A STUDY OF ONE-DIMENSIONAL MOVING BOUNDARY PROBLEMS INVOLVING PHASE CHANGE AND DIFFUSION LOGISTIC POPULATION MODELS

The thesis submitted in partial fulfillment

for the award of degree

DOCTOR OF PHILOSOPHY

by

ABHISHEK KUMAR

DEPARTMENT OF MATHEMATICAL SCIENCES

INDIAN INSTITUTE OF TECHNOLOGY

(BANARAS HINDU UNIVERSITY)

VARANASI-221005

Roll No.: 17121024 **Year of submission : 2023**

CERTIFICATE

It is certified that the work contained in this thesis titled "***A Study of One-Dimensional Moving Boundary Problems Involving Phase Change and Diffusion Logistic Population Models***" by ***Abhishek Kumar*** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. degree.

23/05/23

Dr. Rajeev
Supervisor
Associate Professor
Department of Mathematical Sciences
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005

DECLARATION BY THE CANDIDATE

I, ***Abhishek Kumar***, certify that the work embodied in this thesis is my own bona fide work and carried out by me under the supervision of ***Dr. Rajeev*** from ***July 2017*** to ***December 2022*** at the ***Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi***. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

23.05.2023

Date: 23.05.2023 **Abhishek Kumar**

Place: Varanasi **(Roll No. 17121024)**

CERTIFICATE BY THE SUPERVISOR

It is certified that the above statement made by the student is correct to the best of my/our knowledge.

23-05-2023

Dr. Rajeev
Supervisor
Associate Professor
Department of Mathematical Sciences
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005

23.05.2023

Signature of Head of the Department

विभागाध्यक्ष / Head
गणितीय विज्ञान विभाग
Department of Mathematical Sciences
भारतीय प्रौद्योगिकी संस्थान
Indian Institute of Technology
(काशी हिन्दू विश्वविद्यालय)
(Banaras Hindu University)
वाराणसी / Varanasi-221005

COPYRIGHT TRANSFER CERTIFICATE

Title of the Thesis: ***A Study of One-Dimensional Moving Boundary Problems Involving Phase Change and Diffusion Logistic Population Models.***

Name of the Student: ***Abhishek Kumar***

Copyright Transfer

Date: 23.05.2023

Place: Varanasi

Abhishek
23.05.2023
Abhishek Kumar

ACKNOWLEDGEMENTS

My Ph.D. journey was full of many unforgettable ups and downs accompanied with hardship, encouragement and trust. There are so many people who helped me a lot along this way. It is not possible to say thanks in few words. Still I would like to take this opportunity to express my deepest gratitude for their support, love and affection.

I owe a great debt of thanks to my respected supervisor, ***Dr. Rajeev, Associate Professor, Department of Mathematical Sciences, IIT(BHU), Varanasi***. It has been a great pleasure to work under his mentorship. He listened my queries, with utmost patience and curiosity, and then offered his perceptive that gave a new direction to my research. I am very grateful to him for introducing me a fascinating branch of mathematics. I owe the completion of this thesis to his great wisdom, careful guidance, continuous encouragement, and unending patience over the past years.

Besides my supervisor, I would like to express my deep sense of gratitude to the rest of my RPEC (Research Progress Evaluation Committee) members ***Prof. Ram Sharan Singh*** and ***Dr. Vineet Kumar Singh***, for their precious time, valuable suggestions and encouragement. I owe my special thanks to ***Prof. S. K. Pandey, Head of Department, Prof. T. Som, Prof. L.P. Singh, Prof. S. Das, and Dr. Ashok Gupta, Convener, DPGC of Department*** for their support throughout my research work. I also express my deep sense of gratitude to all faculty members of the department for their assistance and support. I express my thanks to all non teaching staff members of the department for their cooperation and help throughout my research tenure.

I gratefully acknowledge **University Grants Commission, India** for providing the fellowship in form of Junior Research Fellowship and Senior Research fellowship.

I am also grateful to my Institute, IIT(BHU), for providing necessary resources throughout my Ph.D. program.

This acknowledgment would be incomplete if the name of great visionary ***Pt. Madan Mohan Malaviya*** is not mentioned, who made this divine centre of knowledge. Deepest regards to him.

Above all, praises and thanks to the God, the almighty, for his showers of blessings throughout my research work, who has made everything possible.

Varanasi-221005 **Abhishek Kumar**

Dedicated

to

My Parents

Contents

List of Figures

List of Tables

Abstract

This thesis is concerned with the construction and solution of various models of moving boundary problems (MBPs) by different approaches. The main feature of these problems is that they consist variable thermal coefficients or diffusion coefficients (Space-dependent, temperature-dependent) and different types of boundary conditions, due to which analytical solutions of these problems have not been derived yet. Therefore, different approximate and numerical techniques are applied on these problems to derive the almost accurate solutions.

This thesis contains five chapters. In chapter 1, introduction of MBPs, literature survey of MBPs, some definitions and basic concept of some proposed techniques are included.

In chapter 2, a moving boundary problem with moving phase change material and time dependent thermal conductivity is considered in a moving domain. The boundary fixation of this moving boundary has been done by using Landau type transformation. After that, the finite difference discretizations have been done for space and time derivatives, and then explicit scheme has been used to derive the solution. The error analysis, stability analysis and convergence of the have also been established for the considered method. The dependence of temperature distribution in the domain on various parameters is also analyzed.

Next, a space fractional population logistic diffusion model with density dependent dispersal rate has been considered in chapter 3, and explicit scheme is applied on it to get the solution of the problem. The proposed scheme for the above mentioned problem is consistent. It is proved that the scheme is conditionally stable and convergence for the proposed method. The analysis of various parameters of the model on the population growth is also presented through the different figures.

In chapter 4, a time-fractional Stefan problem in a semi-infinite domain with a Robin boundary condition at the first fixed boundary and variable thermal conductivity is presented. The classical approach of the heat balance integral method with quadratic and exponential temperature profiles is applied to the problem to find an approximate solution. To test the validation of the proposed approach, our solution is compared with the analytical solutions for integer-order derivative and fractional order cases when thermal conductivity is a constant. In this study, the effects of variable thermal conductivity and Biot number (Bi) on the temperature distribution and moving interface are also analyzed for the fractional-order system.

In chapter 5, an Operational Matrix Method based on Genocchi polynomials is used to determine the solution of a non-classical Stefan problem with space dependent thermal conductivity, variable latent heat and Robin boundary condition. The proposed method is easy to apply and the results obtained from this method are sufficiently accurate.

In chapter 6, the conclusion of the overall work and the scope for the future work related to this thesis is discussed.