APPLICATION OF PARTICLE SWARM OPTIMIZATION: RESULTS AND DISCUSSION

The particle swarm optimization method was applied to solve the problem of optimal design of earthen channels whose side slopes are riveted with riprap stone and bottom is unlined. Three objective functions were developed for the three classes of riprap stones, i.e., round, subround and subangular, and angular in chapter 3. The constraint equations remain the same for all types of riprap stones. Further, it is essential for any optimization problem to transform its form to fit into the optimization procedure. The transformation procedure of the problem, results obtained by the optimization procedure and discussion of the results are described in following sub-sections.

5.1 Solution Procedure

Eqs.3.58-3.60 were transformed to yield residuals (R_1, R_2, and R_3) arising out of the violation of constraints and they lie in the range of 0 to 1. This process has virtually squeezed the search space and made the computational process more efficient.

$$R_1 = \left| X - 1 \right| \tag{5.1}$$

$$R_2 = \left| \frac{k_{\tau s} y_*}{KD_* \sqrt{1 - \frac{k_\phi}{1 + m_1^2}}} - 1 \right| \qquad (5.2)$$

$$R_3 = \left| \frac{k_{\tau s} y_*}{KD_* \sqrt{1 - \frac{k_\phi}{1 + m_2^2}}} - 1 \right| \qquad (5.3)$$

These residual equations, multiplied by penalty parameters, were added to the objective function to design an augmented cost function for the PSO algorithm to solve a problem.

$$\text{Minimize } F = C_* + \sum_{j=1}^{3} \beta_j \left| R_j \right| \qquad (5.4)$$

where j (=1, 2, and 3) corresponds to constraint Eqs. 33-35, respectively. β_j and R_j are the penalty parameter and residual for the j^{th} constraint, respectively. To ensure sufficient exploration of the search space at the initial stage and fine tune to the search at a later stage, the penalty parameter β_j was defined as a sequentially increasing function:

$$\beta_j = a_1 + a_2 . \left(\frac{N_t}{N_{t\max}} \right) \quad for \ \forall \ j \qquad (5.5)$$

where N_t and N_{tmax} are the number of iteration and the maximum number of iterations, respectively. The values of constants 'a_1' and 'a_2' are given in Table 5.1. No penalty was imposed for an acceptable limit of the tolerance (10^{-6}) for both the flow rate and shear stress constraint violation, since the magnitude of the actual violation becomes insignificant.

A flow chart for the PSO application is shown in Fig. 5.1. Each solution array presents the dimensions of the canal, and size of the riprap.

5.1.1 Range of variables and cost calculation

Based on the preliminary results, a promising range for the width of canal was set to be 2.5 to 15 m. The depth of flow was set in the range of [0.5, 5.0] according to Gupta and Singh (2012). The lower limits for side slopes were set to obtain real values (not imaginary) from Eqs. 3.19-3.20. The upper limit for the side slopes was set to be 3.0 in place of 2.5 (Blacklar and Guo 2009) for acquiring a slightly improved flexibility for search algorithm. The range for the size of riprap stones was adopted in a commercially viable space range of 0.1 to 1.0 m (http://cfpub.epa.gov/npdes/stormwater/menuofbmps/index.cfm?action=browse&Rbutton=detail&bmp=39). Table 5.2 summarizes the range of these variables applied for solving the problem.

The costs of three types of the riprap stones were observed to be different from one company to another. Using the data available on the websites of various companies (http://www.rolfecorp.com/materials/price-list/;http://www.964-rock.com/#cobbles; and http://www.wirelessestimator.com/industryinfoselect.cfm?categid=702), the median costs of the three types of riprap stones (excluding haulage cost from quarry to the construction site) were determined in physical unit *\$/yard*^{3} as 31, 28, and 24 \$/CY for round, subround and subangular, and angular, respectively. The cost of riprap, converted in terms of *\$/m*^{3}, was multiplied by the riprap thickness to obtain the cost of riprap stones per square meter of the coverage area (*\$/m*^{2}). The thickness of riprap placed on the channel side slopes was chosen as 1.5 times D (NCHRP Rep. 568, p. C4). The cost calculated in terms of *\$/m*^{2} was used with the augmented cost function (Eq. 5.4). The costs calculated above were updated according to the trial sizes of the riprap stones involved in the computational process at a given generation. Realistic data for average costs of earth excavation (3.1 \$/CY = 4.052 *\$/m*^{3}) and land acquisition (2.84 \$/m^2) were obtained from an Arizona (USA) canal project under U.S. Bureau of Reclamation.

5.1.2 Parameter setting

The inertia weight was varied linearly, in a range shown in Table 1, for each set of the problem. The value of ψ was chosen to be 4.1 (Clerc and Kennedy 2002), whereas the value of cognitive parameter was adopted as 1.957. The convergence criterion for the algorithm was set on the basis of maximum number of iterations. It was set 100 for round and subround and subangular stones, whereas angular stone needed it to be 220.

To avoid singularity or overshoot, a check on the limit of random numbers in the range of [10^{-5}, 1] was imposed on the solution vectors. The swarm size was 30. The random number generator was initialized with the default setting of the HP probook laptop. A summary of parameter settings for different PSO applications is given in Table 5.1.

5.2 Results and Discussion

The design problem cited by Froehlich (2011a) was adopted to compare the performance of different types of riprap stones for the design of minimum cost channel. Figs. 5.2-5.4 show the convergence characteristics of the mean population cost, best cost particle in the present generation and the global minimum cost obtained so far with iteration number for round, subround and subangular, and angular stones, respectively. The subfigures with suffix (a) refer to the clear water flow condition near channel bottom where a fixed magnitude of Manning's roughness coefficient for channel bed (referred to as Case I hereafter) was applied. The subfigures with suffix (b) associate to the sediment laden flow condition near channel bottom where Manning's roughness coefficient for channel bed was determined using Eq. 3.35 (referred to as Case II). It is clear from the figures that the mean augmented cost of population curves either merge or become asymptotic to the global best cost curves at the end of maximum number of iterations. This alludes to the suitability of values of PSO parameters and it happens, because most of the particles acquire the same cost as the global best. Table 5.1 shows that each set of the problem requires a different set of penalty parameters and range of inertia weight. It may be noted that the maximum number of iterations is relatively smaller for round and subround and subangular stones than the angular stone which reveals that the angular stone requires a relatively larger number of iterations to converge to the population. The reason associates to the uneven and sharp edged surface characteristic of the angular stones, which requires the optimization algorithm to undertake a more rigorous hunt to pick the minimum cost solution in a diverse population.

Tables 5.3-5.5 present the characteristics of the minimum cost channels obtained by the application of round, subround and subangular, and angular stones, respectively.

5.2.1 Side slopes of the channels riveted with different riprap stones

Tables 5.3-5.5 present the characteristics of the minimum cost earthen channels for round, subround and subangular, and angular stones respectively. Tables clearly reveal that the minimum cost channels riveted with different riprap stones acquire the same slope for both the banks of channels (see columns 6-7). The reason appears to associate with the same size of the revetment stones on either bank of the channels, which caused the optimization algorithm to force the residuals R_2 and R_3 to zero for ensuring the full utilization of the stone mass for stability, hence the channels turned out to be symmetric in shape. The same trend continued for all types of the stones.

5.2.2 Discussion relating to the round stone

Table 5.3 presents the characteristics of the minimum cost channels riveted with round stones.

5.2.2.1 *Comparing channels with and without freeboard (both Cases I-II)*

It is observed for clear water flow condition near channel bottom (see Case I in Table 3) that the channels with freeboard possess a relatively reduced bottom width (*b*) and marginally steeper side slopes than the channel without freeboard. It shows that the optimization algorithm tries to narrow down the cross section of channels having freeboard as compared to the channel without freeboard. This finding is in line with the work of Guo and Hughes (1984) on hydraulically efficient channel sections. The similar trend continues for sediment laden flow condition (Case II). The bottom width of the channel with freeboard comes out to be 8.3898 m (relatively smaller) and side slope 1.8272 (relatively steeper) for Case II (see row 1, columns 3, 6, and 7 in Table 5.3) as compared to the 10.0292 m (relatively large) and 1.8680 (flat) (see rows 2-5, columns 3, 6, and7), respectively, for the channels without freeboard (f = 0).

5.2.2.2 *Effect of freeboard scenario options on channel shape (both Cases I-II)*

It is interesting to note that the channels with freeboard possess the same bottom width, flow depth, side slopes and the stone size (see rows 2-5, columns 3, 4, 6-8) for the clear water flow condition near channel bottom (Case I). Thus, the geometric shape of the channels remains the same for all channels having freeboard (see Table 5.3). However, the varying magnitude of the freeboard (see column 5) causes the top width, length of the side slopes and excavated area (see rows 2-5, columns 9-11) to differ in magnitude, hence their costs. Almost the same trend continued for sediment laden flow condition near channel bottom (Case II), hence geometric shape of the channels with freeboard remains independent of the freeboard scenario and flowing water characteristics near channel bottom.

5.2.2.3 *Effect of clear water and sediment laden flow conditions on channel specifications (b, y, A_t, T) and cost*

Inter-comparison of the results for Cases I-II reveals that the channel bottom width (*b*) is relatively larger in Case II (sediment laden flow condition) than that in Case I (clear water flow condition near channel bottom). The flow velocity (*V*) is also higher and costs are lower for Case II than that in Case I. This reveals that the optimization process in Case II reduced the value of Manning's roughness coefficient for the channel bottom which was a function of fluid, flow and sediment characteristics (hydraulic radius, Froude and Reynolds numbers, sediment size and density, channel slope and water viscosity and density), which, in turn, reduced the flow resistance near the channel bottom. Therefore, in order to reduce the flow resistance for higher velocity gain in Case II, the optimal design acquired a larger bottom width and a lower water depth (compare data in columns 3-4 for Cases I-II in Table 5.3). Thus, the excavated area and associated cost also reduce (compare data in columns 11 for Cases I-II in Table 5.3).

The top width also reduced due to steeper side slopes for Case II. Therefore, the total construction cost of channel for sediment laden flow condition near channel bottom (Case II) was relatively lower (see columns 9 and 11 where reduced T and A_t appear) in comparison to that of the channel having a fixed value of Manning's roughness coefficient for Case I.

5.2.3 Discussion relating to the subround and subangular stone

The design alternatives involving subround and subangular stone are presented in Table 5.4.

5.2.3.1 *Comparing channels with and without freeboard (both Cases I-II)*

Table 5.4 reveals that the channels with freeboard (see rows 2-5 for Case I) possess relatively increased bottom widths (b = 8.3697 m), reduced flow depths (y = 3.0335 m), and marginally steeper side slopes (m_1/m_2=1.6401) as compared to those of the channel without freeboard (b = 8.3532 m, y = 3.0396 m and m_1/m_2= 1.6419). The bottom widths and flow depths of the channels having freeboard showed contradictory trends, whereas the side slopes conformed to the feature of the channel whose side slopes were riveted with round stone. The sediment-laden flow condition (see Case II) shows (see Table 5.4) that the bottom widths (b=6.3729, 6.5740, 6.3729 and 6.5740m) of the channels with freeboard provision are higher and flow depths (y=3.2318, 3.1904, 3.2318 and 3.1904m) are lower than that of channel without freeboard (b=6.3596m and y=3.2353m). This characteristic is similar to the clear water flow condition near channel bottom as stated above but opposite to that observed in Case II for round stone (channels having freeboard possess reduced bottom widths and larger flow depths as compared to those of the channel without freeboard).

5.2.3.2 *Effect of freeboard scenario options on channel shape (both Cases I-II)*

The bottom width, flow depth and side slopes of the channels under various freeboard scenarios are observed to be the same for Case I. Hence, similar to the round stone, the geometric shape of the channels is the same (b, y, m_1, m_2 and D) under all freeboard scenarios (excluding f = 0) with the exception of the magnitude of freeboard, which ultimately alters the cost of channel construction. This happens because the channel side slopes (m_1 and m_2) acquire a value (1.6401), which is near to the lower bound 1.45 (applicable to subround and subangular stone). It seems that the optimization algorithm could not find sufficient space to decease it further, which, in turn, eventually restricted/fixed the values of b and y both to give the flow rate of 60 m^3/s. Thus, the channels remained symmetric in shape.

For Case II, marginal variations in side slopes were observed for the channels with freeboard (see columns 6-7 in Table 5.4), hence the magnitudes of bottom width and flow depth varied. This way slight variations in the shape of the channels were observed for Case II. Having utilized the flexibility available with Manning's roughness coefficient (a function of fluid, flow and sediment characteristics-hydraulic radius, Froude and Reynolds numbers, sediment size and density, channel slope, and water viscosity and density) of the channel bottom, the optimization algorithm adjusted the values of both, bottom width (*b*) and flow depth (*y*), for the reduced value of Manning's roughness coefficient of the channel bottom, which resulted in increased flow velocity with consequential reduction in excavation quantum and the length of channel side slopes. Thus, the shapes of channels with freeboard for Case II marginally differed to yield relatively lower cost of channels as compared to Case I.

5.2.3.3 *Effect of clear water and sediment laden flow conditions on channel specifications (b, y, A_t, T) and cost*

Inter-comparison of Cases I-II for subround and subangular stone (see Table 5.4) also shows that the channels with sediment laden flow condition near channel bottom (Case II) have flatter slopes (approximately 1.65) and smaller bottom widths (b = 6.3596, 6.3729, 6.5740, 6.3729, and 6.5740 m) than those (m_1/m_2 = 1.64; b = 8.3532, and 8.3697 m for the remaining four scenarios of Case I. Thus, it reveals that the optimization algorithm yielded a narrower and flatter side slope channel for Case II to have the minimum cost. This happened because the optimization algorithm reduced the value of Manning's roughness coefficient for channel bottom by adjusting the values of bottom width and flow depth, which resulted in the reduced flow area to yield higher flow velocity, hence, the reduced excavated area resulted in the low cost of construction.

The top width also reduced due to smaller width of the channel bottom in Case II. Therefore, the total construction cost of the channel for sediment laden flow condition near the channel bottom (Case II) fell down (see columns 9 and 11 where reduced T and A_t appear) as compared to the channels having a fixed magnitude of Manning's roughness coefficient (Case I).

5.2.4 Discussion relating to the angular stone

Table 5.5 presents the characteristics of the optimal cost channels having angular stones as revetment.

5.2.4.1 *Comparing channels with and without freeboard (both Cases I-II)*

It is observed (see Table 5.5) that bottom width, flow depth and side slopes of the channel with freeboard is almost the same as the channels without freeboard for Case I. This feature does not exactly conform to those for subround and subangular and round stones. Table 5.5 shows that the bottom widths of the channels with freeboard provision (*b*=7.9983, 8.0898, 8.1045, and 8.0891m) are higher and flow depths (*y*=2.9996, 2.9843, 2.9815 and 2.9844 m) are lower than that of the channel without freeboard (*b*=6.8982 m, *y*=3.1960 m) for sediment-laden flow condition near the channel bottom (see Case II in Table 5). This characteristic is in conformity with the subround and subangular stone and in contradiction to that of the round stones. This feature seems to correlate with the angularilty of the stone particles, which promote their stability on the channel side slopes.

5.2.4.2 *Effect of freeboard scenario options on channel shape (both Cases I-II)*

It is also observed (see Case I in Table 5.5) that the channels having freeboard possess the same magnitude of inclinations for either channel bank, hence their geometric shape remains the same for all freeboard scenarios. However, their freeboard magnitudes differ for different scenario options. For Case II, the channels with freeboard provision possess marginal variations in the side slopes, which eventually alter the magnitudes of bottom width and flow depth both. Hence, different freeboard scenario options for channels having sediment-laden flow condition near the bottom cause dissimilarity in the shape of the channels and their costs. The reason of this feature associates to Manning's roughness coefficient of the channel bottom (a function of fluid, flow and sediment characteristics), which was maneuvered by the optimization algorithm to yield the minimum resistance and hence the reduced construction cost.

5.2.4.3 *Effect of clear water and sediment laden flow conditions on channel specifications (b, y, A_t, T) and cost*

Inter-comparison of the results for Cases I-II reveals that the channel bottom widths and flow depths (except for channel without freeboard) are relatively smaller and side slopes are steeper in Case II (sediment-laden flow condition near channel bottom). The flow velocities (*V*) are also higher (8.3754, 8.4862, 8.4600, 8.4535 and 8.4600 %) in Case II and costs are also lower than those in Case I. This reveals that the optimization process in Case II reduced the value of Manning's roughness coefficient for the channel bottom, which was a function of fluid, flow and sediment characteristics. This, in turn, resulted in the reduced excavated area (compare corresponding freeboard scenario data in column 11 for Cases I-II in Table 5.5) cost of construction as well.

The top width (for a given freeboard scenario) also reduces due to steeper side slopes in Case II (see column 9 in Table 5.5 where *T* appear). Therefore, the total cost of channel construction for sediment laden flow condition near channel bottom (Case II) was relatively low in comparison to that of the Case I where channels retain a fixed value of Manning's roughness coefficient.

5.2.5 Selection of an appropriate freeboard scenario for design

In practice, channels are designed with a pre-assigned magnitude of the freeboard (see row 2 in Tables 5.3-5.5 for freeboard scenario 2), therefore, the same process is chosen as a benchmark for comparison. A closer look at the results of three depth dependent freeboard scenarios in Tables 5.3-5.5 reveals that the scenario $f = 0.25+0.25y^{0.25}$, in comparison to $f = 0.5y^{0.25}$ and $f = 0.25+0.25y^{0.5}$ scenarios, gives the cost relatively closer to that of $f = 0.5$. Therefore, equation $f = 0.25+0.25y^{0.25}$ is suggested to be used for the cost optimality when a depth dependent freeboard provision is implemented. It also looks logical to assume the freeboard as a function of flow depth, as higher risk is involved with high flow depth.

5.2.6 Comparing performance of different stones

It is interesting to note that angular stone revetment with sediment laden flow condition near channel bottom offers the highest flow velocity (1.6537, 1.6555, 1.6551, 1.6550 and 1.6551m/s) as compared to those by subround and subangular (1.5848, 1.5848, 1.5875, 1.5848 and 1.5875 m/s) and round (1.5454, 1.5546, 1.5546, 1.5546 and 1.5547 m/s) stones. This is in conformity to the findings of Gupta and Singh (2012). It alludes to the fact that the angular stones can sustain a higher flow induced shear stress and remain stable on steeper side slopes (compare columns 6-7 of Table 5.5 with those of Tables 5.3-5.4) because of the higher angle of repose. The interlocking capability of the angular stones further adds to their stability similar to what it does in road pavements. They (angular stones) yielded the steepest channel banks (see Table 5.6) and caused the channels with freeboard to acquire the lowest land width for sediment-laden flow condition near channel bottom (Case II). This resulted in the smallest length of side slopes for a given flow depth. After acquisition of the steepest slope, a tradeoff seems to exist between the flow depth and bottom width to yield the minimum cost channel. The potential for sustaining higher shear stresses by angular riprap stones allowed the optimization algorithm to yield a design option that can convey water at high flow velocity, which eventually squeezed the cross sectional area to give reduced cost of excavation. Thus, accounting for the shape of the stone particles in design process, which was not given due consideration until so far in side slope revetment of the earthen channels, has been found to be effective for performance enhancement and cost reduction. Thus, the angular shape of the stone particles finds engineering utility, which was potentially not available with round, and subround and subangular stones and only their mass was exploited for stability purposes. It also reveals that the application of Manning's roughness coefficient per Eq. 3.35 offers an additional flexibility to the optimization algorithm to reduce the magnitude of n_b along with the magnitudes of b, y and m for yielding the minimum cost channel. Thus, the channels having side slopes riveted with angular riprap stone and bottom unlined under sediment-laden flow condition near channel bottom resulted in the minimum cost canals with subround and subangular, and round coming the next in sequence from the cost perspective (see Case II in Table 5.6).

5.2.7 Comparison of the cost of channels riveted with different stones

The construction cost of the channels having angular riprap revetment with sediment-laden flow condition near channel bottom (Case II) required 10.5360, 14.8371, 14.7919, 14.8293 and 14.7805% less costs than those having round stone revetment for five different freeboard scenarios. The similar trend was observed for Case I, where Manning's roughness coefficient for channel bottom was considered to be a fixed value of 0.025. The channels with round stone revetment for clear water flow condition (Case-I) need 19.0975, 20.2504, 20.5106, 20.3821 and 20.5607% higher costs than those for angular stones for five different freeboard scenarios, respectively. The cost of hydraulically most efficient canal obtained by Froehlich (2011a) was also calculated by applying subround and subangular stones with unit cost of the items applied in this study as \$ 303.61/m. It is interesting to note that the construction cost of the canal of Froehlich (2011a) comes out to be 4.49 % higher than that (\$ 290.56/m) obtained in the present study with application of the same type of the stone revetment. It is also important to note that the channel area calculated by the Froehlich (2011a) was 46.6 m^2 and the flow velocity was computed to be 1.2875m/s as compared to 40.5591m^2 and 1.4828m/s (see row 1, columns 11 and 13 in Table 5.4 for Case I), respectively, found in this study. Therefore, it can be inferred that a channel designed for the minimum cost by including land acquisition cost along with the costs of earth excavation and riprap lining yields not only a minimum cost section but also a hydraulically efficient channel. It further suggests the type of the riprap stone that needs to be applied for the cost and performance optimality.

5.3 Concluding Remarks

The present investigation involving three types of riprap stones for the design of channels whose side slopes are riveted with riprap stones and bottom is unlined reveals that the angularity of stone particles is a desirable attribute for ensuring a better stability of stone particles on channels' side slopes, hence, the channels with angular stone revetment emerge out to be the best from a cost and performance perspective. The shape of the trapezoidal channel section comes out to be symmetric for cost effectiveness, if the same lot/bulk of stones is used for the revetment of the either bank of the channel. The capability of angular stones to remain stable on steeper side slopes further enabled the channels to acquire a relatively reduced cross section area with consequential higher flow velocity, thus, it makes the channels to be the most hydraulically efficient. Therefore, it is appropriate to design a canal section having side slopes riveted with angular riprap stones for the minimum cost than for the maximum hydraulic efficiency alone, since the minimum cost design formulation incorporating excavation, land acquisition, and riprap costs accounts for both the minimum cost and the hydraulic efficiency. This novel finding is of huge relevance to the engineering domain. The design of channels with sediment-laden flow condition near channel bottom adds another benefit for the optimization algorithm to reduce the construction cost further down by providing the additional flexibility as compared to the fixed value of the Manning's roughness coefficient under clear water flow condition near channel bottom. The freeboard provision methodology has been found to influence the cost of channel construction. In conformity with the findings of Guo and Hughes (1984) for hydraulically efficient channel, the design of round stone riveted earthen channel becomes narrower for channels with freeboard but the same finding does not hold true for angular, and subangular and subround stones. Hence, their finding cannot be generalized as anticipated by the canal designers. Further, the depth dependent freeboard equation $f = 0.25+0.25y^{0.25}$ is suggested for application, as it looks appropriate to adopt a depth dependent freeboard than a fixed magnitude.