ULTIMATE BEARING CAPACITY OF AXISYMMETRIC AND PLANE-STRAIN FOUNDATIONS RESTED OVER SATURATED/UNSATURATED STRATUM USING LIMIT ANALYSIS

A Thesis

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Chapter 10

SUMMARY, CONCLUSIONS, AND FUTURE SCOPE

10.1 SUMMARY

The present thesis demonstrates the utilization of finite element limit analysis in computing the bearing capacity of footings over saturated and unsaturated stratum. In the first part of the thesis, the ring footings over saturated soils and rocky medium are analyzed by using the axisymmetric lower bound finite element limit analysis in association with the three-dimensional yield surfaces and nonlinear optimization. The yield surfaces that are chosen to represent the cohesive-frictional soils, undrained clays, and rocks are based on the smoothened form of the Mohr-Coulomb, Tresca, and Hoek-Brown yield criteria, respectively. The canonical formulations of the lower-bound bearing capacity problem consist of the following components: (a) the linear objective function generated by the integration of the vertical normal stresses below the footing base, (b) the linear equality constraints, which account for the stress equilibrium equations, stress discontinuity conditions, and stress boundary restrictions, (c) the linear and the nonlinear inequality constraints arises out of the soil-footing interface and the C^2-continuous yield functions, respectively. The nonlinear optimization is performed with the help of the interior point method after adding the logarithmic barrier parameter to the objective function. The barrier parameter reduces in each iteration and approaches zero to provide a lower-bound stress field at the optimal point. The technique is first employed to determine the bearing capacity factor for pile footing embedded in saturated normally-consolidated clays whose cohesion increases linearly with depth. Thereafter, the ultimate bearing capacities of ring footing rested over homogenous as well as layered geomaterials are rigorously evaluated by duly considering the influence of different loading positions, surcharge, and footing roughness. The different layered soils considered in this thesis are (a) sand over sand, (b) clay over clay, and (c) sand over clays. The combined effects of various geometrical factors (such as radius ratio, upper-layer thickness), material properties (i.e. layer strength, footing roughness), loading configurations (i.e. loading positions over the ring), and boundary constraints (i.e., surcharge pressure) are extensively investigated, and the corresponding design charts are prepared. The failure patterns which illustrate the extent and intensity of the inelastic regime are drawn for a few cases on the basis of the closeness of the Mohr circle pertaining to the current state of stress to the Mohr yield circle.

In the second part of the thesis, the strip footings over unsaturated soils are analyzed by using the plane strain upper bound finite element limit analysis in association with the Mohr-Coulomb planar yield surface and linear optimization. The water flow in the vadose zone (i.e. above the groundwater table) is idealized by Darcy's linear flow law and Gardner's one-parameter hydraulic conductivity function. The strength analysis is performed by considering the suction stress-based effective stress formulation conceptualized by Prof. Lu and his co-workers in the early twenty-first century. Considering the steady-state water flow, the unsaturated bearing capacity of strip footing is computed for vertical, concentric loading and inclined, eccentric loading. The unsaturated property functions are incorporated in the numerical simulation by employing van-Genuchten's soil water characteristics curve. An attempt has also been made to perform the bearing capacity analysis of strip footing considering the transient flow of water in the vadose zone. Richards one-dimensional transient flow equation and Gardner's soil water characteristics curves capture the time-dependent suction stress profiles. The nodal velocity contours provide valuable insights into the collapse pattern.

In summary, the thesis has a twofold contribution: (a) developing an axisymmetric LB-FELA technique based on nonlinear optimization and applying it to compute the load-carrying capacity of ring foundation, and (b) proposing a suction-stress-based plane-strain UB-FELA technique and applying it for strip footing analysis on unsaturated soils by considering the steady-state and transient water flow. Three-noded linear triangular elements are invariably used for discretizing the domain, and the computational codes are developed in MATLAB (version R2019a).

10.2 CONCLUSIONS

Based on the findings presented in the thesis, the following conclusions can be drawn:

- The bearing capacity analysis of the pile foundation embedded in linearly varying cohesive soils reveals that there is a nonlinear enhancement of N_c with the normalized pile length upto a certain dimension; beyond that, any further increment of pile length (for a constant pile diameter) hardly makes any influence on the N_c factor. Furthermore, the roughness along the pile base and shaft had a noticeable impact on the bearing capacity, with the roughness along the pile shaft playing a more significant role than the pile base.

- The improvements in the bearing capacity due to the inclusion of a sandy layer in between the ring footing and undrained clays are rigorously examined and reported in terms of the efficiency factor (η); where η>1 indicates the benefit of the insertion of sand between the footing and the weak clays. It seems that η increases with the increase in surcharge pressure, footing roughness (δ), and sand-layer thickness but decreases with the increase in radius ratio (r_i/r_o). Even including a thin sand layer between the footing and the weak clays enhances the load-carrying capacity manifold. The benefits are much more prominent for perfectly rough ring footing with a relatively large annulus section. With the increment of footing roughness, the load withstanding capacity enhances up to a certain δ value; beyond that η remains to be constant. Depending on the surcharge loading, layer strength, and ring geometry, there exists an optimum sand layer thickness (h_{opt}) up to which the η-parameter either increases or decreases. The normalized optimum thickness increases with the increase in sand friction, surcharge pressure, and soil-footing roughness, whereas it decreases with the increase in r_i/r_0 and undrained cohesive strength of the underlying layer. The η profiles with respect to r_i/r_0 take different shapes. Due to the variation of the layer strength and sand layer thickness, the η versus r_i/r_0 may show (i) an increasing trend, (ii) a decreasing trend, or (iii) an inverted 'V' shape profile. The extent of the plastic zone and the size of the elastic wedge developed beneath the footing substantiate the numerical outcomes.

- By considering complete loading (L_0) and three partial loadings, the bearing capacity of ring footing over two layered soil systems is rigorously computed. The chosen partial loading positions are the inner half (L_1), middle half (L_2), and outer half (L_3). For the specific strength parameters and surcharge loadings, irrespective of the layer profile, the allowable loading intensity is noticeably higher for any partial load arrangement in comparison to the complete loading system. Amongst the chosen partial loadings, the UBC is maximum for L_1 and becomes least for the L_3 position. The results are presented in terms of normalized bearing capacity ($p_u/\gamma(r_0–r_i)$) and efficiency factor (η). The efficiency factor and optimum top layer thickness are found to be independent of loading positions. The key observations pertaining to the three different two-layered systems are as follows:

- The study on sand-sand layered system identifies the existence of optimum top layer thickness (h_{opt}/r_0), critical radius ratio ($r_i/r_0|_{cr}$), and specific radius ratio ($r_{i,sp}/r_{0,sp}$). The magnitude of h_{opt}/r_0 and $r_{i,sp}/r_{0,sp}$ are based on the condition where the strength of the bottom layer has an insignificant role in determining the bearing capacity. The h_{opt}/r_0 is found to be smaller for smooth ring footing in comparison to its rough counterpart. The $r_i/r_0|_{cr}$ is the radius ratio at which the bearing capacity is found to be maximum especially, for higher friction angle with surcharge loading. The existence of $r_i/r_0|_{cr}$ is not visible for L_1 condition. The existence of $r_i/r_0|_{sp}$ becomes more prominent when a weaker sand layer overlies strong sand deposits. Beyond $r_i/r_0|_{sp}$, the UBC curves start to converge for both smooth as well as rough footing. The numerical solutions give an impression that the annular footing is far more beneficial than the circular one if the ring footing is placed on the strong (top)-weak(bottom) sandy system with surcharging and under the action of L_2 or L_3 load type.

- A rigorous investigation is also carried out to evaluate the UBC of the ring footing placed over two different type of layered clays: weak clays over strong clays (Type B1) and strong clays over weaker clays (Type B2). The p_u curves appear to be sharply decreasing, almost constant, and noticeably increasing for L_1, L_2, and L_3 loadings, respectively. The effect of surcharge pressure appears to be more significant for soil Type B2 and L_2 loading position but does not get affected by the r_i/r_0 ratio. The undrained cohesive strength of the bottom layer does not influence the bearing capacity of soil Type B1, but plays a significant role in determining the UBC of soil Type B2.

- The lower bound ultimate bearing capacity is also computed for the ring footing lying over drained sand, which is underlain by fully cohesive strata and subjected to various loading arrangements. The variations in loading arrangement result in a notable discrepancy in the trend of the UBC curves, especially when the bottom clay layer is relatively weak and the inserted sand layer is thin ($h/r_0 \leq 0.5$). The $r_i/r_0|_{cr}$ becomes more prominent for L_3-type loading and in the presence of surcharge pressure. The study reveals that ring footings are preferable to circular ones if the cohesive strength of the underlying clay layer, lying below a relatively thinner sandy layer ($h/r_0 \leq 0.5$), is less than some specific cohesive strength ($c_u/(\gamma r_0)|_{sp}$); the magnitude of $c_u/(\gamma r_0)|_{sp}$ depends on the load type, surcharge pressure, and footing roughness. In most cases, with the increase in sand layer thickness, there is a significant improvement in UBC, especially when the added sand layer is of higher strength. However, when a thin ring footing is placed over relatively weaker sands, without surcharging, and underlain by stronger clays, the efficiency factor decreases.

- The impact of partial loading on the UBC of ring footing resting on a homogenous rock layer has been analyzed by varying the rock strength parameters, overburden pressure, and ring geometry. Similar to the soil profile, the UBC of the partially-loaded ring footing rested on rocks is reported to be maximum for L_1 and minimum for L_3-type loading. The L_0-type loading generates lower UBC than any of its partial loading counterparts. The ring footing is most effective for outer half loading, while a solid circular footing yields the highest ultimate bearing capacity for inner half loading. The variation of *GSI* parameter increases the nonlinearity of UBC profiles in comparison to the variation of m_i parameter. The UBC of the ring footing increases drastically if the *GSI* of the underlying rock is greater than 60. The UBC curves corresponding to various ring sizes are distinctive and unique in nature if the applied loading is of L_1-type, whereas the gap between the UBC curves emerges due to L_3-type of loading is relatively minimal. The L_2- and L_3-type loading exhibits either a pronounced peak point or a range of radius ratio at which the UBC is maximum; the prominence of the pronounced peak radius ratio becomes more visible with the incorporation of the overburden pressure. In a general sense, the circular footings are favorable for L_1 loading; whereas, ring footings are found to be advantageous for L_2 and L_3 loadings.

- By accounting for the suction in the vadose zone lying over the groundwater table, the upper bound limit analysis formulations are modified for evaluating the unsaturated UBC of the strip foundation. Due to the consideration of the gradient of the yield functions, the constraints arising from the elements and the shared interfaces are exactly the same as the saturated formulation, but the power dissipation (within the elements and along the discontinuity lines) terms in the objective function are adequately modified to consider the effect of water table fluctuations, seasonal changes, and the steady-state water flow within the variably saturated zone. Instead of matric suction, the suction stress is used as the fundamental stress state variable in the effective stress formulation as well as in formulating the MC yield envelope and the MC yield circle.

- By employing the modified UB-FELA technique, the UBC of strip footing resting on unsaturated homogenous sands is investigated by varying the flow conditions, hydromechanical properties, groundwater table position, and unsaturated properties. The partially saturated soils exhibit greater bearing capacity factors than fully saturated ones. Depending on the soil strength and soil-footing roughness condition, a critical water table position (h_{wcr}) is reported that maximizes the computed bearing capacity. The h_{wcr} appears earlier for the ponderable, surcharge-less soils than the weightless unsaturated soils with surcharge pressure. Regardless of the frictional strength of the sand, the UBC is reported to be higher for well-graded sand (characterized by lower pore spectrum number) and high air-entry value. For soils with low air entry values, the saturated coefficient of permeability and the flow conditions hardly impact the computed UBC factors. This study reveals that the asymmetry parameter of the vG SWRC model has a significant influence on the computed UBC. The nature and the magnitude of the nodal velocity vectors in the velocity contours further substantiate the variation of UBC with respect to the change in the mechanical properties, unsaturated soil properties, hydrological boundary conditions, and unsaturated soil properties.

- The extensive design charts are prepared to show the impact of the eccentric/ oblique loading on the computed UBC of strip footing rested on unsaturated sands. The load inclination and the eccentricity reduce the load-bearing capacity nonlinearly and to a significant extent. The reduction of UBC due to load inclination hardly gets impacted due to the variation of air-entry value; nevertheless, the desaturation rate has an appreciable influence on the inclination-induced UBC reduction. The narrow distribution of the pore-size and deeper water table depth diminishes the impact of air-entry value on the load-carrying capacity. The size and shape of the normalized *V-H* envelopes corresponding to unsaturated soil properties and groundwater table position are also critically scrutinized. Nodal velocity contours for specific cases provide insight into the velocity vectors and extent of the failure zone.

- The consideration of transient flow in the vadose zone shows how the load-withstand capacity of the strip foundation rested on unsaturated soil gets impacted by the combined effect of infiltration rate and duration. The results are presented in terms of normalized bearing capacity. No matter whatsoever the sand's hydro-mechanical properties, the computed UBC always appears to be higher for a lower infiltration rate. Not only the magnitude of UBC, the overall trend of the time-dependent UBC profiles gets drastically impacted by the location of the groundwater table. The combined effect of infiltration duration and rate, unsaturated soil properties, soil's shear strength on the transient bearing capacity are discussed elaborately. The fluctuation of the water table has more impact on bearing capacity than the infiltration rate. With the lowering of the groundwater table, there exists a critical time (t_{cr}), which is defined as the time at which the intensity of the bearing capacity becomes maximum. The value of t_{cr} increases for the slower infiltration rate.

10.3 LIMITATIONS AND FUTURE SCOPE

The present numerical study possesses limitations that are inherently associated with the assumptions of limit theorems. The limit theorem is based on perfect plasticity theory with the consideration of associativity and coaxiality. Most of the soil either hardens or softens after the yielding; hence, in the future, one can explore the possibility of incorporating the strain/work hardening models in the realm of the limit analysis technique. The assumption of associativity overestimates the collapse load (Drescher & Detournay 1993) due to the prediction of excessive dilation of the soil (Rowe & Davis 1982; Borst & Vermeer 1984; Lade 1992). Numerous experimental results on frictional soils indicate that only the change in the principal stress orientations can cause plastic deformations, and there are several occasions where the principal plastic strain rate axes do not align with the axes of the principal stresses (Towhata and Ishihara 1985, Miura et. al. 1986, Gutierrez et. al. 1991). Hence, the collapse load can be further predicted by considering the non-associativity and non-coaxiality rule of plasticity.

The numerical computations also neglected the impact of stress and strain levels on the peak friction and dilatancy angles. Few researchers (Golder 1941; De Beer 1965, 1970; Fukushima and Tatsuoka 1984; Bolton and Lau 1989; Cerato and Lutenegger 2007; Kumar and Khatri 2008) earlier emphasized the effect of confining stress, relative density, and the mobilized strain on the friction and dilatancy angles. Therefore, additional investigations are to be done in the future by incorporating the effect of stress dependency and progressive shear failure.

The study in the present thesis assumes that the material is isotropic. However, cohesive soils and rocks behave mostly in an anisotropic way. Hence, the anisotropic failure criterion can be used further with due consideration of joint spacing and joint orientations (for the rocky stratum).

Moreover, in this thesis, the load-bearing capacities are examined solely through the use of strength parameters. Essentially, this study focuses exclusively on the ultimate limit state. The analysis procedure does not account for the serviceability limit state and therefore the stiffness parameters were not induced during the computational; process. However, there are many instances where the design of the geotechnical structures follows the settlement-based criteria and not the shear failure criteria. By adopting proper constitutive model and employing the displacement-based finite element model, the settlement at any stage before the attainment of ultimate failure state can be predicted.

In the case of unsaturated analysis, the model assumes that flow characteristics and strength parameters are homogeneous, uniform, and independent of net-normal stress. This simplification may not accurately capture the real behavior of the soil, which can exhibit variations and dependencies on stress conditions. Also, the model considers unidirectional flow, which may not account for multidirectional flow conditions that can occur in practical scenarios. The SWRC model assumes that the variation of saturation in the soil is solely dependent on matric suction during drained loading. However, in reality, the degree of saturation can vary even for a constant matric suction, which can impact the suction stress value. Therefore, further works need to be carried out to incorporate the effect of net-normal stress-induced volumetric deformation on the suction stress.

The formulations also assume the uniqueness of soil retention curves in numerical simulations, disregarding the hysteresis between drying and wetting curves observed in reality (Tang et al., 2017; Yuan and Du, 2018, 2020; Fathipour et al., 2023). The drying phase is reported to generate higher suction stress and maximum bearing capacity. The soil-water retention curves are influenced by initial density and water content, with hysteresis being relatively lesser in highly dense coarse-grained sands after multiple drying-wetting cycles. There is a future scope to improve the formulations by considering these aspects for the realistic modelling of soils.

The unsaturated problems in this dissertation considers the soil above the water table to have relatively higher frictional strength and very low air entry value ($\leq$ 10kPa).

Consequently, the characteristics of the chosen soil in the vadose zone bear resemblance to sandy soils. The analysis can be further extended for evaluating the load bearing capacity of the strip foundation by considering the variably saturated soil as fine-grained clayey soils. For such execution, alongwith the cohesive strength, a very high value of air-entry value (higher than 1000 kPa) needs to be incorporated in the analysis.

While performing the axisymmetric formulations in the lower bound theorem, the equilibrium conditions are satisfied only at the centroid of each element and not throughout the domain. In this regard, further modifications can be done by computing the true lower-bound solutions. Moreover, in the present thesis, only the upper bound theorem has been modified to incorporate the effect of suction stress and that too by linear optimization. The formulations and the computations of the lower bound solutions of the stability of geotechnical structures in unsaturated soils provide a vast scope for future work. There is also a scope to improve the proposed upper-bound formulation by considering axisymmetric analysis and employing nonlinear optimization.