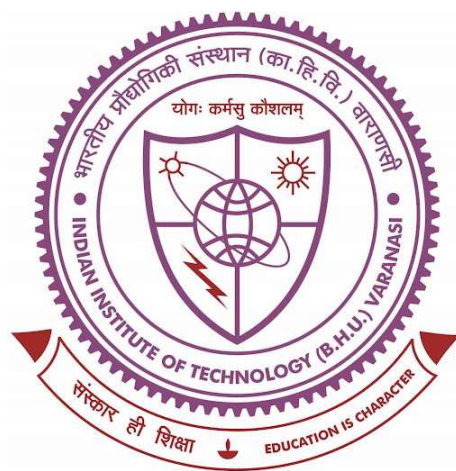

On Hermite Expansions



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by

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Chapter 4

Conclusion

Let $g_a(x) = e^{-ax^2/2}$. Suppose that function and its Fourier transform are bounded above by g_a , and g_b respectively. In the case, when $ab \geq 1$, Hardy characterized the functions explicitly. Inspired by Hardy's result, in the case $ab < 1$, and $a = b$, Vemuri characterized the sharp decay of Hermite coefficients. In this thesis, we have extended and generalized these results. We give the conclusions chapter-wise.

- In chapter 1, with the same hypothesis as those of Vemuri, we prove that certain combinations of Hermite coefficients have a better decay. It is expected that the result could be used to obtain a sharp Gaussian decay of the solution of time dependent Harmonic oscillator Schrodinger equation.
- In chapter 2, we remove the condition $a = b$ assumed by Vemuri and prove the sharp decay rate for the Hermite coefficients. There can be several interesting questions in this context. For instance, one can ask, can we prove analogous result in more general settings (higher dimension Euclidean spaces, Heisenberg group, symmetric spaces, etc).