To

my beloved parents

and

other family members

for their endless love, support and encouragement

# CERTIFICATE

It is certified that the work contained in the thesis titled **Robust Static Output Feedback Controller Design Using LMIs** by **Jitendra Kumar Goyal** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree. It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. Degree.

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# DECLARATION

I, **Jitendra Kumar Goyal**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Dr. Sandip Ghosh** and **Dr. Shyam Kamal** from July-2016 to March-2021, at the Departement of Electrical Engineering, Indian Institute of Technology (BHU), Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, theses, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

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It is certified that the above statement made by the student is correct to the best of my/our knowledge.

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# Abstract

Static output feedback (SOF) controller design for both the continuous-time and discrete-time systems are open problems in control theory due to the absence of computable necessary and sufficient conditions. Numerous methods have been developed for the design, yet there are scopes for improvement. This thesis work concerns the development of new sufficient conditions for designing the SOF controller ensuring some performance measure in terms of robustness and transient behavior. The work has been divided into three main parts.

The first part deals with designing SOF controller for continuous-time systems. This development involves suitable decomposition of Lyapunov matrices and deriving Linear matrix inequality (LMI) criteria that ensures $H_\infty$ performance. LMI criteria for pole-placement in LMI region is also considered. Next, the SOF design condition is exploited for designing PID controller for higher-order MIMO systems. Further, extension of the matrix decomposition concept for more realistic system representation (uncertain systems with parametric perturbations and actuator saturation) is considered and new sufficient conditions for SOF controller design for a class of polytopic systems both with and without actuator saturation are derived. Numerical examples are presented to demonstrate the effectiveness of the proposed results.

The second part presents new sufficient LMI conditions for designing static $H_2$ and $H_\infty$ controllers for discrete-time systems. Two design criteria are developed. First one is developed through appropriate use of a bounding inequalities. It involves certain relaxations of the LMI variables leading to less conservative design criterion. The second design criterion involves decomposition of an auxiliary matrix. The decomposition facilitates linearization of nonlinear term of reduced size to obtain LMI criteria. This leads to less conservative results than the design conditions developed previously. A transformation framework is also proposed for designing dynamic output feedback controller using

the SOF design method. Pole-placement is one of the well-known technique to improve the transient behavior of the systems. To this end, the constant damping ratio locus for discrete-time systems leads to a non-convex region. A new convex approximation of the constant damping ratio loci by an elliptical segment is proposed here. Note that, being convex, the ellipse segment can be defined as an LMI region. Then, LMI criteria for controller (state feedback and SOF) synthesis are derived for linear discrete-time polytopic systems. A numerical example illustrates the efficacy of the proposed design approach.

The third part considers applications of the developed theory. Laboratory scale coupled tank and Twin Rotor MIMO System (TRMS) are considered for validation of some of the theoretical results in experimental setups. A boost converter widely used in power electronics applications, is also considered to be an application problem. System models of these applications are nonlinear in nature. Due to the nonlinearities involved in their dynamics, designing controllers using linear control theory is challenging. The design technique adopted in this work is based on considering the nonlinear terms involved, in the form of structural or model uncertainties. The nonlinear models of the systems is represented in the form of a polytopic systems that allows design and implementation of a linear controller. The nonlinear term is treated as an uncertain parameter in the system model. A $\mathcal{L}_2$ based Proportional Integral (PI) controller is designed combined with pole placement in a desired LMI region to ensure better transient behavior of the system. Experimental results have been provided and compared with conventional design to illustrate the efficacy of the proposed design method. Design of a quasi-Linear Parameter Varying (qLPV) PI controller for TRMS is also considered. The nonlinear model is represented as a qLPV polytopic plant with an affine dependence on a nonlinear parametric function of the pitch angle. This representation retains the exact model as opposed to the conventional linearization around an operating point. Due to the availability of the pitch angle measurement, the nonlinear parameter can be obtained in real-time and the controller is designed using qLPV technique. To deal with limited control input for such systems, the proposed controller design also considers the actuator saturation that yields controller with practical gains without any additional gain bound criterion. Further, the transient tracking performance is also considered in the design by using closed-loop eigenvalues assignment in desired damping regions. Finally, to demonstrate the effectiveness of the proposed design, a comparative study along with experimental results are provided. For

the discrete-time design, the pole-placement criteria in constant damping region is applied on a power electronic boost converter system in order to improve its transient behavior subject to variation in the input voltage and load resistance. Corresponding simulation results are provided to validate the results.

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# Nomenclature

### List of Greek and Roman Symbols

| | |
|---|---|
| $\mathbb{R}$ | Set of real numbers |
| $\mathbb{R}^n$ | $n-$ dimensional vector in Eucledian space |
| $\mathbb{N}$ | Set of positive numbers |
| $*$ | Symmetric terms of a matrix |
| $Co\{.\}$ | Convex hull |
| $Sym\{\mathcal{Z}\}$ | $\mathcal{Z} + \mathcal{Z}^T$ |
| $(\cdot)^{-1}$ | Inverse of a matrix |
| $(\cdot)^T$ | Transpose operation |
| $I$ | Identity matrix |

**Abbreviation**

| | |
|---|---|
| LMI | Linear Matrix Inequality |
| BMI | Bilinear Matrix Inequality |
| LQR | Linear Quadratic Regulator |
| CT | Continuous-time |
| DT | Discrete-time |
| AREs | Algebraic Riccati Equations |
| PI | Proportional Integral |
| PID | Proportional Integral Derivative |
| LTI | Linear Time Invariant |
| SOF | Static Output Feedback |
| DOF | Dynamic Output Feedback |
| 2-DOF | Two Degree of Freedom |
| MRAC | Model Reference Adaptive Control |
| ZOH | Zero-order hold |
| MIMO | Multi-input multi-output |
| PSO | Particle Swarm Optimization |
| APSO | Adaptive Particle Swarm Optimization |
| SMC | Sliding mode control |
| ROA | Region of attraction |
| LFT | Linear fractional transformation |
| CLF | Common Lyapunov function |
| PDLF | Parameter dependent Lyapunov function |