PREFACE

One of the important discoveries about the Euclidean Fourier transform during the 20^{th} century was that under appropriate curvature assumptions on the support of a measure, the Fourier transform of the measure vanishes at infinity. Moreover, it belongs to some L^p space under appropriate dimension assumptions. In this thesis, we will prove quantum analogs of these results.

The Weyl transform is a non-commutative or quantum analog of the Euclidean Fourier transform. Various results about the Euclidean Fourier transform have simple analogs for the Weyl transform. For instance, the analog of the Riemann-Lebesgue lemma is that the Weyl transform of an L^1 function is a compact operator. Thus, the quantum analog of continuity together with decay at infinity is compactness. Similarly, the Schatten classes S^p are quantum analogs of the L^p spaces. Indeed, the quantum analog of the Hausdorff-Young theorem is that the Weyl transform maps $L^p(\mathbb{R}^{2n})$ to $S^{p'}(L^2(\mathbb{R}^n))$, where $1 \leq p \leq 2$ and $1/p + 1/p' = 1$. We will prove that under appropriate curvature and dimension assumptions, the Weyl transform of a measure is compact, and belongs to some Schatten class.

Chapter 1 is introductory in nature, and contains the appropriate background and history of the subject. In Chapter 2, we will consider a smooth measure supported on a compact hypersurface of positive Gaussian curvature in $\mathbb{R}^{2n}$, and prove that the Weyl transform of such a measure is compact and belongs to the p-Schatten class if $p > n \geq 6$. In Chapter 3, we will consider a smooth measure supported on a real-analytic submanifold of arbitrary codimension, and prove that the Weyl transform of such a measure is compact provided that the submanifold is not contained in any affine hyperplane. In Chapter 4, we will consider the Weyl transform of a compactly supported distribution, and prove a more general result in this direction. We will prove that the Weyl transform of a compactly supported distribution on $\mathbb{R}^{2n}$ is in $S^p(L^2(\mathbb{R}^n))$ if and only if its Fourier transform is in $L^p(\mathbb{R}^{2n})$, $1 \leq p \leq \infty$. Moreover, we will prove that the Weyl transform of a compactly supported distribution is a compact operator if and only if its Fourier transform vanishes at infinity. Finally, in Chapter 5, we will give some applications of the results obtained in the thesis. We will describe the conditions under which the quantum translates of a non-zero Schatten class operator are linearly independent. Moreover, we will provide an analog of the Fourier restriction theorem for the Fourier-Wigner transform.