

Evolution of Some Mathematical Models of Atmospheric Vortices

Exploration of Steady and Unsteady States



**Thesis submitted in partial fulfillment for the award of
the degree of**

Doctor of Philosophy

by

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DECLARATION BY THE CANDIDATE

I, **Kriti Yadav**, hereby declare that the research work embodied in this thesis is my own bona fide work carried out under the supervision of **Prof. Sanjay Kumar Pandey** from **July 2019 to June 2025**, at the **Department of Mathematical Sciences, Indian Institute of Technology (BHU), Varanasi**. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma.

I further declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in this thesis. I further declare that I have not willfully copied any work, such as paragraphs, texts, data, results, etc., reported in journals, books, magazines, reports, dissertations, theses, etc., or available at websites and included them in this thesis and cited as my own work.

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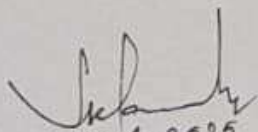
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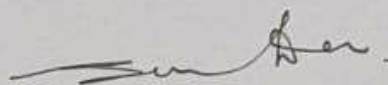
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Dedicated

To

My parents,

Mr. Matukuti Yadav and Mrs. Ranjana Yadav,

and my grandparents,

Late Mr. Raghunath Prasad and Late Mrs. Prabhawati,

whose boundless devotion lights my way,

whose encouragement lifts my soul,

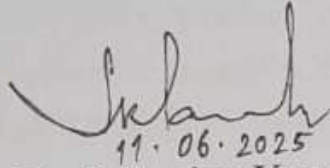
whose selflessness weaves my dreams,

with deep gratitude, this work I lay.

CERTIFICATE

It is certified that the research work contained in this thesis titled "*Evolution of Some Mathematical Models of Atmospheric Vortices: Exploration of Steady and Unsteady States*" by **Ms. Kriti Yadav** has been carried out under my supervision and that it has not been submitted elsewhere for a degree.

It is further certified that Ms. Kriti Yadav has fulfilled all the requirements such as Comprehensive Examination, Candidacy, SOTA etc. for the award of Ph.D. Degree.



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Date:

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(Kriti Yadav)

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