OPTIMAL DESIGN OF EARTHEN CHANNELS: PROBLEM DOMAIN, RATIONALE, DEFINITION AND MODELS

Technical soundness, feasibility and efficient performance, and economic viability are the all-pervasive needs of engineering projects. Construction of canals in irrigation projects attracts a substantial proportion of the capital investment in agricultural sector. Massive investments on the construction of the channels, therefore, draw a diligent attention of researchers for the reduction of cost and improved performance of the canals. A marginal percentage curb on the cost of such channels lead to a significant savings. Therefore, criterion to design a channel for hydraulic efficiency alone seems to be insufficient unless the design procedure attempts to minimize the cost for bringing it below, or equal to, the economic constraint.

3.1 Problem Domain and Rationale

A bulk of literature is available on canal design where attempts were made to either minimize the cost (Guo and Hughes 1984; Loganathan 1991; Kacimov 1992; Swamee et al. 2000b; Das 2000, 2007a, 2007b, 2008; Jain et al. 2004; Bhattacharya 2006; Bhattacharya and Satish 2007, 2008; Chahar and Basu 2009; Blackler and Guo 2009; Easa 2009) of a canal construction or to increase the hydraulic efficiency (Guo and Hughs 1984; Monadjemi 1994; Anwar and de Vries 2003; Atmapoojya and Ingle 2003; Kacimov 2004) of the conveyance system. The literature revealed that these two paramount objectives were always given separate treatment. This scenario led to a scope for the present work where a design methodology needs to be proposed for a unified objective that simultaneously accounts the issue of hydraulically efficiency and cost effectiveness as well. Different combinations of objective functions and constraints were imposed to design the channels for irrigation purpose. The cost effectiveness of earthen channels relative to the lined canals makes them a preferred option. In line with Froehlich (2012), the present work involves the application of riprap stones to protect the erosion of side slopes of the earthen channel. The construction of earthen channels having side slopes riveted with loose riprap and bottom unlined is easy and it does not entail high level of skill set for supervision during construction. The application of riprap stones also results in a relatively cheaper channel construction cost. It is not only easily detectable for any damage but cheaper in repair cost (USACE 1998). It also offers a favorable aquatic environment for living organisms to survive and grow. However, literature on the identification of the most suitable type of riprap stones for the revetment of earthen canal sidewalls from cost perspective is not available. The macro roughness caused due to revetment of channel side slopes by riprap stones need not be of much concern, as the benefits overshadow the efficiency issue. The anticipated adverse impact on hydraulic efficiency needs to be investigated by comparing the results of the present study with the available data in literature. Swamee et al. (2002) estimated that more than

50% of water supplied at the head of the canal is lost through seepage. In fact, seepage loss is viewed as a major problem in earthen irrigation canals, but appropriate means, if devised, can turn it into a boon for watersheds under scarcity. A series of deep wells constructed along the course of rivers/canals in their vicinity can contribute significantly to the ground water recharge during flood seasons, which may be retrieved through those networks of wells during lean period; and the same subsurface storage possess potential to contribute to the base flow during lean period. Swamee et al. (2000b) also stated that the withdrawal from a well near a stream could be derived from the storage in the aquifer as well as from infiltration from the nearby stream. Allowing seepage through earthen canal networks is a debatable issue at this stage until ground level implementation of the suggested provision proves its worth, but the provision seems to add a new dimension to the water resources management area. In the present era of eco-friendly projects leading to sustainable green earth, earthen water conveyance systems may find a significant utility, if appropriate mechanism to alleviate the associated problems is evolved. Therefore, improvisation in design process of riprap riveted earthen channels seems justified.

Various geometric shapes- rectangular, trapezoidal, round cornered trapezoidal, triangular, semi-circular, parabolic, parabolic–bottomed triangle, power-law sections (Swamee and Bhatia 1972; Guo and Hughes 1984; Flynn and Marino 1987; Loganathan 1991; Monadjemi 1994; Swamee

1995; Strelkoff and Clemmens 2000; Babaeyan- Koopaei et al. 2000; Swamee et al. 2000 and 2002; Anwer and de Vries 2003; Godara 2003; Anwar and Clarke 2005; Aksoy and Altan-Sakarya 2006; Chahar 2005, 2006, and 2007; Froehlich 1994 and 2008; Das 2000, 2007, and 2008; Hussein 2008; Easa 2009; and Chahar and Basu 2009) for non-erodible channels were adopted for optimal design with different combinations of objective functions such as hydraulic efficiency, cost, seepage, evaporation, and constraints such as flow rate, side slope stability (Bhattacharjya and Satish 2007), width of right way, sub criticality, and non-scouring etc. Swamee et al. (2000a) applied their formulated equations for assessing seepage loss through the channels, and they found that the trapezoidal channel permits the least seepage in comparison to other channels they investigated; hence a trapezoidal channel is adopted for the optimal design purpose. No literature was found where a non-symmetric trapezoidal channel was adopted, therefore, a non-symmetric trapezoid section is adopted for the present investigation. Having observed the particle swarm optimization to be a relatively recent development, it was chosen for application to the present study.

The performance comparison of different types of the riprap stones (round, subround, subangular, and angular) are available in literature (Gupta and Singh 2012) from hydraulic efficiency perspective but their performance from cost perspective is not available. Hence, Froehlich (2012) expressed his concern for investigation into this aspect of riprap stones. Swamee et al.

(2000b) also suggested that the cost of land acquisition be incorporated in the optimization formulation. Literature on the optimal design of channels incorporating land acquisition cost is also sparse, therefore this aspect has been included in the present study. Designers normally include freeboard as a fixed magnitude of depth (typically 0.5 m); therefore, the present study incorporates the same magnitude as freeboard. In order to assess the impact of freeboard on optimal dimensions of a channel, another channel without freeboard was also designed for the same flow rate. Das (2008) provided a depth dependent freeboard equation and the same was also applied to create different freeboard scenarios to assess the impact of freeboard provision methodologies on optimal channel dimensions and the cost of channel construction.

A canal section normally carries sediment laden flow near channel bottom and it significantly affects the flow characteristics, and hence roughness coefficient in flow equation. Further, the effects of the clear water and sediment-laden flows near channel bottom are also investigated for the optimal channel dimensions and cost. Inclusion of this aspect adds to another novelty in the present work.

3.2 Problem Definition

A design procedure for the minimum cost trapezoidal earthen channel whose side slopes are riveted with riprap stone and bottom is unlined is to be evolved applying an optimization method. Figure 3.1 shows a schematic diagram of the channel that needs to be designed. Several alternative designs are required for investigation into the performances of different types of riprap stones (round, subround, subangular, and angular) applied for the revetment of side slopes. Minimization of the construction cost of the channel will be the objective function of the nonlinear model, and it will include the costs of excavation, riprap revetment and land acquisition for unit length of the channel. To assess the relative performances of different types of riprap stones from cost and stability perspectives, the optimal dimensions of the channels need to be compared. Different freeboard scenarios, including a fixed magnitude and depth-dependent cases, shall be created to generate several alternative designs for riveted earthen channels to assess the impact of freeboard provision methodologies on optimal channel dimensions and cost. For each type of riprap stone, channels without freeboard shall also be designed to act as a benchmark for comparison purpose. In order to investigate the effect of flowing water characteristics on optimal channel dimensions and cost, the clear water and the sediment-laden flow conditions in the channel are considered.

In addition to the above investigations into the optimal designs of earthen channels, a novel optimization algorithm that can handle all types of riprap stones simultaneously in its single computational program to identify the most suitable type of riprap stone for revetment is also developed. The results obtained by the new optimization algorithm shall be compared with those obtained by Particle Swarm Optimization method.

3.3 Model Formulation

In order to determine the mean flow velocity through a prismatic channel, two equations are available. The first uniform flow formula for a prismatic channel having constant shape, longitudinal slope, and roughness was given by Chezy (Rouse and Ince 1963) in 1769 and it relies on the premise that the flow resisting force per unit area of the channel bed is proportional to the square of the mean flow velocity. Another equation, as presently known, is Manning's equation. The Chezy equation appears simple but it has limited applications in practice because the Chezy's coefficient depends on the both, the flow conditions and the channel roughness, and evaluation of its magnitude is difficult (Akan 2006, p.71). Therefore, in the present study, Manning's equation has been chosen for application, which is given as:

$$V = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \tag{3.1}$$

where, n is Manning's roughness coefficient, R is hydraulic radius and S is the longitudinal slope of the channel.

3.3.1 Flow rate through channel

Flow rate through a prismatic open channel having homogeneous wetted surface roughness is obtained by Manning's equation as:

$$Q = \frac{k_n}{n} A_f^{5/3} P_f^{-2/3} S_o^{1/2} \tag{3.2}$$

where k_n, n, A_f, P_f and S_o are the unit conversion constant (1.0 for SI units and 1.486 for U.S. customary units), Manning's roughness coefficient, flow cross-section area, wetted perimeter, and longitudinal channel bed slope, respectively. The geometric features of the channel are determined as:

$$A_f = by + (m_1 + m_2)\frac{y^2}{2} \tag{3.3}$$

$$P_f = \left[\left(\sqrt{1+m_1^2} + \sqrt{1+m_2^2}\right)y + b\right] \tag{3.4}$$

where m_1, m_2, b and y represent the slopes of the canal banks, bottom width, and flow depth, respectively.

The application of riprap stones on side slopes with unlined channel bottom causes roughness differential. It requires an appropriate treatment for determining the flow rate through such channels. Composite roughness and divided channel methods are the two available ways to treat such non-homogeneity in channel surface roughness characteristics. They are briefly described below:

3.3.1.1 *Composite roughness method*

This method proposed by Horton (1933) and Einstein (1934) relies on the presumption that the mean flow velocity through the channel is equal to the subarea mean flow velocity. The equivalent roughness coefficient (n_c) for differentially rough surface characteristics channel is given by:

$$n_c = \left(\frac{n_b^{3/2} P_b + n_s^{3/2} P_{s1} + n_s^{3/2} P_{s2}}{P} \right)^{2/3} \tag{3.5}$$

$$P = P_b + P_{s1} + P_{s2} \tag{3.6}$$

where

$$P_b = b \tag{3.7}$$

$$P_{s1} = y\sqrt{1 + m_1^2} \tag{3.8}$$

$$P_{s2} = y\sqrt{1 + m_2^2} \tag{3.9}$$

where m_1, m_2, b, y, P_b, P_{s1} *and* P_{s2} represent the channel side slopes, bottom width, flow depth, and wetted perimeters corresponding to bottom, left and right side banks of the channel, respectively. n_s and n_b represent Manning's roughness coefficients for channel side slopes and bottom, respectively.

3.3.1.2 *Divided cannel method*

This method entails the division of the channel cross section area into subareas (Yen 1991) with each subarea treated individually as a small channel. Flow through each subarea is determined by involving their respective flow area, wetted boundary, associated roughness coefficient, and slope (refer Figs. 3.1 of Yen 2002). The flows through each subarea are added together to find the flow rate through channel cross section area.

The application of protective riprap stones to canal side slopes induces complicated flow patterns near side walls, which, in turn, cause a large variation in the shear stress distribution on the wetted perimeter of the canal. To adequately address the large variation in surface roughness characteristic of channel side walls and bottom for determining the flow rate through the canal, the divided channel method (DCM) suggested by Yen (1991, 2002) was applied with Manning's equation instead of applying a composite roughness coefficient method (Horton 1933; Einstein 1934). Figs. 3.1 shows a schematic diagram for divided channel method (DCM) applied to the canal. Figs. 3.1 also represents the divided sub-sections of the channel and their areas are:

$$A_I = (m_1 y)\frac{y}{2} \tag{3.10}$$

$$A_{II} = by \tag{3.11}$$

$$A_{III} = (m_2 y)\frac{y}{2} \tag{3.12}$$

The discharges through the three subareas shown in Figures 3.1 are determined as:

$$q_I = \frac{k_n \sqrt{S_o}\left[m_1 y^2\right]^{5/3}}{2^{5/3} n_s \left[y\sqrt{1+m_1^2}\right]^{2/3}} \tag{3.13}$$

$$q_{II} = \frac{k_n \sqrt{S_o}\left[by\right]^{5/3}}{n_b b^{2/3}} \tag{3.14}$$

$$q_{III} = \frac{k_n \sqrt{S_o}\left[m_2 y^2\right]^{5/3}}{2^{5/3} n_s \left[y\sqrt{1+m_2^2}\right]^{2/3}} \tag{3.15}$$

Therefore, the total discharge through the channel is calculated as the sum of discharges through individual sub-areas of the canal section (see Figs. 3.1) as:

$$Q = q_I + q_{II} + q_{III} \tag{3.16}$$

$$\therefore Q = \frac{k_n \sqrt{S_o}\left[by\right]^{5/3}}{n_b b^{2/3}} + \frac{k_n \sqrt{S_o}\left[m_1 y^2\right]^{5/3}}{2^{5/3} n_s \left[y\sqrt{1+m_1^2}\right]^{2/3}} + \frac{k_n \sqrt{S_o}\left[m_2 y^2\right]^{5/3}}{2^{5/3} n_s \left[y\sqrt{1+m_2^2}\right]^{2/3}} \tag{3.17}$$

where n_s is Manning's roughness coefficient for either side slope of the channel and q_I, q_{II} and q_{III} are flows through three subareas of the channel cross section.

3.3.2 Manning's roughness coefficient for channel side slopes

Manning's roughness coefficient for loose riprap protected canal side slopes is related with the mean size of stones as:

$$n_s = \frac{k_n k_D D^{1/6}}{\sqrt{g}} \tag{3.18}$$

where k_D, D and g represent the coefficient depending on the rock shape and angularity, riprap stone median size (size larger than 50% of the particles in the mixture), and gravitational acceleration, respectively. Maynord (1991) conducted experimental studies at Hydraulics Laboratory of Waterways Experiment Station, U. S. Army Corps of Engineers, USA (Technical Report HL-88-4), and suggested k_D = 0.145 for determining flow resistance of crushed quarry stones which vary in their shape from angular to subangular, whereas Strickler (1923) proposed it to be 0.148 for natural channels constituted by gravels and cobbles. The protective stones for the construction of channels are normally procured from stone quarries, therefore, value of the k_D suggested by Maynord (1991) as 0.145 for flow resistance of crushed quarry stones is adopted in the present study in tune with Froehlich (2011a).

3.3.3 Shear stresses for incipient motion of riprap stone particles

In order to determine the critical shear stresses at which motion of a riprap stone on channel side slope is about to occur, the tractive force approach (Carter et al. 1953; Lane 1955; Terrell and Borland 1958), recommended by U.S. Bureau of Reclamation (USBR) and National Cooperative Highway Research Program (NCHRP) report 568 for design of riprap lined channel is adopted. Froehlich (2011a) also applied the same to obtain the critical shear stresses (τ_{cs1} and τ_{cs2}) exerted by fluid flow on the riprap stones of diameter *D riveted on the side slopes* of the channel.

3.3.3.1 *Left side slope*

The critical shear stress exerted by fluid flow on riprap stones laid on the left bank of the channel is:

$$\tau_{cs1} = N_s(G-1)\rho g D\sqrt{1-\frac{k_\phi}{1+m_1^2}} \tag{3.19}$$

3.3.3.2 *Right side slope*

The critical shear stress exerted by fluid flow on riprap stones laid on the right bank of the channel is:

$$\tau_{cs2} = N_s(G-1)\rho g D\sqrt{1-\frac{k_\phi}{1+m_2^2}} \tag{3.20}$$

where

$$k_\phi = 1/\sin^2\phi \tag{3.21}$$

where N_s , G, and ϕ represent the critical Shields number for the flow over a horizontal bed, stone particle specific gravity, and rock mass angle of repose, respectively. The fully rough flow conditions prevailing on riprap-lined channel side slopes yields the critical Shields number (Froehlich 2011a; and Millar 2005) as:

$$N_s = 0.048 \times \tan\phi \tag{3.22}$$

Further, the same bulk of riprap stones obtained from a single quarry is presumed to be used for the revetment of channel side slopes, therefore, the same size (D) riprap stone is involved in the expressions of critical shear stresses acting on a riprap stone.

3.3.4 Flow induced shear stresses on canal side slopes

The flow induced shear stresses acting on side slopes and bottom of a canal are found to be non-uniform (Chien et al. 1997, p. 335; Akan 2006, p.164), and their magnitudes are needed to evaluate the stability of riprap stones on channel side slopes.

The magnitude of maximum shear stress (τ_s) exerted by fluid flow on channel side slope is expressed in terms of the longitudinal shear stress acting on a horizontal bed of an infinitely wide channel as:

$$\tau_s = k_{\tau s}\rho\, g\, y\, S_o \tag{3.23}$$

where ρ is the fluid density and $k_{\tau S}$ is dimensionless bank shear-stress multiplier respectively. The dimensionless bank shear-stress multiplier ($k_{\tau S}$) for subcritical flow condition is needed to apply the above equation. By using the boundary shear stress data measured in an experimental trapezoidal channel, built at The University of Birmingham, UK, that had side slopes roughened with gravels and bottom smooth (Al-Hamid 1991), Knight et al. (1992, 1994) gave an expression for the dimensionless bank shear-stress multiplier ($k_{\tau s}$) for subcritical flow condition as:

$$k_{\tau s} = 0.01 \frac{R}{y} C_{sf} e^{\alpha} \left[2.37 + \left\{ \frac{P_b}{P_s} \right\}^{0.85} \right] \tag{3.24}$$

where $R = A / P$

$$P_s = y \left(\sqrt{1 + m_1^{\,2}} + \sqrt{1 + m_2^{\,2}} \right) \tag{3.25}$$

$$C_{sf} = \begin{cases} 1.0 & if \ \left(\dfrac{P_b}{P_s} \right) \le 6.546 \\ 0.5857 \times \left(\dfrac{P_b}{P_s} \right)^{0.2847} & otherwise \end{cases} \tag{3.26}$$

$$\alpha = -3.230 \log \left(\frac{P_b}{C_2 P_s} + 1 \right) + 4.605 \tag{3.27}$$

where $C_2 = 1.5 \left(\dfrac{k_{sw}}{k_{cb}} \right)^{0.21}$ (3.28)

where k_{sw}, and k_{cb}, represent the Nikuradse equivalent grain roughness heights for the side slopes and channel bottom; P_s shows wetted perimeter corresponding to the canal side slopes, respectively; and R denotes the hydraulic radius. Since Manning's roughness coefficient relates to the Nikuradse sand grain roughness (Mostafa and McDermid 1971; Dooge 1991; Yen 1991, 1992, and 2002) as:

$$n \propto k_s^{1/6} \tag{3.29}$$

where k_s is the Nikuradse sand grain roughness.

Therefore, Eq. 3.28 can be expressed in terms of Manning's roughness coefficients as:

$$C_2 = 1.5 \times (n_s / n_b)^{1.26} \tag{3.30}$$

where n_s and n_b are Manning's roughness coefficients for canal side slopes and bottom respectively.

3.3.5 Factor of safety against flow induced shear stresses

In order to ensure the stability of a riprap stone particle on channel side slope, the critical shear stress (τ_{cs}) at which a riprap stone particle is onset to move, requires to be higher than the flow induced maximum shear stress (τ_s) by an adequate factor of safety as:

$$f = \frac{\tau_{cs}}{\tau_s} \tag{3.31}$$

3.3.6 Stability of riprap stones

The stability of the riprap stones can thus be ensured by allowing the stabilizing forces to exceed the flow-induced shear stresses. Therefore, the following equations need to be satisfied at limiting condition by riprap stones riveted on the left and right banks of the channel, respectively:

$$\tau_{cs1} - f\tau_s \geq 0 \tag{3.32}$$

$$\tau_{cs2} - f\tau_s \geq 0 \tag{3.33}$$

3.3.7 Types of riprap stones and their characteristics

United States Bureau of Reclamation (USBR 1990, p. 208) classified four types of riprap stones (round, subround, subangular and angular) based on their angularity feature, and Froehlich (2011b), by conducting experiments with a large heap of stones, determined the mass angles of repose for these stones. He classified these stone particles in three groups as per their mass angles of repose obtained by his experiments. Thus, his experimental finding constitutes them into three classes as:

$$\phi = \begin{cases} 31.8^0 & \text{for round stone} \\ 34.6^0 & \text{for subround and subangular stone both} \\ 38.4^0 & \text{for angular stone} \end{cases} \tag{3.34}$$

These three types of riprap stones are adopted in this present study to compare their performances from cost and stability perspectives. Since this classification is based on the angle of repose, therefore, it appropriately describes the shape-feature of the stone particles that need to be investigated in the present study for optimal design purpose.

3.3.8 Flowing water characteristics

The published literature on optimal design problems always considered clear water flow through channels, and hence adopted a fixed value of Manning's roughness coefficient for channel design procedure. Since the flow in an earthen canal carries substantial sediment proportion particularly near the canal bottom, therefore, the present study investigates the effect of flowing water characteristic by considering the flow to be clear water and sediment-laden similar to an alluvial channel. Thus, Manning's roughness coefficients (Yen 1991, 2002) for the two different cases shall vary, and they are handled as described below.

3.3.8.1 Clear water flow

The clear water flow condition in the channel needs a constant value of Manning's roughness coefficient for canal bed (n_b). It is adopted as 0.025 in tune with Froehlich (2011a).

3.3.8.2 *Sediment-laden flow*

In order to account the effect of sediment-laden flow in channels, the bed form characteristics near the bottoms of canal and alluvial channel were treated alike and Manning's roughness coefficient (n_b) for canal bed was determined by the expression suggested by Yen (2002) as:

$$n_b = \begin{bmatrix} R^{1/6} \dfrac{1.16 T^{*0.175}}{c_n \mathrm{Re}^{0.19}} \; for\ F < 0.4 \\ d_{50}^{1/6} \dfrac{0.054}{c_n} \left(\dfrac{d_{50}}{R} \right)^{-0.04} \dfrac{\mathrm{Re}^{0.05}}{F^{0.88}} \; for\ 0.4 \le F < 0.7 \\ d_{50}^{1/6} \dfrac{0.17}{c_n F^{0.15}} \; for\ 0.7 \le F < 1 \\ d_{50}^{1/6} \dfrac{0.17}{c_n F^{0.45}} \; for\ 1 < F < 2 \end{bmatrix} \tag{3.35}$$

$$T^* = S_w \frac{\left(R / d_{50} \right)}{\left(\frac{\rho_s - \rho}{\rho} \right)} \tag{3.36}$$

$$\mathrm{Re} = VR/\nu \tag{3.37}$$

where dimensionless constant's (c_n) value is 3.132 when k_n value is 1 $m^{1/2}/s$ for the SI unit system in Manning's equation. V, Re, d_{50}, F, ρ_s, v and S_w are the mean flow velocity, Reynolds number, mean sediment size, Froude number, density of sediment particles, kinematic viscosity of water, and water surface slope relative to the horizontal (= S_o for uniform flow), respectively.

3.3.9 Freeboard scenario

A three-parameter freeboard (*f*) equation proposed by Das (2008) was adopted in the present study to include the effect of freeboard in the optimal design of an earthen canal. It creates different options of freeboard provision either as a function of flow depth or as a fixed value according to the values of the three involved parameters. The same expression was applied to create different freeboard scenarios as:

$$f = k_1 + k_2 y^x \tag{3.39}$$

where k_1 (m), k_2 (m^{1-x}), y (m) and x represent dimensional constants, depth of flow and dimensionless constant, respectively The values of parameters in Eq. 3.39 were varied (Easa 2009) to obtain five different freeboard scenarios. Comparison of the results obtained for various freeboard scenarios enables one to know how the cost optimality is influenced by different freeboard provisions. The freeboard scenario with values of $k_1 = k_2 =0$ yields a channel without freeboard and $k_1 = 0.5$ and $k_2 = 0$ leads to a channel with a fixed magnitude of freeboard.

3.4 Minimization Problem Statement

Design of a channel having riprap riveted side slopes and unlined bottom constitute a non-linear optimization problem, and is formulated for the minimization of its construction cost with associated constraints due to riprap stone stability hereunder.

3.4.1 Objective function

Since the costs of different types of riprap stones are not the same, therefore, three classes of riprap stone require different objective functions to be formulated with their respective costs, and thus they constitute three different problems to be solved separately using an optimization technique. The minimization type objective function in terms of cost of unit length ($/m) for three respective classes of riprap stone are given below.

3.4.1.1 Cost model for round riprap stone application

The costs of earth excavation, riprap revetment and land acquisition have been added together to get the cost of channel construction per unit length of channel. Land acquisition and earth excavation costs are the same for all cases. The cost of per unit length of channel with round riprap stone revetment needs to be minimized and is determined as:

$$\text{Minimize } C_{round}(b, m_1, m_2, y, D) = c_1 A_t + c_r P_s + c_3 T \tag{3.40}$$

3.4.1.2 Cost model for subround and subangular riprap stone application

Likewise, cost of per unit length of the channel with subround and subangular riprap stone revetment that needs to be minimized is:

$$\text{Minimize } C_{subround \ and \ subangular}(b, m_1, m_2, y, D) = c_1 A_t + c_{sr} P_s + c_3 T \tag{3.41}$$

3.4.1.3 Cost model for angular riprap stone application

The cost of per unit length of the channel with angular stone revetment is determined as:

$$\text{Minimize } C_{angular}(b, m_1, m_2, y, D) = c_1 A_t + c_a P_s + c_3 T \tag{3.42}$$

where c_1, c_r, $c_{sr,}$ $c_{a,}$, and c_3 are the earth excavation cost per unit volume (\$/m^3), costs of three types of riprap stones (\$/m^2), and the land acquisition cost (\$/m^2) per unit length of the canal, respectively. The bulk mass of riprap stones is commercially available in USD per cubic meter or cubic yard. Therefore, its cost needs to be converted to the per meter square area of the inclined surface associated with channel side slopes. The costs of the three types of the riprap stones are commercially available in physical unit *$/yard^3*. It was first converted to *$/m^3*, and multiplied by the thickness of riprap stone surface to determine its cost in terms of *$/m^2*. A_t, P_s, and T denote the total excavated area, wetted perimeter corresponding to the side slopes, and top width (T) of the canal, including freeboard, as:

$$A_t = b(y+f) + (m_1 + m_2)\frac{(y+f)^2}{2} \tag{3.43}$$

$$P_s = \left[\left(\sqrt{1+m_1^{\,2}} + \sqrt{1+m_2^{\,2}}\right)(y+f)\right] \tag{3.44}$$

$$T = b + (y+f)(m_1 + m_2) \tag{3.45}$$

3.4.1.4 Constraints

The objective functions (Eqs.3.13-3.15) are associated with each of the two flow rate constraint equations separately as:

$$\frac{Q}{k_n\sqrt{S_o}} - \left[\frac{[by]^{5/3}}{n_b b^{2/3}} + \frac{\left[m_1 y^2\right]^{5/3}}{2^{5/3} n_s \left[y\sqrt{1+m_1^2}\right]^{2/3}} + \frac{\left[m_2 y^2\right]^{5/3}}{2^{5/3} n_s \left[y\sqrt{1+m_2^2}\right]^{2/3}}\right] = 0 \tag{3.46}$$

The shear stresses acting on stable riprap stones laid on the canal side slopes constitute other constraints, and they were formulated using Eqs. 3.19-3.22 as:

$$N_s(G-1)\rho g D\sqrt{1-\frac{k_\phi}{1+m_1^{\,2}}}-f_s\tau_s=0 \tag{3.47}$$

$$N_s(G-1)\rho g D\sqrt{1-\frac{k_\phi}{1+m_2^{\,2}}}-f_s\tau_s=0 \tag{3.48}$$

A summary of the sets of problem formulated with different combination of equations are given in Table 1.

3.5 Non-Dimensionlization of Equations

The sets of non-linear optimization problem were formulated in dimensionless equations. Non-dimensionalization exercise translated the problems independent of units in addition to scaling down of actual search space into a compressed search space which makes the search easier and computationally efficient. Problem associated parameters were used to develop a scaling parameter λ for non-dimensionalization purpose, and decision variables were transformed dimensionless as:

$$b_* = b/\lambda;$$

$$y_* = y/\lambda;$$

$$D_* = D/\lambda;$$

$$f_* = f / \lambda$$

$$A_{t*} = b_*(y_* + f_*) + (m_1 + m_2)\frac{(y_* + f_*)^2}{2} \tag{3.49}$$

$$P_{s*} = \left[\left(\sqrt{1 + m_1^{\ 2}} + \sqrt{1 + m_2^{\ 2}}\right)(y_* + f_*)\right] \tag{3.50}$$

$$T_* = b_* + (y_* + f_*)(m_1 + m_2) \tag{3.51}$$

where λ is a scaling parameter expressed as:

$$\lambda = \left(\frac{Q^2}{gs_o}\right)^{1/5} \tag{3.52}$$

The cost terms in Eqs. 3.19-3.22 were non-dimensionalized using costs of earth excavation (c_1) and scaling parameter (λ) as:

$$C_* = \frac{Cost}{c_1\lambda^2};$$

$$c_{r*} = \frac{c_r}{c_1\lambda};$$

$$c_{sr*} = \frac{c_{sr}}{c_1\lambda};$$

$$c_{a*} = \frac{c_a}{c_1\lambda};$$

$$c_{3*} = \frac{c_3}{c_1\lambda}$$

Manning's roughness coefficients for channel side slope and bottom were also made dimensionless in tune with the Froehlich (2012) as:

$$n_{s*} = \frac{n_s\sqrt{g}}{k_n\lambda^{1/6}} = k_D D_*^{1/6} \tag{3.53}$$

and

$$n_{b*} = \frac{n_b\sqrt{g}}{k_n\lambda^{1/6}} \tag{3.54}$$

3.5.1 Dimensionless objective function

Hence, the dimensionless objective functions (Eqs. 3.40-3.42) take the following form:

Model 1*: round stone*

$$\text{Minimize} \quad C_{r*}\left(b_*, m_1, m_2, y_*, D_*\right) = A_{t*} + c_{r*}P_{s*} + c_{3*}T_* \tag{3.55}$$

Model 2*: subround and subangular stone*

$$\text{Minimize} \quad C_{sr*}\left(b_*, m_1, m_2, y_*, D_*, \phi\right) = A_{t*} + c_{sr*}P_{s*} + c_{3*}T_* \tag{3.56}$$

Model 3*: angular stone*

$$\text{Minimize} \quad C_{a*}\left(b_*, m_1, m_2, y_*, D_*, \phi\right) = A_{t*} + c_{a*}P_{s*} + c_{3*}T_* \tag{3.57}$$

3.5.2 Dimensionless constraint equations

The application of dimensionless terms defined above also transforms Eqs. 3.46-3.48 as:

$$1 = \left[\frac{(b_* y_*)^{5/3}}{n_{b*} {b_*}^{2/3}} + \frac{\left(m_1 {y_*}^2\right)^{5/3}}{2^{5/3} k_D D_*^{1/6} \left(y_* \sqrt{1+m_1^2}\right)^{2/3}} + \frac{\left(m_2 {y_*}^2\right)^{5/3}}{2^{5/3} k_D D_*^{1/6} \left(y_* \sqrt{1+m_2^2}\right)^{2/3}} \right] \quad (3.58)$$

Thus, the dimensionless flow rate constraint model relating to the non-linear optimization problem can be written as:

$$\Psi_2(b_*, y_*, m_1, m_2) = 1 - Y = 0 \quad (3.59)$$

Where' $Y = \left[\frac{(b_* y_*)^{5/3}}{n_{b*} {b_*}^{2/3}} + \frac{\left(m_1 {y_*}^2\right)^{5/3}}{2^{5/3} k_D D_*^{1/6} \left(y_* \sqrt{1+m_1^2}\right)^{2/3}} + \frac{\left(m_2 {y_*}^2\right)^{5/3}}{2^{5/3} k_D D_*^{1/6} \left(y_* \sqrt{1+m_2^2}\right)^{2/3}} \right]$

The dimensionless constraint equations due to riprap stone stability on side slopes are:

$$\Psi_3(b_*, y_*, m_1, m_2, D_*) = k_{\tau s} y_* - KD_* \sqrt{1 - \frac{k_\phi}{1+m_1^2}} = 0 \quad (3.60)$$

$$\Psi_4(b_*, y_*, m_1, m_2, D_*) = k_{\tau s} y_* - KD_* \sqrt{1 - \frac{k_\phi}{1+m_2^2}} = 0 \quad (3.61)$$

where $K = \frac{N_s(G-1)}{f_s s_o}$ is a dimensionless critical side slope shear-stress coefficient (Froehlich 2011a).

An optimization technique needs to be applied to solve the sets of problem related to three classes of riprap stones. A pre-assigned type of riprap stone for the design of riprap riveted channel is used to solve a given set of problem. Hence, three separate programs are to be coded.

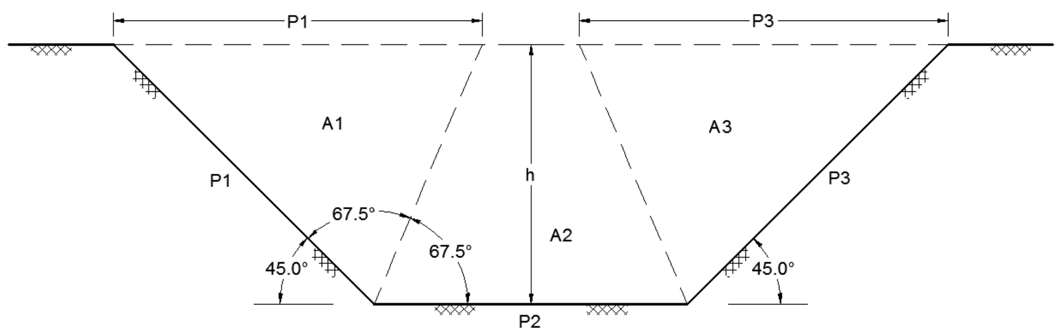

Fig. 3.1 (a) Divided cannel method (Yen 2002)

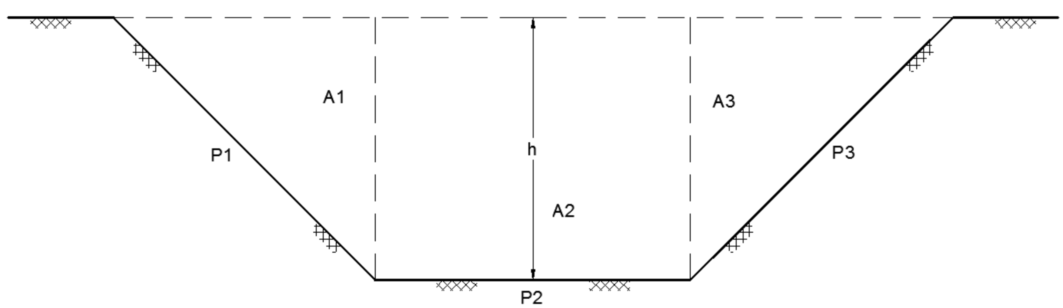

Fig. 3.1 (b) Alternative way of dividing cannel (Yen 2002)