PREFACE

The thesis consists of five chapters. **Chapter 1** provides the introduction of the thesis. Basic definitions that are being used throughout this thesis are collectively provided. It also presents the literature survey and recent works on fractional partial differential equations (FPDEs). The motivation and objectives of the thesis are also explained.

Chapter 2 presents the stable numerical schemes for generalized time-fractional diffusion equations (GTFDEs) with smooth and non-smooth solutions on the non-uniform grid. In time, the generalized Caputo derivative is discretized by a difference scheme of order $(2-\alpha)$ on a non-uniform grid where $0 < \alpha < 1$. Stability and convergence for smooth as well as non-smooth solutions are obtained in L_2-norm and L_∞-norm, respectively. Several numerical results are presented to show how the grading of meshes is essential. Also, numerical results validate the efficiency and effectiveness of proposed schemes and show how a non-uniform grid produces better results.

Chapter 3 is devoted to develop new finite difference schemes with higher order of convergence for the generalized time-fractional diffusion equations (GTFDEs) with a weight function $\omega(t)$. Three different discrete analogs with different order of approximations are designed for the generalized Caputo derivative. The major contribution of this chapter is the development of a L2 type difference scheme that results in $(3-\alpha)$ order of convergence in time. The spatial direction is discretized using a second order difference operator. Fundamental properties of the coefficients of the L2 difference operator are examined and proved theoretically. The stability and convergence analysis of the developed L2 scheme is established theoretically using the energy method. An efficient algorithm is developed and implemented on numerical test problems to prove the numerical accuracy of the scheme.

Chapter 4 presents a stable multistep numerical scheme for the non-linear generalized time-fractional diffusion equations (GTFDEs) with non-smooth solutions. Mesh grading technique is used to discretize the temporal direction which results in $2-\alpha$ order of convergence ($0 < \alpha < 1$). The spatial direction is discretized using a second order difference operator and the non-linear term is approximated using Taylor's series. Theoretical stability and convergence analysis is established in the L_2-norm. Moreover, some random noise perturbations are added to investigate the numerical stability of the developed scheme. Finally, numerical simulations are performed on three test examples to verify the robustness and efficiency of the scheme.

Chapter 5 Solutions of Robin boundary value problem for a generalized diffusion equation with a non-smooth solution are studied in Chapter 5. The Caputo derivative in the generalized sense has been discretized using a difference scheme of order $(2-\alpha)$ on a non-uniform mesh with $0 < \alpha < 1$ in the temporal direction. Test example shows how the grading of the mesh is essential for non-smooth solution and using such kind of mesh generate stronger results.