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It is certified that the work contained in this thesis titled "**Study of Non-Linear Wave Propagation Phenomena in Gas dynamics**" by **Shobhit Kumar Srivastava** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. degree.

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DECLARATION BY THE CANDIDATE

I, **Shobhit Kumar Srivastava**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Dr. L. P. Singh** from **July, 2017** to **May, 2022** at the **Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi**. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

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It is certified that the above statement made by the student is correct to the best of my/our knowledge.

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This thesis is dedicated

with love and deepest gratitude

To my parents

Mr. Shreesh Kumar Srivastava and Mrs. Sarita Srivastava

To my supervisor

Prof. L. P. Singh

ACKNOWLEDGEMENTS

At the outset I express my homage to **Baba Vishwanath Ji** and **Sankat Mochan Hanuman Ji** whose blessings was always felt by me. I am also express my reverence to **Pt. Madan Mohan Malaviya Ji**, the founder of the Capital of Knowledge that is Banaras Hindu University.

It is of immense pleasure to convey my respect and gratitude to my revered supervisor **Prof. L. P. Singh**, Former Head of Department of Mathematical Sciences, Indian Institute of technology (Banaras Hindu University), Varanasi, for his persistent guidance and inspiration to complete my research work. I am deeply grateful to him for his negotiation, motivation towards research work, inspiring guidance, untiring help and valuable suggestions given by him at every moment where I feel some difficulty, during the progress of research work. I thank him for his enormous patience that he shown towards me and also for having faith in my capabilities and making me realize that physical reality provides such amazing, interesting and a seemingly never ending supply of problems. He always cared and always encouraged me like a loving parent. He understand me and my problem beyond my imagination.

I would like to express my gratitude to **Prof. S. K. Pandey**, Head, Department of Mathematical Sciences, Indian Institute of technology (Banaras Hindu University), Varanasi, for his co-operation, constructive suggestion and providing the necessary facilities in the Department.

I offer my sincere gratitude to **Dr. Lavanya Selvaganesh**, Assistant Professor, Department of Mathematical Sciences, Indian Institute of technology (Banaras Hindu University), Varanasi, for her affectionate help and motivation throughout my tenure as a research scholar. I am thankful for her gratefulness, motivations, and inspiring words.

I would like to express my deep sence of gratitude to **Dr. V. K. Singh**, Convener DPGC, Department of Mathematical Sciences for encouraging me during my research work. I also acknowledge my sincere thanks to DPGC members of the Department for their encouragement and moral support. I am also thankful to my RPEC members **Dr. R. K. Pandey** and **Prof. P. K. Singh**, Department of civil engineering Indian Institute of technology, (Banaras Hindu University), Varanasi, for their valuable suggestions throughout the period of research work.

I acknowledge my sincere thanks to **Prof. T. Som, Prof. S. K. Upadhyay, Prof. S. Mukhopadhyay, Prof. S. Das and Prof. M. K. Vemuri** for their encouragement and moral support. My sincere thanks are also to **Dr. Ashok Gupta, Dr. Rajeev, Dr. Sunil Kumar, Dr. Debdas Ghosh, Dr. Anuradha Banerjee and Dr. Abhash Kumar Jha** for their valuable suggestions and encouragement during my research work.

I offer thankful acknowledgement to my seniors, **Dr. Akmal Husain, Dr. D. B. Singh, Dr. Mithilesh Singh, Dr. Subedar Ram, Dr R. Singh, Dr. Triloki Nath, Dr. R. K. Gupta** and **Dr. J. P. Chaudhary**.

I would like to thank my close friends **Dr. Sandeep Kumar Maurya, Rajneesh Dwivedi, Pawan Prajapati, Abhilash Rai, Ajay Kumar, Abhay Rajpoot** for their continued encouragement, appreciation, stimulating help and criticism. I am also thankful to my colleagues **Dr. R. K. Chaturvedi, Dr. Pooja Gupta, Pradeep, Dheeraj Shukla, Shweta** and **Gaurav** for their sincere help.

From the bottom of my heart, I owe my special thanks my father **Shri Shreesh Kumar Srivastava** and to my mother **Smt. Sarita Srivastava**, my mama **Shri Satya Prakash Srivastava** and other family members. I have no words to record my respect for their unconditional support and it was not possible for me to come at this stage.

I am also thankful to non-teaching staffs of the Department for their co-operation and support. Last but not least, I acknowledge the financial assistance provided by the University Grant Commission (UGC), Government of India, in the form of UGC-JRF and SRF, without which this thesis would not have been possible.

Shobhit Kumar Srivastava

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PREFACE

Gas dynamics is a particular branch of fluid dynamics evolved at the end of the 19^{th} century from attempts to understand the fundamentals of high-speed compressible flow through nozzles and passages. It has grown with the development of high-speed flight and has became an area of research for physicists, chemists, applied mathematicians and astrophysicists. The term gas dynamics suggests the idea that the field is exclusively related to the state of gaseous matter. Historically, one of the most important areas of application of gas dynamics was also the theoretical treatment of detonation in gases, in particular the description of the fundamental properties of the detonation wave, a reactive shock wave and the unsteady motion of the detonation products. However, modern gas dynamics takes a wider, more general view than in the pioneering days and is nowadays concerned with the causes and effects arising from high-speed flows, steady and unsteady, viscous and non-viscous, conducting and non-conducting for which compressibility is physically significant.

A wave can be described as a disturbance that travels through a medium, accompanied by a transfer of energy, from one point to another but the particles in the medium are not displaced by the wave. They consist of vibrations around almost fixed locations. They are broadly classified by Whitham [1] as, dispersive waves and wave motions formulated mathematically as hyperbolic system of partial differential equations. In waves represented by hyperbolic PDEs, the disturbance is not felt by every point in the space at once unlike the dispersive waves where the disturbance is felt at once by all the particles of the medium. For the hyperbolic equations, every characteristic curve in the (x, t) space represents a moving wavelet, and the behavior of the solution on a characteristic curve corresponds to the idea that information is carried by that wavelet [2].

In 1860, Bernhard Riemann published a study titled "On the propagation of plane air waves of finite amplitude", the first mathematical analysis of the Euler equations of gas dynamics [3]. In that work, among many contributions, is a study of what came to be known as the Riemann problem which is a hyperbolic partial differential equation (PDE) with initial data consisting of two constant states separated by a discontinuity. In the construction of the solution to the Riemann Problem, there are certain kind of waves known as elementary waves (shock wave, rarefaction wave and contact discontinuity) described by the characteristics of the governing system, constitute the solution of the problem. It's exact solution provides the real physical properties with several wave families and their propagation. Across a rarefaction wave the flow properties are continuous. The shock wave and contact discontinuity both are the discontinuous solutions but there is a slight difference between them. Across the shock, all the flow variables experience sudden change but in case of contact discontinuity the velocity and pressure will be continuous whereas the density, temperature, entropy etc. experience a sudden change.

The present thesis, embodies the results of researches carried out by me at the Department of Mathematical sciences, Indian Institute of Technology (BHU), Varanasi, during the period July 2017 to May 2022 under the supervision of Prof. L. P. Singh. The present thesis is devoted for the study of the propagation of non-linear waves in various gaseous medium governed by the quasilinear system of partial differential equations. The main purpose of this study is to understand the wave propagation and its evolutionary behavior under certain differential constraints by using some analytical methods such as progressive wave method, method of characteristic, method of wavefront analysis etc. Also, we have studied the Riemann Problem for the non-homogeneous system for a dusty gas under the influence of external force. The entire thesis is divided into seven chapters as follows:

Chapter - 1 is introductory and gives an overview of work done in the field of non-linear wave propagation including their applications and methodology. The mathematical theory and their fundamental properties have also been briefly discussed. The basic features of the non-linear waves and its propagation is described. The physical properties of ideal gases, non-ideal gases, dusty gases are also briefly reviewed. Some results, which we shall need in subsequent chapters, have also been included in this chapter.

Chapter - 2 concerns with the study of progressive wave solution for the waves of finite and moderately small amplitude in the mixture of the non-ideal gas and dust particles governed by quasilinear hyperbolic system of PDEs. Using the progressive wave method, we derive the transport equation which provides the conditions of the shock formation, and equation to determine the shock strength. Also, it is shown that how the presence of the parameter of non-idealness and mass fraction of dust particles influence the shock formation and shock strength for the planar and non planar cases. We also discuss the consequence of variation in the value of the non-ideal parameter and the mass fraction of small solid dust particles on the evolutionary process of the shock.

In **Chapter - 3**, we have studied the various parameter effects on the propagation of weak discontinuities by using the method of characteristics. Analytical solutions of the quasi-linear system of hyperbolic PDEs are obtained and examined the evolutionary behavior of shock in the characteristic plane. The general behavior of solutions to the Bernoulli equation, which determines the evolution of weak discontinuity in a non-linear system, is studied in detail. Also, we discuss the formation and distortion of compressive and expansive discontinuities under the effect of van der Waals parameter and small particles for planar and cylindrically symmetric flow.

The comparison between planar flow and cylindrically symmetric flow is studied under the influence of non-idealness and mass fraction of dust particles. It is found that the compressive waves become shock after a certain lapse of time. The medium considered here is the mixture of van der Waals gas with small dust particles.

Chapter - 4 presents the study of the evolutionary behavior of plane and cylindrically symmetric acceleration discontinuities along the characteristic path under the effect of dust particles in a non-ideal magnetogasdynamic flow. Implications regarding the propagation of disturbances in planar and cylindrically symmetric flows have been shown. Using the characteristics of the governing quasilinear system as a reference coordinate system, we transform the fundamental equations and find the solution. It is explored how the dust particles, along with the non-ideal parameter, will influence the steepening or flattening of the propagating waves in magnetic and non-magnetic cases. The transport equation leading to the evolution of acceleration discontinuities is determined, which provides the relation for the occurrence of shock. The impact of non-idealness of the gas and dust on the evolutionary process of propagating waves for the magnetic and non-magnetic cases are discussed. The comparison between the flow patterns and distortion of the propagating waves for planar and cylindrically symmetric flows is demonstrated under the various parameter effects.

In **Chapter - 5**, influence of the magnetic field on the propagation of shock waves in radiation gasdynamics is analyzed by using method of wavefront analysis. We examined the behavior of the waves propagated into the two-dimensional (2-D) steady supersonic magnetogasdynamic flow of non-ideal gas with radiation. The transport equation is derived, which determines the condition for the shock formation. The effect of non-idealness and thermal radiation and their consequences under the influence of magnetic field is studied and examined how the flow patterns of the disturbance vary with respect to the variation in the parameters of the flow. It is found that the presence of magnetic field plays an essential role in the wave propagation phenomena. The nature of the solution with respect to the Mach number is analyzed, and it is examined how the shock formation distance changes with an increase or decrease in the value of the Mach number. Also, the effect of non-idealness on the shock formation distance is elucidated and examined how the shock formation is affected by the increase in the value of the non-ideal parameter in the presence of magnetic field with thermal radiation.

In **Chapter - 6**, we have studied the Riemann Problem (RP) for a non-homogeneous system governing the one-dimensional compressible flow of dusty gas, where the external forces are assumed to be continuous function of time. The elementary wave solutions such as rarefaction wave, shock wave and contact discontinuity are rigorously determined. The effect of dust particles on the density and flow velocity and shock speed is examined, and their consequences on the solution of the Riemann Problem are discussed. Moreover, we discussed the condition for the existence of vacuum that appears in the solution of the Riemann Problem.

Lastly, in **Chapter - 7**, we have summarized the complete analysis of the work done in this thesis. Major contributions made in the thesis are briefly discussed followed by a discussion on the future scope.