shown that a recent 95.4 GeV excess reported by the Large Hadron Collider can be easily accommodated within the framework of Standard Hierarchical VEVs model.

List of Publications

- [1] G. Abbas and N. Singh, [arXiv:2412.08523 [hep-ph]].
- [2] G. Abbas, A. K. Alok, N. R. S. Chundawat, N. Khan and N. Singh, Phys. Rev. D **110** (2024) no.11, 115015 doi:10.1103/PhysRevD.110.115015 [arXiv:2407.09255 [hep-ph]].
- [3] G. Abbas, R. Adhikari, E. J. Chun and N. Singh, Eur. Phys. J. Plus **140** (2025) no.1, 73 doi:10.1140/epjp/s13360-025-06008-6 [arXiv:2308.14811 [hep-ph]].
- [4] G. Abbas, V. Singh, N. Singh and R. Sain, Eur. Phys. J. C **83** (2023) no.4, 305 doi:10.1140/epjc/s10052-023-11471-5 [arXiv:2208.03733 [hep-ph]].

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- [5] G. Abbas, V. Singh and N. Singh, Springer Proc. Phys. **304** (2024), 1228-1230 doi:10.1007/978-981-97-0289-3_346
- [6] G. Abbas and N. Singh, doi:10.31526/ACP.BSM-2023.3 [arXiv:2404.03232 [hep-ph]].

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