7 SURFACE MINER TOTAL FACTOR PRODUCTIVITY

7.1 Introduction

Production, defined broadly, entails creating new goods and services through the utilisation of resources (Eğilmez, 2016; Songur and SaraçElmas, 2017). Resources termed as production factors, enable the production process. The transformation of these production factors into output is symbolised by a mathematical expression known as production function in neoclassical economics. Economic theory describes the production function as a technical relationship between production factors, such as labour, capital and technology, and output (Philip Wicksteed 1894; Cheng & Han, 2017). In alignment with the neoclassical tradition, this study excludes 'energy' as a production factor due to its perception as an intermediate product.

Nevertheless, in recent years a lot of efforts have been put to define and quantify productivity and identify the relationship between the constituent elements of the total factor productivity. These efforts mostly concentrated on developing either partial labour productivity or machine utilisation indices. Labour productivity, gauged in tonnes per labour shift time, is most commonly used in the coal mining industry as a measure of changes in labour efficiency of the coal production process. The productivity of a mining method in an Indian mine is measured using output per man shift (OMS) index, as explained in Equation 7.1.

$$Q = f(\mathrm{MS}, \mathrm{OMS}) \qquad \text{(Equation 7.1)}$$

where, Q is the coal production (t), MS is the man shift time (hr), OMS is the output per man shift as a positive constant.

However, this productivity index has drawn criticism from several researchers (Salter, 1960; Good *et al.*, 1997; Coelli *et al.*, 1998). As a single factor productivity index, the observed change in productivity might result from a different mix of production factors utilisation and/or other modifications in the production environment. Discrepancies in OMS may stem from variations in capital investment or production scale rather than differences in the effectiveness of 'labour' input (Beri, 1982).

For instance, although OMS as a productivity index in the Indian coal mining industry has exhibited a consistent upward trend, particularly since the 1980s, this increase cannot solely be attributed to enhanced "labour" productivity. Due to rising production targets during this period, policy directives focused predominantly on bolstering production through capital accumulation (BICP, 1988). It directed large-scale investments towards the mechanisation of mining operations by adopting foreign technologies (Chari, 1988). Therefore, selection of a function to appropriately capture the underlying production process and selection of the independent variables used in a function to capture all the actual production drivers become crucial for evaluating total factor productivity.

Machine shift time (MST) is identified as the most commonly used input to develop machine utilisation index, as a measure of changes in machine productivity for the production of coal in opencast mines (Singh *et al.*, 2023a). However, MST-based machine utilisation index indicated changes only in machine utilisation efficiency and did not account for changes in machine cost efficiency. This scenario exposed the research gap for the requirement of economic analysis of SM to comprehend the relationship between its economic inputs and output.

Economic analysis of excavators can be computed in terms of ownership and operating (O&O) costs using Caterpillar method (Caterpillar, 2008), Corps of Engineers method (Atcheson, 1993; US Army, 2003), Association of General Contractors of America

(AGC) method (Popescu, 1992), and Peurifoy method (Peurifoy *et al.*, 2006). None of these methods provide exact O&O costs of SM due to the numerous variables involved and the uncertain nature of mining. Nonetheless, Finch and Fidler (1981), Schimm (1997), Aykul *et al.* (2007), Pathak and Rodkar (2011), and Nolan and Kecojevic (2014) have attempted to assess the O&O costs of SM using various modified approaches.

In this study, machine shift cost (MSK) is introduced as a production factor, calculated in terms of O&O costs for a 24-hour scheduled shift of SM using modified 'Corps of Engineers' method and gauged in lac ₹. In this method, except for the one-time initial capital cost of machine purchase, ownership cost is considered as a fixed cost to incur annually, irrespective of machine usage. It comprised expenses related to depreciation, interest, taxes, and insurance on the initial capital cost of SM. Conversely, operating cost is considered as a variable cost to incur daily during machine usage. It comprised expenses related to repair and maintenance, fuel, lubricating oil, grease, manpower, picks, holders and administrative overhead. Primary data sources for calculating O&O costs included coal production schedules, sales invoices and balance sheets related to SM.

SM total factor productivity is evaluated as a positive index based on production factors utilised in the production process. Two production factors, namely, machine shift time (MST) as a 'labour' input and machine shift cost (MSK) as a 'capital' input, along with the coal production (Q) as a net output of SM are aggregated on the monthly basis. A dataset from 45 field-scale SM coal production trials is analysed to develop a production function based on machine shift time (MST) and machine shift cost (MSK). A confirmation test based on root mean square error (RMSE) exhibited that the Cobb-Douglas production function (CDPF) has the highest predictive accuracy, outperforming the bivariate exponential production function (BEPF) at 0.05 significance level.

Moreover, study revealed that the SM coal production adhered to the CDPF relationship with increasing returns to scale, suggesting when MST and MSK increase, SM coal production rises at an accelerating rate. Different isoquant curves representing a unique production level for each curve are also observed to develop coal production classification system based on SM total factor productivity during the quantitative modelling of SM coal production.

7.2 Production functions

Equation 7.2 represents the coal production model based on machine shift time (MST) as a 'labour' input using bivariate exponential production function (BEPF). It is used to understand the relationship between coal production and machine shift time related to SM in opencast mines.

$$Q = A_0 \times (MST)^{\alpha} \qquad \text{(Equation 7.2)}$$

where, Q is the coal production (t), MST is the machine shift time (h), A_0 is the productivity parameter, α is the output elasticity with respect to 'labour', and A_0 and α are positive constants.

Equation 7.3 shows the extended form of a BEPF model, i.e., the Cobb-Douglas production function (CDPF) model based on the machine shift time (MST) as 'labour' input and the machine shift cost (MSK) as 'capital' input to measure SM total factor productivity in Indian opencast coal mines.

$$Q = A \times MST^{\alpha} \times MSK^{\beta} \qquad \text{(Equation 7.3)}$$

where, Q represents the coal production (t), A signifies the total factor productivity, MST is the machine shift time (h), MSK represents the machine shift cost (lac ₹), α and β signifies output elasticities to labour and capital respectively, and A, α and β are positive constants.

The prevalent method for measuring total factor productivity based on production factors related to a production function entails the linear, logarithmic transformation of Equations 7.4 and 7.5 before applying multiple linear regression.

$\ln(Q_{it}) = \beta_0 + \alpha \ln(MST_{it}) + \mu_{it}$ (Equation 7.4)

$\ln(Q_{it}) = \beta_0 + \alpha \ln(MST_{it}) + \beta \ln(MSK_{it}) + \mu_{it}$ (Equation 7.5)

where, β_0, α, β are parameters, μ_{it} is the error term and $\beta_0 = \ln(A)$.

An extra experiment was undertaken to bolster confidence in the study results by developing a coal production model based on MSK using BEPF, as given in Equations 7.6 and 7.7.

$Q = A_0 \times (MSK)^{\beta}$ (Equation 7.6)

$\ln(Q_{it}) = \beta_0 + \beta \ln(MSK_{it}) + \mu_{it}$ (Equation 7.7)

where, Q represents the coal production (t), A_0 is the total factor productivity parameter, MSK is the machine shift cost (lac ₹), β_0 and β are parameters, and μ_{it} is the error term.

The coefficient of determination (R^2), F-statistic, t-statistic, and p-level values of the regression model were used to indicate the model's goodness of fit. The plot between predicted and actual values of coal production was used to validate SM coal production regression models. Lastly, A confirmation test based on root mean square error (RMSE), frequently utilised for the robust assessment of model fit on data, was employed to compare the efficacy of developed regression models. RMSE was calculated using Equation 7.8.

$\mathrm{RMSE} = \frac{1}{N}\sqrt{\sum_{i=1}^{N}\left(\frac{A_i - E_i}{A_i}\right)^2}$ (Equation 7.8)

where, A_i is the actual value measured for sample 'i', E_i is the predicted value by the model for sample 'i' and N is the project sample size.

7.3 Research data

Table 7.1 details the dataset of 45 field-scale SM coal production trials to measure total factor productivity of SM in opencast coal mines.

Table 7.1: Input dataset to measure total factor productivity of SM in opencast coal mines

S. N.	**Sample**	**Q (t)**	**MST (h)**	**PL (n)**	**DL (l)**	**O&O (₹/t)**	**MSK (lac ₹)**	**ln Q**	**ln MST**	**ln MSK**
1	P1-a	171892.5	279.5	112	18564.4	21.09	36.25	12.05	5.63	3.59
2	P1-b	39600	180	38	7326	59.65	23.62	10.59	5.19	3.16
3	P1-c	36115.2	158.4	36	7764.8	66.15	23.89	10.49	5.07	3.17
4	P2-a	134531.64	225.8	83	12565.3	19.82	26.66	11.81	5.42	3.28
5	P2-b	34916.4	190.8	25	5876.4	62.59	21.85	10.46	5.25	3.08
6	T3-a	249541.48	357.1	120	25453.2	18.36	45.82	12.43	5.88	3.82
7	T3-b	174278.8	364.6	85	17776.4	21.97	38.29	12.07	5.9	3.65
8	T3-c	68027	230.6	68	10544.2	42.17	28.69	11.13	5.44	3.36
9	B_m2-a	239346.28	357.1	132	20344.4	19.18	45.91	12.39	5.88	3.83
10	B_m3-b	185068.8	288	109	17581.5	21.79	40.33	12.13	5.66	3.7
11	B_m4-d	161179.2	302.4	105	17729.7	26.31	42.41	11.99	5.71	3.75
12	B_m5-b	162500	312.5	98	22262.5	30.53	49.61	12	5.74	3.9
13	B_m5-c	41580	252	42	8149.7	84.89	35.3	10.64	5.53	3.56
14	R_m3-a	232560	342	93	22093.2	20.08	46.7	12.36	5.83	3.84
15	R_m3-b	40614	193.4	30	6904.4	67.91	27.58	10.61	5.26	3.32
16	R_m2-a	271656	352.8	120	23090.8	17.57	47.73	12.51	5.87	3.87
17	R_m2-b	72468	237.6	45	10145.5	44.25	32.07	11.19	5.47	3.47
18	H_m4-a	244673	342.2	115	22020.6	19.28	47.17	12.41	5.84	3.85
19	H_m4-b	368971.2	388.8	144	27672.8	14.59	53.83	12.82	5.96	3.99
20	H_m4-c	58212	237.6	37	7276.5	48.83	28.42	10.97	5.47	3.35
21	H_m5-a	152334	334.8	76	19041.8	30.50	46.46	11.93	5.81	3.84
22	H_m5-b	103723.2	302.4	83	14521.2	40.12	41.61	11.55	5.71	3.73
23	H_m5-c	22377.6	201.6	20	4251.7	119.8	26.82	10.02	5.31	3.29
24	H_m5-d	30203.5	208.3	32	5587.6	94.72	28.61	10.32	5.34	3.35
25	H_m6-a	316108.8	403.2	136	24656.5	16.39	51.81	12.66	6	3.95
26	H_m6-b	349048	379.4	140	26876.7	15.12	52.78	12.76	5.94	3.97
27	H_m6-c	47520	216	40	7128	60.34	28.67	10.77	5.38	3.36
28	L1-a	180682.3	386.9	108	21139.8	23.88	43.15	12.1	5.96	3.76
29	L1-b	64800	288	55	13089.6	56.89	36.86	11.08	5.66	3.61
30	L1-c	48384	252	46	10450.9	67.45	32.64	10.79	5.53	3.49
31	L2-a	215119.8	379.4	110	23232.9	20.32	43.71	12.28	5.94	3.78
32	L2-b	85449.6	331.2	64	12646.5	40.36	34.49	11.36	5.8	3.54
33	UK1-a	123161.5	319.9	64	13547.8	30.67	37.77	11.72	5.77	3.63
34	UK1-b	122709.6	280.8	77	12025.5	24.40	29.94	11.72	5.64	3.4
35	UK1-c	121586.4	280.8	71	12401.8	25.09	30.51	11.71	5.64	3.42
36	UK1-d	41832	252	40	6065.6	69.25	28.97	10.64	5.53	3.37
37	LK2-a	183768	372	72	16539.1	22.57	41.48	12.12	5.92	3.73
38	LK2-b	231927.5	319.9	93	17858.4	17.24	39.98	12.35	5.77	3.69
39	LK2-c	33805.35	245.5	31	5408.9	85.17	28.79	10.43	5.5	3.36
40	UK3-a	262008	432	121	23056.7	19.58	51.3	12.48	6.07	3.94
41	UK3-b	209664	327.6	113	18031.1	19.91	41.74	12.25	5.79	3.73
42	UK3-c	34914	253	33	6703.5	94.23	32.9	10.46	5.53	3.49
43	LK4-a	99792	259.2	63	14270.3	34.25	34.18	11.51	5.56	3.53
44	LK4-b	130046.4	316.8	74	12744.5	27.65	35.96	11.78	5.76	3.58
45	LK4-c	35358.4	200.9	31	6081.6	77.73	27.48	10.47	5.3	3.31

where, Q is the monthly coal production by SM (t), MST is the machine shift time of SM on monthly basis (h), PL is the pick loss during coal production on monthly basis (n), DL is the diesel loss during coal production on monthly basis (l), O&O is the ownership and operating costs of SM (₹/t), MSK is the machine shift cost of SM on monthly basis (lac ₹).

7.4 Results and discussion

7.4.1 SM productivity using MST-based BEPF model

Multiple linear regression was conducted to evaluate the log-linear BEPF model for coal production based on MST. The regression model outcomes are presented in Table 7.2. The model comprised of 43 degrees of freedom, was indicating a strong fit between variables with a high R^2 value as 0.77 at 95% significance level.

Table 7.2: Regression statistics of log-linear BEPF model based on MST

Regression Statistics	
Multiple R	0.877
R Square	0.769
Adjusted R Square	0.764
Observations	45

The statistical significance of the deviation from linearity for the aforementioned regression model was evaluated using ANOVA. The critical value for F (1,45) was calculated as 4.06 at a 95% significance level. However, the observed value of F (1,45) was 143.38 at the same significance level, as presented in Table 7.3. As the observed value exceeded its critical value and the significance of the F-statistic was less than 0.05, the relationship was deemed significant in terms of linearity.

Table 7.3: ANOVA test results of log-linear BEPF model based on MST

Model	Degrees of freedom	Sum of Squares	Mean Squares	F-statistic	Significance
Regression	1	21.68	21.68	143.38	0.000
Residual	43	6.5	0.15		
Total	44	28.18			

Table 7.4 displays the intercept value and the significance of coefficients of the log-linear BEPF model based on MST. The critical t-statistic value was calculated as 2.01, whereas the observed t-statistic value (1,45) was 11.97 at a 95% significance level. Since the observed t-statistic was higher than its critical value, a significant difference in the value of a regression coefficient from zero was confirmed.

Table 7.4: Descriptive statistics of log-linear BEPF model based on MST

Particulars	Coefficients	*t*-statistic	Significance	Lower 95%	Upper 95%
Intercept	-4.5637	-3.385	0.001	-7.282	-1.845
ln (MST)	2.8552	11.974	0.000	2.374	3.336

The productivity parameter (A_o) was estimated by taking the anti-log value of intercept, and the output elasticity with respect to 'labour' input (α) was observed as a regression coefficient of ln (MST), as given in Table 7.4. Equation 7.9 shows the SM coal production model based on MST using BEPF.

$$Q = 0.01 \times (MST)^{2.8552} \qquad \text{(Equation 7.9)}$$

where, Q is the coal production (t), and MST is the machine shift time (h).

Figure 7.1 illustrates a plot confirming that the predicted values fall near the straight line against the actual values of coal production, which validated the statistical significance of MST-based BEPF model.

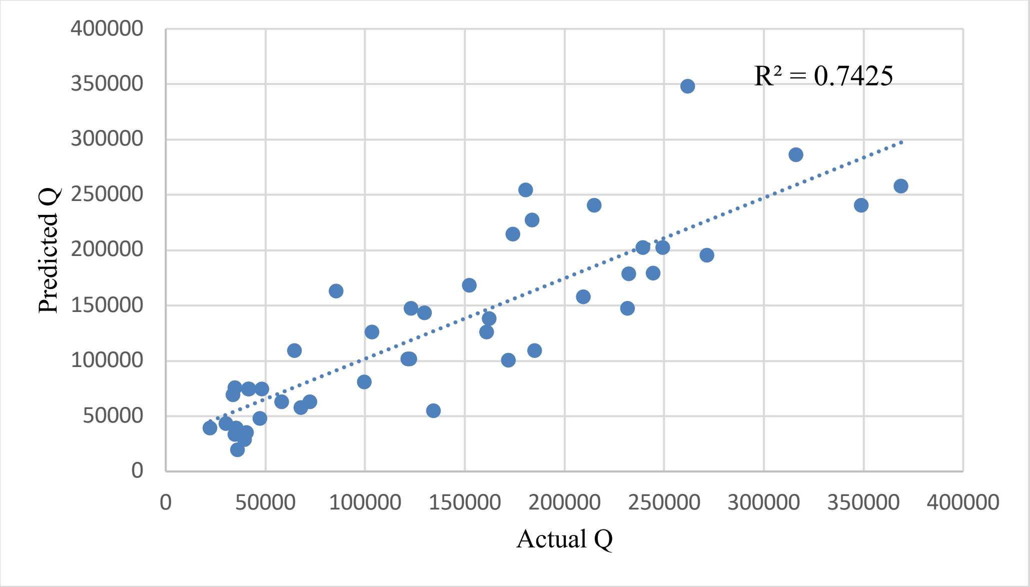

Figure 7.1: Plot of actual against predicted coal production for MST-based BEPF model

7.4.2 SM productivity using MSK-based BEPF model

Multiple linear regression was conducted to assess the log-linear BEPF model for coal production based on MSK. The R^2 value of this model was observed as 0.75, indicating a strong fit between variables at 95% significance level, as given in Table 7.5.

Table 7.5: Regression statistics of log-linear BEPF model based on MSK

Regression Statistics	
Multiple R	0.865
R Square	0.749
Adjusted R Square	0.743
Observations	45

The observed value of F (1,45) was 128.06, as presented in Table 7.6. Given that the observed value again exceeded the critical value, and the significance of F-statistic was less than 0.05, the relationship between variables was determined to be significant in terms of linearity at 95% significance level.

Table 7.6: ANOVA test results of log-linear BEPF model based on MSK

Model	Degrees of freedom	Sum of Squares	Mean Squares	F-statistic	Significance
Regression	1	21.1	21.1	128.06	0.000
Residual	43	7.08	0.16		
Total	44	28.18			

Table 7.7 displays the intercept value and the significance of coefficients of the log-linear BEPF model for coal production based on MSK. The observed value of t-statistic (1,45) was 11.32, which was greater than the critical value. A significant difference in the value of a regression coefficient from zero was confirmed, indicating a linear relationship between variables at 95% significance level.

Table 7.7: Descriptive statistics of log-linear BEPF model based on MSK

Particulars	Coefficients	*t*-statistic	Significance	Lower 95%	Upper 95%
Intercept	1.3432	1.484	0.145	-0.482	3.168
ln (MSK)	2.8494	11.316	0.000	2.342	3.357

The productivity parameter (A_o) was estimated by taking the anti-log value of intercept, and the output elasticity with respect to 'capital' input (β) was observed as a regression coefficient of ln (MSK), as given in Table 7.7. Equation 7.10 shows the coal production model based on MSK using BEPF.

$$Q = 3.831 \times (MSK)^{2.8494} \qquad \text{(Equation 7.10)}$$

where, Q is the coal production (t), and MSK is the machine shift cost (lac ₹).

Figure 7.2 illustrates a plot confirming that the predicted values fall near the straight line for actual values of coal production, which validated the statistical significance of BEPF model based on MSK.

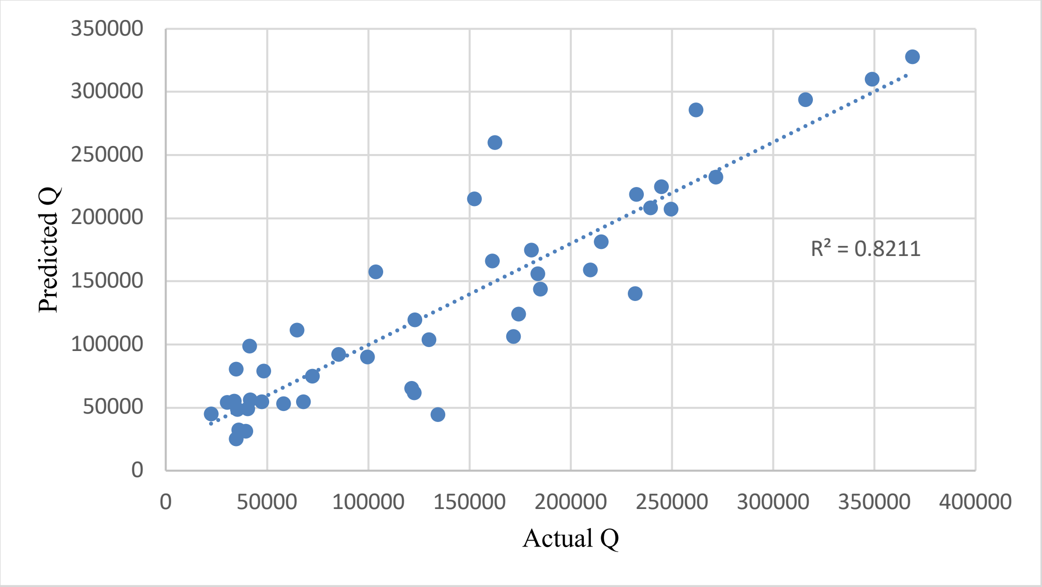

Figure 7.2: Plot of actual against predicted coal production for MSK-based BEPF model

7.4.3 SM total factor productivity using CDPF

Multiple linear regression analysis was conducted to examine the log-linear CDPF model for coal production based on MST and MSK. The R^2 value of the model was observed as 0.79, indicating a strong fit between variables at 95% significance level, as presented in Table 7.8.

Table 7.8: Regression statistics of log-linear CDPF model for coal production

Regression Statistics	
Multiple R	0.89
R Square	0.792
Adjusted R Square	0.782
Observations	45

ANOVA test results represents that the observed value of F (1,45) was 79.87, as given in Table 7.9. Given that the observed value exceeded its critical value and the significance of F-statistic was less than 0.05, the relationship between variables was found to be significant in terms of linearity.

Table 7.9: ANOVA test results of log-linear CDPF model for coal production

Model	Degrees of freedom	Sum of Squares	Mean Squares	F-statistic	Significance
Regression	2	22.31	11.16	79.87	0.000
Residual	42	5.87	0.14		
Total	44	28.18			

Table 7.10 details the intercept value and the significance of the coefficients of a log-linear CDPF model. The p-levels, indicating the probability of error involved in accepting observed results as valid for “representative of the population”, were 0.005 for MST and 0.039 for MSK, which are smaller than 0.05, indicating a linear relationship between independent variables and the dependent variable at a 95% significant level.

Table 7.10: Descriptive statistics of log-linear CDPF model for coal production

Particulars	Coefficients	*t*-statistic	Significance	Lower 95%	Upper 95%
Intercept	-2.6273	-1.66	0.104	-5.821	0.566
ln (MST)	1.7162	2.952	0.005	0.543	2.89
ln (MSK)	1.2537	2.131	0.039	0.067	2.441

The total factor productivity (A) was estimated by taking the anti-log value of intercept and output elasticities with respect to ‘labour’ input (α) and ‘capital’ input (β), which were observed as respective regression coefficients, as detailed in Table 7.10. Equation 7.11 shows the coal production model based on MST and MSK using CDPF.

$$Q = 0.072 \times (MST)^{1.7162} \times (MSK)^{1.2537} \qquad \text{(Equation 7.11)}$$

where, Q is the coal production (t), MST is the machine shift time (h), and MSK represents the machine shift cost (lac ₹).

Figure 7.3 illustrates a plot confirming that the predicted values fall near the straight line for actual values of coal production, which validates the statistical significance of coal production CDPF model.

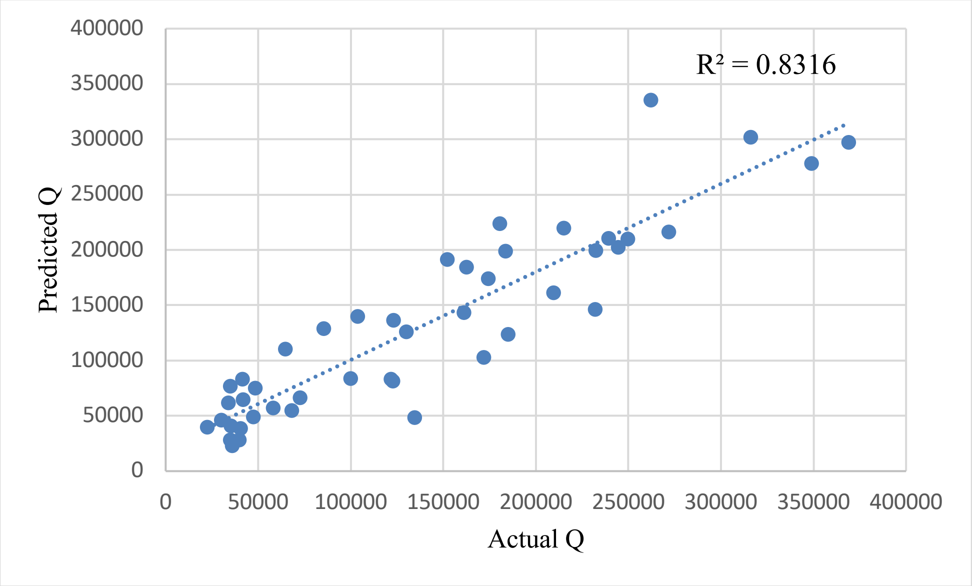

Figure 7.3: Graph between actual against predicted values for coal production CDPF model

A confirmation test based on the root mean square error (RMSE) revealed that the CDPF coal production model was found to be more significant with the least RMSE than the BEPF coal production models at a significance level of 0.05. The RMSE values were observed as 0.352, 0.451 and 0.329 for MST-based BEPF model, MSK-based BEPF model and CDPF model for coal production, respectively. The lowest RMSE value indicated that the coal production model using CDPF showed an improvement over BEPF models.

A 3D surface plot has been developed between coal production and MST as ‘labour’ input and MSK as ‘capital’ input for SM operating in a 24-hour scheduled shift time to classify different categories for SM coal production based on SM total factor productivity, as shown in Figure 7.4. Table 7.11 details the proposed range for five different SM-based coal production categories using CDPF-based isoquants.

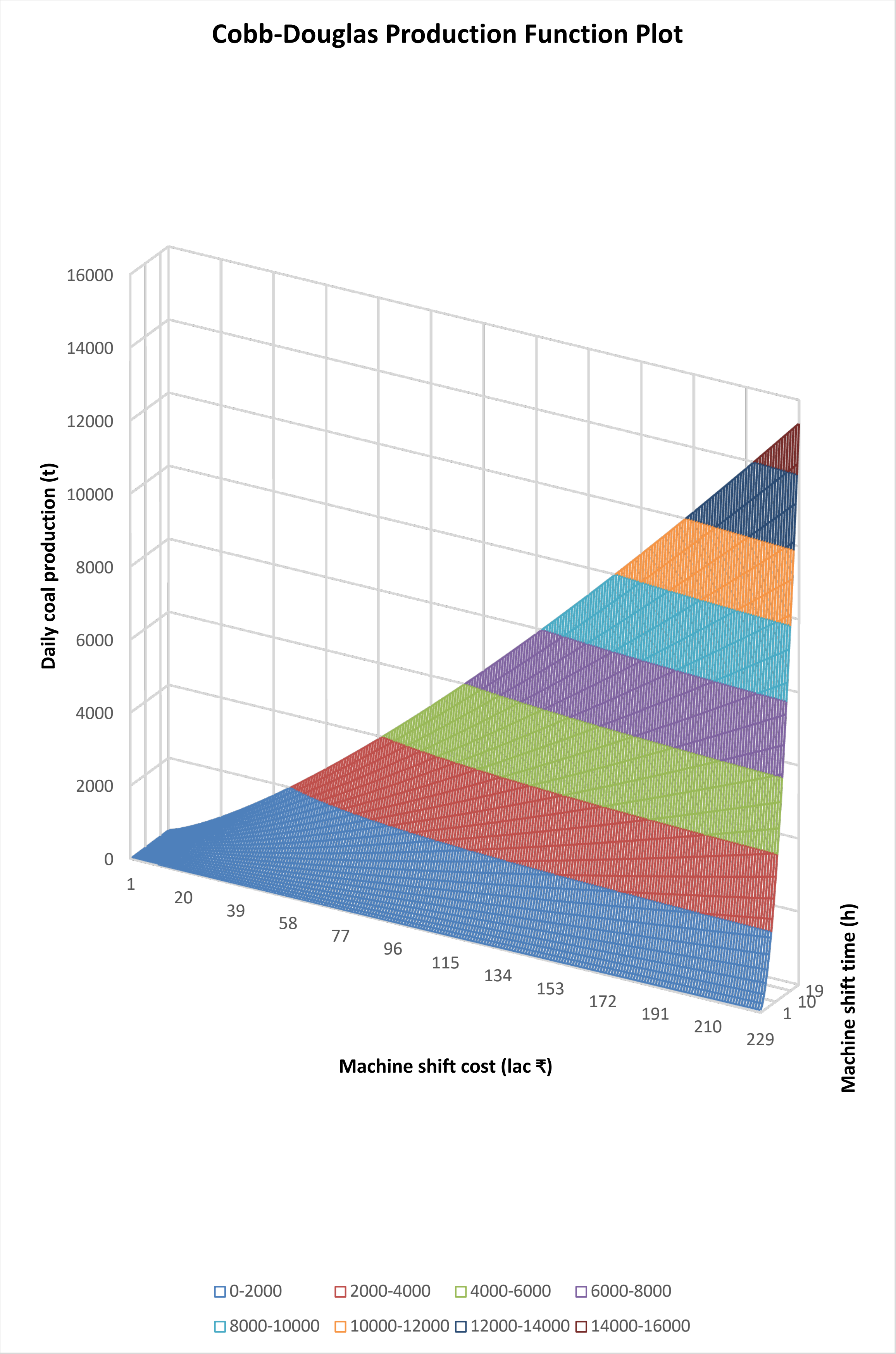

Figure 7.4: CDPF plot for coal production using SM in Indian opencast mines

Table 7.11: Coal production classification system based on SM total factor productivity

S. No.	Daily coal production range (t)	Category	Class
1	More than 14000	Excellent	I
2	10000 – 14000	Very good	II
3	6000 – 10000	Good	III
4	2000 – 6000	Moderate	IV
5	Less than 2000	Poor	V

7.5 Conclusions

7.5.1 Conclusions

- Coal production model based on two production factors, namely, machine shift time (MST) as ‘labour’ input and machine shift cost (MSK) as ‘capital’ input using Cobb–Douglas production function (CDPF) was found to be statistically significant for the measurement of SM total factor productivity at 0.05 significance level. The model was expressed as follows:

$$Q = 0.072 \times (MST)^{1.7162} \times (MSK)^{1.2537} \qquad \text{(Equation 7.11)}$$

- The output elasticities of labour (α) and capital (β), i.e., 1.7162 and 1.2537, gauged the utilisation intensity of MST and MSK, respectively. A higher value of α than β suggested that SM leans towards the MST-intensive technology. Moreover, the sum of α and β exceeded unity, indicating an increasing return to scale economy. It highlighted the need for mine managers to exercise caution while utilising MST and MSK as production factors to avoid irregularities and cost overruns during SM coal production, since, doubling either or both factors shall increase the SM coal production for more than double.

7.5.2 Suggestion for future work

This study considered Cobb–Douglas production function to simulate SM coal production with multiple inputs and single output, which paves the way for using other production functions like translog production functions with both single output and multiple outputs. Moreover, application of data envelopment analysis (DEA) and heuristic rank aggregation approach like Borda method may address the well-known problem of ranking mines using SM for coal production.