CHAPTER – VII

DESIGN OF FRACTIONAL ORDER PHASE SHAPER

INTRODUCTION

PID controller so popular that the 90% of the process industry rely on this controller. PID controller provides satisfactory references tracking, good output disturbance rejection and robustness of the control systems (Cominos et al (2002)). Certain tuning techniques of PID extends control quality of closed loop control system such as loop shaping techniques proposed by Gracia et al (2006). cheng et al (2004) proposed a auto tuning technique for PID controller which achieves the “flat phase” around the gain crossover frequency. Here the PID controller is accompanied by the fractional order phase shaper.

Chen et al (2003) Proposed an analytical tuning method for PID controller to achieve the flat phase and also used the fractional operator of the form s^{α} in series with the PID controller to adjust the width of the frequency range (flat phase frequency interval). Monje et al (2004) taken the advantage of fractional integration action α, and fraction differentiation action β in order to design the Fractional order PID. With the help of results he achieved flat phase around the gain cross over frequency as well as iso-damping step response under varying gain of the plant. As the FOPID controllers are irrational functions (Fractional order transfer function) we can’t directly apply rational functions (Integer order transfer functions) it is necessary to approximate these irrational function to rational functions. In literature several researchers are developed their own techniques for the same, however the rational function are applicable only certain frequency range. Fractional Operator element can be realized by a lossy capacitor by using microelectronic approach Vinagre et al (2007) by analog circuit realization Bohannan et al (2002) or by integer order approximation for example Carlson representation Carlson et al (1964) or CRONE approximation. In this chapter Carlson approximation proposed by Carlson et al (1964) is applied to approximate Fractional order functions in to integer order function with a first order numerator and first order denominator.

Our motto is to design the Fractional shaper, which enables the systems to accomplish open loop phase flat and automatically established the integer approximated of fractional order phase shaper as a rational function, which is effective for a frequency range around the gain crossover frequency.

In order to make flat phase around the gain crossover frequency, Bode's integral formula based Fractional Order Phase shaper is designed accompanied with the plant and the classically designed PID controller. This feature, coupled with the fact that the methodology does not assume any specific characteristic of the PID controller used, makes it practically usable in existing control loops. The phase shaper assures the closed loop system exhibit iso-damping step response with constant over shoot under variation of the system gain within a particular interval. Enhanced parametric robustness with respect to certain gain variation concept is used for the systems where the plant gain varies depending on the regime of operation e.g. non-linear systems controlled by piece-wise linear approximation, requiring switching of controllers.

Here we are considering First Order plus Time Delay (FOPTD) process which is a reasonable approximation for most process plants and assumed to be tuned by any method suitable for tuning industrial PID controllers.

The rest of the chapter is organized as follows; section 6.1 describes design methodology used for modeling of fractional order phase shaper. In section 6.2, the non-linear minimization problem is explained. To illustrate the results some numerical examples have been given in section 6.3 and section 6.4. at the end section 6.5 concludes the chapter.

7.1 MODELING OF FRACTIONAL ORDER PHASE SHAPER

In this section we presents the design of fractional order phase shaper with a FO differ-integrator for a given system G(s) comprising a plant P(s) and its PID controller C(s)

$$G(s) = P(s)*C(s) \tag{7.1}$$

The motto of designing the Fractional order Phase shaper is to the advantage of the fractional power α so that the closed loop system exhibit the constant overshoot for certain gain variations in the plant. The design methodology applied here makes the open loop G(s) phase constant around the gain crossover frequency (i.e., for a particular frequency interval), assuring us a constant phase margin around gain crossover frequency and constant overshoot (iso-damping property) for closed loop time response of the control system. A constant phase around the gain cross over frequency also enhances the robustness performance of the control system.

$$G(s) = C_{ph}(s)*C(s)*P(s) \tag{7.2}$$

7.1.1 ASSUMPTIONS MADE FOR MODELING OF FRACTIONAL ORDER PHASE SHAPER

- Here for designing the Fractional Order phase shaper the plant must be of the form of FOPTD
- The accompanying integer order PID can be designed by any classical designing technique, which guarantees stable closed loop system. Here PID controller tuned by classical Z-N technique.
- It is to be noted that the designed phase shaper doesn’t add neither extra gain nor net phase to the system.

As mentioned in the introduction the methodology applied here for designing the phase shaper uses the Bode’s integral formula (Karimi et al (2002)). The bodes integral formula gives the information about the gain and phase derivative of the system with respect to frequency without the specific model of plant. Here the combination of both plant and PID controller is considered as a single system for the designing Fractional Order phase shaper, so that it can be applied to any existing control loop.

According to bodes integral formula the derivative of the phase of G(s) around gain cross over frequency (ω_{cg}) is approximated. For stable minimal phase system around frequencies bodes integral formula (Karimi et al (2002)) is defined as

$$\omega \frac{d\angle G(j\omega)}{d\omega} = \angle G(j\omega) + \frac{2}{\pi}[\ln|k_g| - \ln|G(j\omega)|] \tag{7.3}$$

Where k_g is the static gain of G(s).

It is proved in Karimi et al (2003) that above formula is applicable for both minimum phase and non-minimum phase system

Putting $\omega = \omega_{gc}$ in the Equation (7.3) yields

$$\left.\frac{d\angle G(j\omega)}{d\omega}\right|_{\omega=\omega_{gc}} = \frac{\varphi_m - \pi}{\omega_{gc}} + \frac{2}{\pi\omega_{gc}}\ln|k_g| \tag{7.4}$$

Where φ_m- is the phase margin of G(s)

ω_{gc} – is the gain crossover frequency

Our objective is to make the phase flat around gain crossover frequency (ω_{gc}) using the phase shaper $C_{ph}(s)$ then the relationship should be valid in frequency interval $\Delta\omega$ in the neighborhood of gain crossover frequency (ω_{gc})

$$\frac{d}{d\omega}\angle G(j\omega) + \frac{d}{d\omega}\angle G_{ph}(j\omega) = 0 \tag{7.5}$$

It is observed that for First Order Plus Time Delay Systems $\left.\frac{d\angle G(j\omega)}{d\omega}\right|_{\omega=\omega_{gc}}$ is negative. For Equation (7.5) to be valid around ω_{gc}, $\left.\frac{d\angle G(j\omega)}{d\omega}\right|_{\omega=\omega_{gc}}$ should be positive. This drawback overcomes with the phase shaper of the form

$$C_{ph}(\mathrm{s}) = \frac{(1+\mathrm{as}^q)}{s^q} \tag{7.6}$$

With $\frac{1}{a} \le \omega_{gc}^q$ And (7.7)

$$0 \le q \le 1 \tag{7.8}$$

The phase of the fractional order phase shaper is derived as

$$\frac{d\angle C_{ph}(j\omega)}{d\omega} = \frac{aq\omega^q \sin\frac{q\pi}{2}}{\omega(1+2a\omega^q \cos\frac{q\pi}{2} + a^2\omega^{2q})} \tag{7.9}$$

Using Equation (7.6), Equation (7.9) and Equation (7.5) it can be written as

$$\frac{\phi_m - \pi}{\omega_{gc}} + \frac{2}{\pi\omega_{gc}} \ln\left|k_g\right| + \frac{aq\omega^q \sin\frac{q\pi}{2}}{\omega(1+2a\omega^q \cos\frac{q\pi}{2} + a^2\omega^{2q})} = 0 \tag{7.10}$$

It is observed from the above Equation (7.10) that the derivative of the open loop phase with respect to frequency ω is zero, this condition makes the open loop phase constant around ω_{gc}. The objective of the phase shaper is to allow flexibility in making the open loop phase curve flat over certain frequency interval by adjusting the parameters a and q. it is to be noted that by addition of $C_{ph}(s)$

Will alter overall phase of the open loop system at gain crossover frequency (ω_{gc}) and net phase of $G(s)$ at ω_{gc} is given by

$$\phi\big|_{\omega=\omega_{gc}} = \phi_m - \pi - \frac{q\pi}{2} + \tan^{-1}\left(\frac{a\omega^q \sin\frac{q\pi}{2}}{1+a\omega^q \cos\frac{q\pi}{2}}\right) \tag{7.11}$$

From above Equation (7.11) it follows that the phase of at ω_{gc} is less than the phase of G(s) at the same frequency. Since the phase shaper flatness the phase curve of $C_{ph}(s)*C(s)*P(s)$ around ω_{gc}, it follows that the phase margin may reduce with the introduction of the phase shaper. Thus if the minimum desired phase margin with the phase shaper be ϕ_{md}, then it follows that the constraint shown below must also be satisfied.

$$\phi_{md} - \phi_m + \frac{q\pi}{2} - \tan^{-1}\left(\frac{a\omega^q \sin\frac{q\pi}{2}}{1 + a\omega^q \cos\frac{q\pi}{2}}\right) \le 0 \tag{7.12}$$

7.1.2 NON-LINEAR MINIMIZATION PROBLEM

According to the Equation (7.6), Equation (7.7.), Equation (7.8) and Equation (7.12) mentioned in the above a set of four non linear equations in terms of two unknown parameters (a,q) is found. Therefore, it is difficult to solve these four non linear equations in a easy and direct way. An optimization toolbox in MATLAB is used to solve these non linear equations with minimum error. Here we are employing FMINCON function that calculates the non-linear constraints minimum of a function of several valuables.

$$\min_x \; f(x) \text{ such that } \begin{cases} c(x) \le 0 \\ ceq(x) = 0 \\ A.x \le b \\ Aeq.x = beq \\ Lb \le x \le ub \end{cases}$$

It solves the problems of the form MIN f(x) subjected to C*(x)≤0*, *Ceq=0, LB≤x≤UB*, where f is the function to minimize; *C* and *Ceq* represent the non linear constraints and linear constraints. *X* is the minimum looked for *LB* and *UB* define a set of lower and upper bounds on the design variables, x.

Here Equation (7.6) is considered as the objective function to minimize and other specifications are taken as non linear constraints for the minimization and all of them subjected to the optimization parameters defined within the function FMINCON.

7.2 ILLUSTRATIVE EXAMPLE - I

A FOPDT system is considered for the design of phase shaper. The simulation results comprise of frequency response with and without phase shaper are presented. The system with phase shaper is subjected to gain variations and the resultant closed loop step response is shown to exhibit iso-damping over the certain gain interval.

The FOPTD system is chosen as

$$P(\mathrm{s}) = \frac{316.7}{s+2} e^{-0.5s} \tag{7.13}$$

The accompanying PID controller for the above FOPTD system is tuned by Z-N technique as

$$C(\mathrm{s}) = 0.00739 + \frac{0.00739}{s} + 0.00184s \tag{7.14}$$

The open loop transfer functions with *P(s)*C(s)* is found to have gain margin of 1.7161, phase margin of 86.05^0 at gain cross over frequency of 1.440

Now, we will design the FO phase shaper the steps given below for $\phi_{md} = 50^0$ (desired phase margin)

I. Given for $\phi_{md} = 50^0$ (desired phase margin), and obtained ω_{gc}

II. With these specifications we write Equation (7.6), Equation (7.7.), Equation (7.8) and Equation (7.12) it will give us four non linear equations in terms of *a, q*.

III. Determine the values of these two parameters by FMINCON optimization tool box in MATLAB. The values comes out to be, *a*=0.394 , *q*=0.8182.

IV. With the obtained values of *a, q* obtain the transfer function of the fractional order Phase shaper.

The fractional order phase shaper comes out to be

$$C_{ph} = \frac{1 + 0.394 s^{0.8182}}{s^{0.8182}} \tag{7.15}$$

V. Approximate the Fractional order phase shaper to integer order by Carlson approximation technique

Figure 7.1 shows the frequency response curves for the plant and the controller with and without the phase shaper. It is seen that the phase margin reduces to 50^0 from 86^0, as predicted, and the gain margin increases to 14.986 dB from 1.76 dB. The gain crossover frequency reduces from 1.440 rad/sec to 0.4431 rad/sec.

In Figure 7.1, it is seen that the phase margin reduces with the introduction of the phase shaper, which is evident from the increased overshoot due to a step input as shown in Figure 7.2. Again, it is seen from Figure 7.1, if the plant gain is increased, the gain crossover frequency shifts towards a higher frequency value, but the phase margin remains constant. Figure 7.2 shows that increase in system gain reduces the rise-time while maintaining the same overshoot. The system gain, as shown in Figure 7.2 can be varied by keeping the overshoot constant. The advantage of the phase shaper becomes evident considering the fact that the PID controller alone cannot handle such large variation in gain. The closed loop system, with the PID controller alone becomes unstable when the system gain is increased by a factor more than two.

It is observed that for a given FOPTD plant, with phase shaper proposed in this chapter, phase margin reduces, leading to slight increase in overshoot but the response exhibits iso-damping. Further, the controller output becomes lower than the system controlled by PID controller, which is highly desirable for reduction in actuator size. This, however, assumes that a nominal phase shaper is used.

Frequency Response of plant with controller and with and without phase shaper

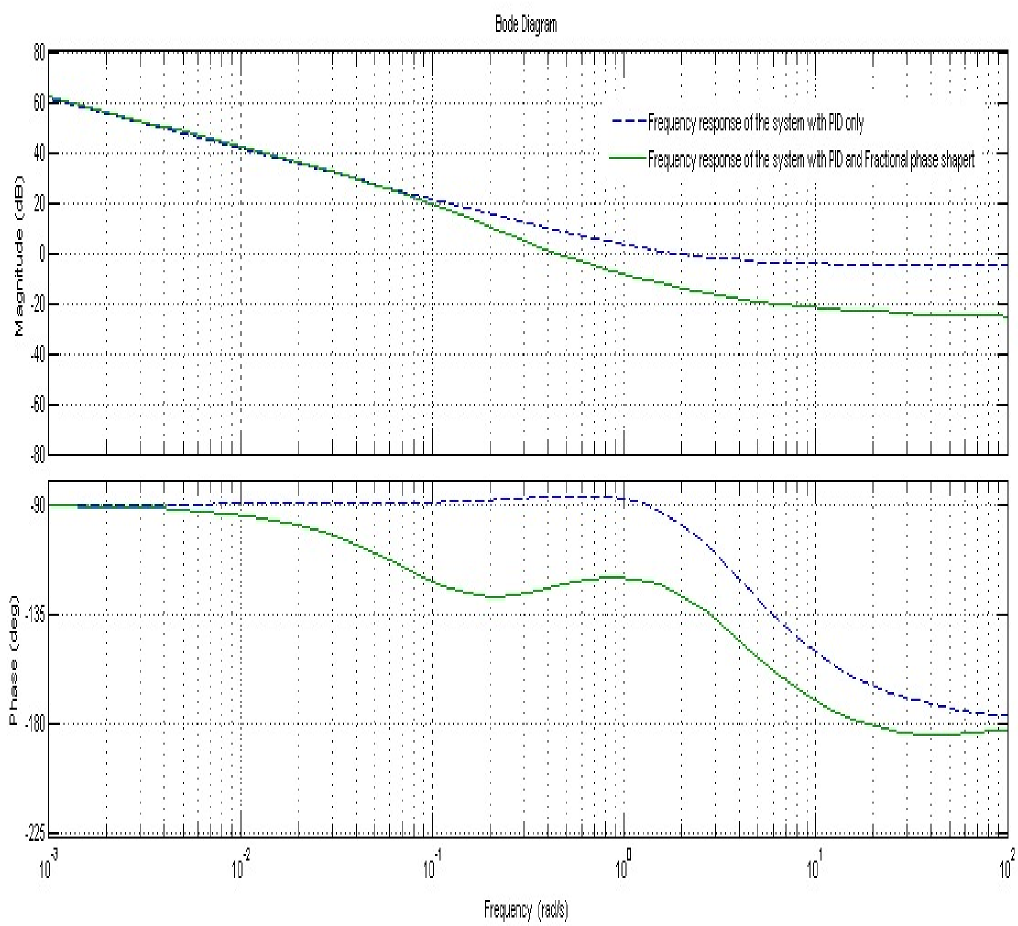

Figure 7.1- frequency response of plant with controller with and without phase shaper

Step Response of plant with controller and with and without phase shaper under varying loop

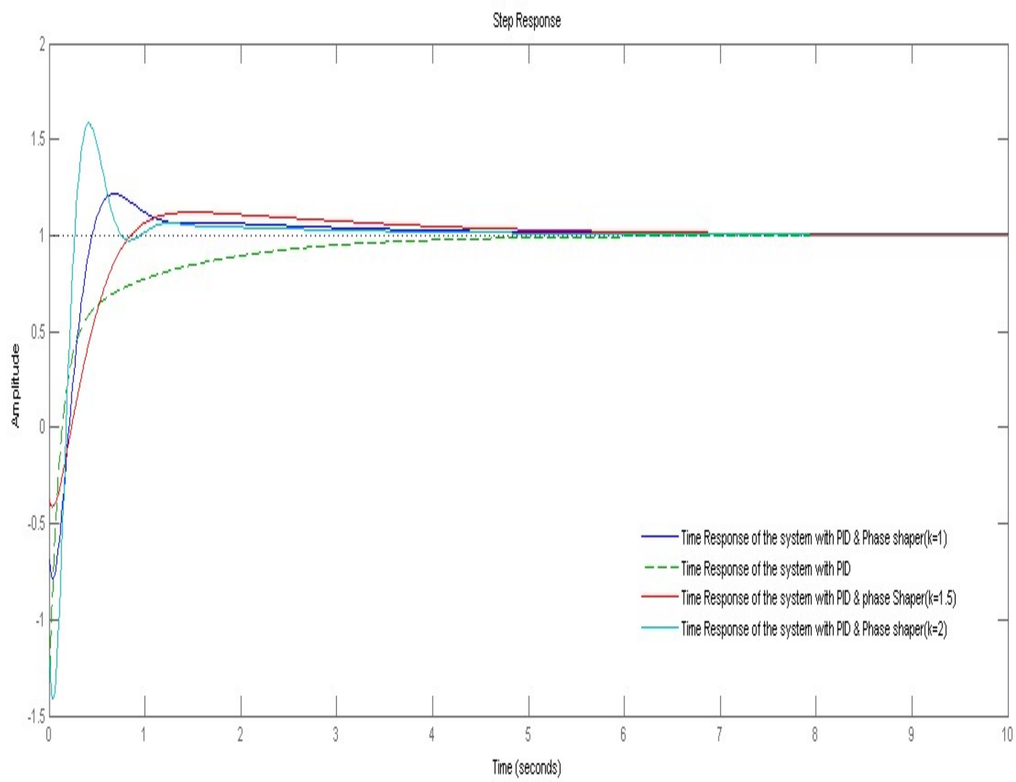

Figure 7.2- Step response of plant with controller with and without phase shaper under varying loop gain

7.3 ILLUSTRATIVE EXAMPLE - II

The Lag dominated FOPTD system is chosen as

$$P(\mathrm{s}) = \frac{2}{80s+1}e^{-1s} \tag{7.16}$$

The accompanying PID controller for the above FOPTD system is tuned by Z-N technique as

$$C(s) = 112 + \frac{62.29}{s} + 29.49s \tag{7.17}$$

The open loop transfer functions with *P(s)*C(s)* is found to have gain margin of 2.17, phase margin of 20.22^0 at gain cross over frequency of 1.382

Now, we will design the FO phase shaper the steps given below for $\phi_{md} = 50^0$ (desired phase margin)

I. Given for $\phi_{md} = 10^0$ (desired phase margin), and obtained ω_{gc}

II. With these specifications we write Equation (7.6), Equation (7.7.), Equation (7.8) and Equation (7.12), it will give us four non linear equations in terms of *a, q*.

III. Determine the values of these two parameters by FMINCON optimization tool box in MATLAB. The values comes out to be, *a*=0.084 , *q*=0.1001.

IV. With the obtained values of *a, q* obtain the transfer function of the fractional order Phase shaper.

The fractional order phase shaper comes out to be

$$C_{ph} = \frac{1 + 0.084s^{0.1001}}{s^{0.1001}} \tag{7.18}$$

V. Approximate the Fractional order phase shaper to integer order by Carlson approximation technique

Figure 7.3 shows the frequency response curves for the plant and the controller with and without the phase shaper. It is seen that the phase margin reduces to 10^0 from 20^0, as predicted, and the gain margin increases to 15.5986 dB from 2.17 dB. The gain crossover frequency reduces from 1.372rad/sec to 0.3962 rad/sec.

In Figure 7.3, it is seen that the phase margin reduces with the introduction of the phase shaper, which is evident from the increased overshoot due to a step input as shown in Figure 7.4. Again, it is seen from Figure 7.3, if the plant gain is increased, the gain crossover frequency shifts towards a higher frequency value, but the phase margin remains constant. Figure 7.4 shows that increase in system gain reduces the rise-time while frequency response producing the flat phase around the gain cross over frequency. The system gain can be varied keeping the open loop phase constant. The advantage of the phase shaper becomes evident considering the fact that the PID controller alone cannot handle such large variation in gain. The closed loop system, with the PID controller alone becomes unstable when the system gain is increased by a factor more than two.

It is observed that for a given FOPTD plant, with phase shaper proposed in this chapter, phase margin reduces, leading to slight increase in overshoot but the response exhibits iso-damping. Further, the controller output becomes lower than the system controlled by PID controller, which is highly desirable for reduction in actuator size. This, however, assumes that a nominal phase shaper is used.

Frequency Response of plant with controller and with and without phase shaper

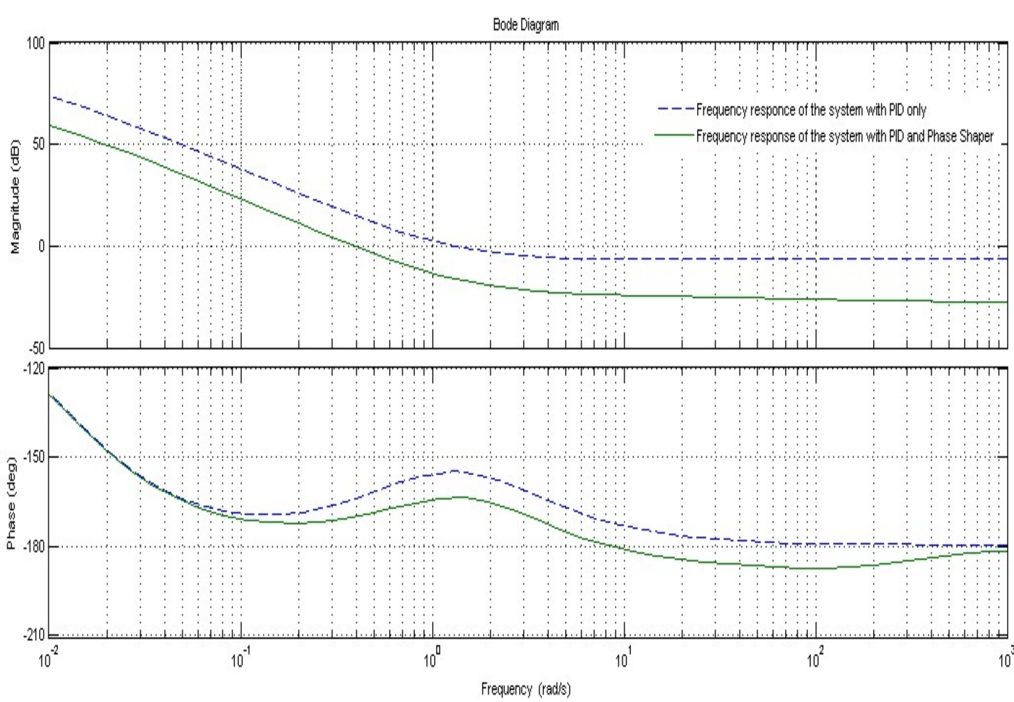

Figure 7.3- frequency response of plant with controller with and without phase shaper

Step Response of plant with controller and with and without phase shaper under varying loop

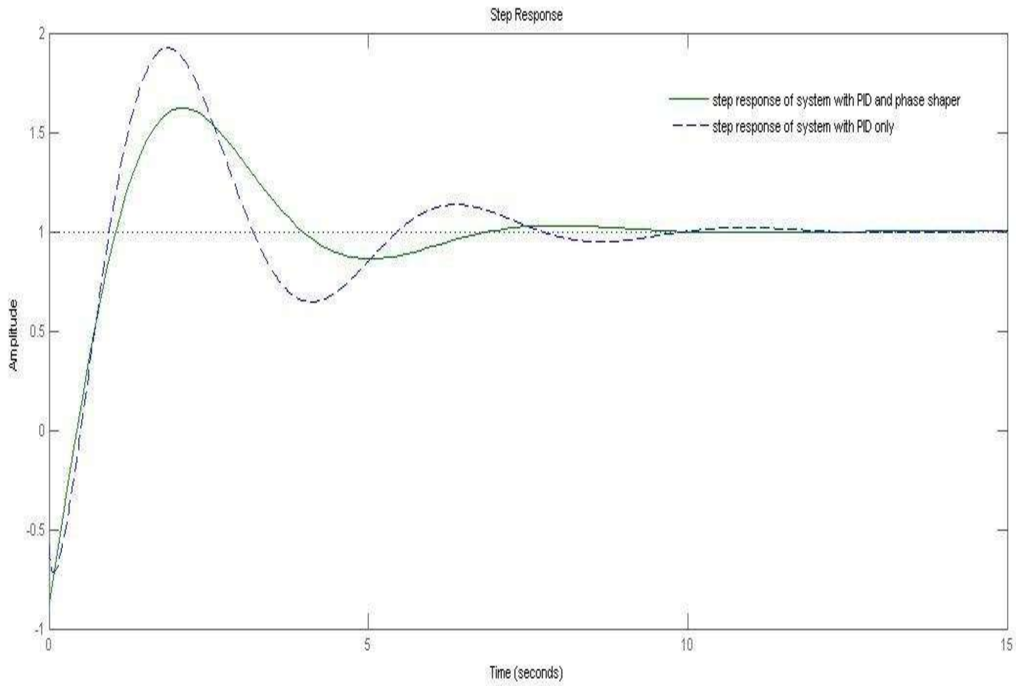

Figure 7.4-Step response of plant with controller with and without phase shaper under varying loop gain

7.4 CONCLUSION

The technique used in the process of designing of fractional order phase shaper can provide the parametric robustness of PID control loop to gain variations. The control quality enhancement of any PID controller control loop with fractional phase shaper provides a very useful for process control applications under the variation of the plant parameters (gain like) with respect to time. The designed phase shaper fulfills the iso-damping property which makes the open loop phase constant at gain crossover frequency and time response maintain constant overshoot followed by other specifications.

The advantage of the phase shaper is that it is of lower order and practically realizable. This concept allows design of a simple hardware element (to design phase shaper), which is employed with classically designed PID controller. This methodology having the ability of controlling the non-linear plants such as varying system gain in different operating conditions. The main drawback by employing the fractional order phase shaper is that the phase margin is reduces in order to achieve the good robustness. to overcome this drawback it is recommended to applied for the systems with high damping ratio thus phase shaper assure the high phase margin. The concept of minimum phase margin provides the designing to specify maximum overshoot while flattening the open loop phase around the gain cross over frequency.