

STUDY ON FINITE AND FIXED-TIME STABILITY OF NONLINEAR IMPULSIVE DYNAMICAL SYSTEMS AND THEIR APPLICATIONS



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by

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Dedicated

to

My Beloved Parents

Mrs. Aasia Begum

and

Late Md. Haroon Rashid

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PREFACE

The mathematical model that describes impulsive systems is inspired by impulse phenomena that occur in nature, engineering, physics and the social sciences. The models of impulsive action are interesting to the researchers to capture the instantaneous changes in the environment in which the system is operating and the dynamics of the system depending on its parameters and structure. The theory of impulsive systems not only provides a bridge between continuous and discrete systems but also provides theoretical support to impulsive control techniques which are powerful discontinuous control approaches in the field of control theory. Since the pioneering work of Milman and Myshkis [1] in 1960 on impulsive systems and impulsive control theory, these theories have found widespread uses across various domains, contributing to numerous practical implementations. Notable examples of such applications include the development of cryptographic schemes for secure communication, effective population control in ecological systems, satellites, air traffic control, collective dynamics of complex networks, and the modelling of economic phenomena. In recent years, the fundamental principles of impulsive control theory have been applied to a broader range of problems. For instance, the development of mathematical tools for the analysis of dynamics of discontinuous control systems, the design of intelligent control for different plants with impulses, and the practical applications of impulsive systems in complex environments.

The study of nonlinear systems greatly depends on the qualitative analysis of the solution, such as their existence and stability. The majority of the physical phenomena

represent the nonlinear dynamical systems along with the initial data. The initial data often derived from various measurements inherently contain errors, so it is crucial to understand how a small disturbance in initial data can impact the intended behaviour of the solutions. Thus, the investigation of conditions that prevent solutions from deviating from the desired behaviour holds great significance. The area of mathematics focusing on such issues is termed as stability theory. To study the analysis of dynamical systems, a noteworthy method is the Lyapunov direct method, initially introduced by Alexander M. Lyapunov. This method is employed to investigate the stability theory of a system of ordinary differential equations without finding its solution.

The main objective of the present research work is to study the stability behaviour of nonlinear dynamical systems under the effects of impulses. This study mainly focused on the investigation of different types of stabilities like finite-time and fixed-time stability for nonlinear impulsive dynamical systems around the equilibrium point based on the Lyapunov stability theory. The finite-time stability is nothing but finding an upper bound of convergence time that depends on the initial conditions, known as settling-time; whereas, fixed-time stability is defined by an upper bound on convergence time that remains constant and is independent of the initial conditions. In applications of neural networks, stability analysis of their mathematical model is an important part. For example, neural networks that are designed to implement in content addressable memory must have “stable” equilibrium points and achieve synchronization between neural networks so that the error system must be “stable”. Therefore, we focused on investigating the problem of stability of nonlinear impulsive systems and extended the derived theoretical results to the synchronization of different types of neural networks affected by impulses.

This thesis has two main parts based the work on two types of stability theories. The first part, covered in **Chapters 2-5**, analyzes the fixed-time stability theory and its applications to various neural networks; whereas, the second part contains **Chapter**

6, which gives a detailed discussion of finite-time stability theory under the delayed impulses. The work carried out under the thesis is organized into **Seven Chapters** as follows:

Chapter 1 introduces a brief introduction of impulsive dynamical systems, classification of impulsive dynamical systems, delay differential equation, Lyapunov stability theory, and synchronization. Further, we briefly examined the architectures of different neural networks, such as Cohen-Grossberg, bidirectional associative memory, memristor and complex-valued neural networks. Detailed discussion about the state of art on the present work is also given here and the objective of the thesis is provided at the end of this chapter.

Chapter 2 discusses the theoretical results of fixed-time stability of impulsive dynamical systems under the effects of impulses. This chapter is divided into two different subchapters. In **Subchapter 2.1**, the fixed-time stability of nonlinear impulsive dynamical systems has been studied. Stabilizing and destabilizing impulsive effects have been separately investigated to estimate the fixed-time convergence precisely by using the concept of Lyapunov functional and average impulsive interval. The theoretical derivation shows that the estimated fixed-time in this study is less conservative and more accurate as compared to the existing fixed-time stability results. **Subchapter 2.2** presents the fixed-time stability of Cohen-Grossberg bidirectional associative memory neural networks with desynchronizing impulsive effects. Two types of controllers: one with signum terms and another without signum terms have been derived based on a proposed Lyapunov inequality to study the fixed-time stability of Cohen-Grossberg bidirectional associative memory neural networks. The settling-time functions obtained in this chapter depend on the parameters of the impulsive sequences.

Chapter 3 describes on achieving fixed-time synchronization of inertial Cohen-Grossberg neural networks with time-varying delays and desynchronizing impulsive effects. Firstly, a new lemma is presented which involves fewer parameters as compared

to the existing works to achieve fixed-time stability of the impulsive systems with destabilizing impulses. Based on the proposed lemma, sufficient conditions are established to achieve fixed-time synchronization of inertial Cohen-Grossberg neural networks with desynchronizing impulses by designing a unified controller.

Chapter 4 explores the fixed-time synchronization of neural networks under the effects of synchronizing and desynchronizing impulses. In this chapter, we have considered two different types of neural networks: one is memristor neural networks and another is complex-valued inertial neural networks to investigate the synchronization within some fixed-time. **Subchapter 4.1** aims to establish some theoretical results on fixed-time synchronization of memristor neural networks with time delay and impulsive effects. Some sufficient conditions are derived to achieve the fixed-time synchronization of memristor neural networks under the effects of synchronizing and desynchronizing impulses. Moreover, the estimated settling-time is observed to be less conservative than the existing works on memristor neural networks for the case of synchronizing impulses. **Subchapter 4.2** provides the fixed-time synchronization of complex-valued inertial neural networks with time-delay and impulsive effects. To achieve the synchronization, firstly the original complex-valued inertial neural networks are transformed into first-order complex-valued neural networks using the variable transformation. Then, based on the average impulsive interval and comparison method, certain sufficient conditions are established to ensure the fixed-time synchronization of complex-valued inertial neural networks for synchronizing and desynchronizing impulses.

Chapter 5 demonstrates the fixed-time pinning impulsive synchronization of coupled neural networks with mixed-delays. The pinning impulsive control mechanism is designed to merely control partial nodes, and not all the nodes of the system, which not only saves resources but also improves the communication efficiency. Then, based on the appropriate Lyapunov function and average impulsive interval, a new sufficient condition of the fixed-time synchronization criteria for coupled neural networks through

the pinning impulsive control has been derived

Chapter 6 addresses the issue of finite-time stability for a general class of nonlinear systems under the influence of stabilizing and destabilizing delayed impulses. Using impulsive control theories and the Lyapunov stability concept, some Lyapunov-type theorems on finite-time stability along with settling-time estimations are established for two different kinds of delayed impulses. The obtained settling-time functions for the nonlinear impulsive system are proven to be dependent on impulse time sequences as well as the initial conditions

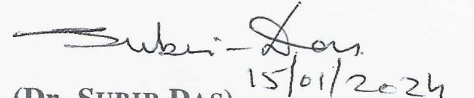
Chapter 7 summarizes the research that has been conducted under the present thesis and provides the possibility of future research in the related area.

CERTIFICATE

It is certified that the work contained in the thesis titled “STUDY ON FINITE AND FIXED-TIME STABILITY OF NONLINEAR IMPULSIVE DYNAMICAL SYSTEMS AND THEIR APPLICATIONS” by MD ARZOO JAMAL has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. Degree.


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I, **MD ARZOO JAMAL**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Prof. SANTWANA MUKHOPADHYAY** and the co-supervision of **Prof. SUBIR DAS** from January, 2019 to January, 2024 at the **Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi**. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, theses, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

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ABBREVIATIONS

FTS	Finite-Time Stability
FxTS	Fixed-Time Stability
BAM	Bidirectional Associative Memory
CGNN	Cohen-Grossberg Neural Network
CGBAMNN	Cohen-Grossberg Bidirectional Associative Memory Neural Network
ICGNN	Inertial Cohen-Grossberg Neural Network
MNN	Memristor Neural Network
CVNN	Complex-Valued Neural Networks
CVINN	Complex-Valued Inertial Neural Networks

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