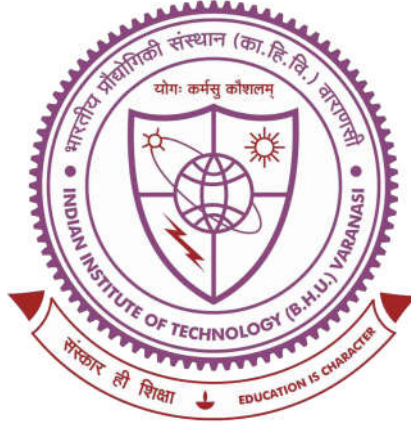


ON QUALITATIVE PROPERTIES OF FRACTIONAL  
INTEGRO-DIFFERENTIAL EQUATIONS



*The thesis submitted in partial fulfilment*

*for the Award of Degree*

*DOCTOR OF PHILOSOPHY*

*by*

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December 2024

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It is certified that the work contained in this thesis titled “**On Qualitative Properties of Fractional Integro-Differential Equations**” by **Pratima Tiwari** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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I, *Pratima Tiwari*, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of *Prof. Rajesh K. Pandey* from *July, 2018 to December, 2024* at the *Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi*. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, theses, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

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It is certified that the above statement made by the student is correct to the best of my/our knowledge.

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**Pratima Tiwari**

*Dedicated*

*to*

*My Grandmother*

**Swargiya Chameli Devi**